

Duration: **50 minutes**  
 Aids Allowed: **NONE** (in particular, no calculator)

Student Number:

Last (Family) Name(s):

First (Given) Name(s):

*Do **not** turn this page until you have received the signal to start.*  
 In the meantime, please read the instructions below *carefully*.

This term test consists of 3 questions on 10 pages (including this one), printed on both sides of the paper. *When you receive the signal to start, please make sure that your copy of the test is complete, fill in the identification section above, write your student number where indicated at the bottom of every odd-numbered page (except page 1), and write your name on the back of the last page.*

Answer each question directly on the test paper, in the space provided, and use the reverse side of the pages for rough work. If you need more space for one of your solutions, use the reverse side of a page and *indicate clearly the part of your work that should be marked.*

In your answers, you may use without proof any result or theorem covered in lectures, tutorials, homework, tests, or the textbook, as long as you give a clear statement of the result(s)/theorem(s) you are using. You must justify all other facts required for your solutions.

Write up your solutions carefully! In particular, use notation and terminology correctly and explain what you are trying to do — part marks *will* be given for showing that you know the general structure of an answer, even if your solution is incomplete.

If you are unable to answer a question (or part), you will get 20% of the marks for that question (or part) if you write “I don’t know” and nothing else — you will *not* get those marks if your answer is completely blank, or if it contains contradictory statements (such as “I don’t know” followed or preceded by parts of a solution that have not been crossed off).

MARKING GUIDE

# 1: \_\_\_\_\_/13

# 2: \_\_\_\_\_/11

# 3: \_\_\_\_\_/ 9

BONUS

MARKS: \_\_\_\_\_/ 6

TOTAL: \_\_\_\_\_/33

*Use this page for rough work — clearly indicate any section(s) to be marked.*

**Question 1.** [13 MARKS]

Prove the correctness of the algorithm below. Follow the format given in class. (HINT: This is not complicated but it *is* long — you may want to leave out the proof of the loop invariant at first.)

```
# Pre:  $x \in \mathbb{N}$ 
 $r = 0; s = x$ 
# LI:  $r \leq s$  and  $r^2 \leq x < (s + 1)^2$ 
while  $r < s$ :
     $m = \lceil (r + s)/2 \rceil$  # Fact:  $r < m \leq s$ . (You may use this without proof in your solution.)
    if  $m * m \leq x$ :  $r = m$ 
    else:  $s = m - 1$ 
# Post:  $r$  is the “integer square root” of  $x$ , i.e.,  $r^2 \leq x < (r + 1)^2$ 
```

*Use this page for rough work — clearly indicate any section(s) to be marked.*

**Question 2.** [11 MARKS]

Recall that the value of a string in balanced ternary ( $s = d_k d_{k-1} \dots d_1 d_0$  where each  $d_i \in \{-, 0, 1\}$ ) is defined as  $\#s = d_k 3^k + d_{k-1} 3^{k-1} + \dots + d_1 3^1 + d_0 3^0$  (e.g.,  $\#11-01 = 81 + 27 - 9 + 1 = 100$ ).

**Part (a)** [5 MARKS]

Give a DFA for language  $\text{ODD} = \{s \in \{-, 0, 1\}^* : \#s \text{ is odd}\}$ , and justify briefly that your DFA is correct (clearly give the “meaning” of each state).

**Part (b)** [3 MARKS]

Give a RE for language  $\text{ODD}$ , and explain how you obtained your RE.

**Part (c)** [3 MARKS]

Give a RE for language  $\text{EVEN} = \{s \in \{-, 0, 1\}^* : \#s \text{ is even}\}$  and explain how you obtained your RE.

*Use this page for rough work — clearly indicate any section(s) to be marked.*

**Question 3.** [9 MARKS]

Prove that language  $L_2 = \{0^{2^k} : k \in \mathbb{N}\} = \{0^1, 0^2, 0^4, 0^8, \dots\} = \{0, 00, 0000, 00000000, \dots\}$  is *not* regular.

*Use this page for rough work — clearly indicate any section(s) to be marked.*

**Bonus.** [6 MARKS]

**WARNING!** This question is difficult and will be marked harshly: credit will be given only for *significant* progress toward a correct answer (in particular, “I don’t know” will be worth zero). Please attempt this only *after* you have completed the rest of the test.

Give a context free grammar for language  $L_a = \{ s \in \{a, b\}^* : s \text{ contains strictly more } a\text{'s than } b\text{'s} \}$ , and justify briefly that your CFG is correct.

On this page, please write nothing except your name.

**Last (Family) Name(s):** \_\_\_\_\_

**First (Given) Name(s):** \_\_\_\_\_