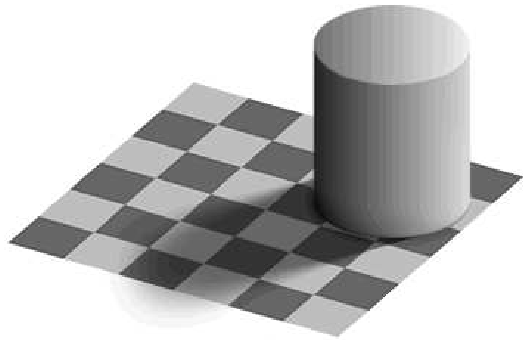
Lighting and Reflectance Models



Basic principles of illumination and reflectance are introduced. They are central to understanding the dependence of image colour and intensity on material reflectance, geometry, and lighting.

Basic Radiometry:

- irradiance and radiance
- point light sources and extended illuminants

Reflectance Models:

- BRDF: Bidirectional Reflectance Distribution Function
- diffuse (Lambertian) and specular reflectance
- Phong model

Suggested Reading: Fleet, D. and Hertzmann, A.: Chapters 8 and 12 of notes for CSC418/D18.

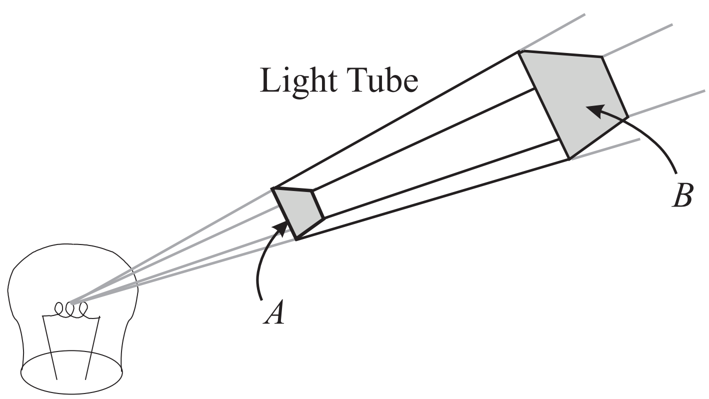
Matlab Tutorials: phongDemo.m, phongShade.m, and colourTutorial.m

Light Tube Analogy

Light density: Treat light as a continuous quantity (density) as opposed to discrete photon counts.

Assume a non-absorbing, non-scattering, uniform medium, such as a vacuum or (apprx) air. Then geometrical optics models light as travelling along straight lines called rays.

Ignore time-delays due to the finite speed of light, and ignore the diffraction of light.



Given a collection of rays that lie within a tube and which cross both ends of the tube (i.e. A and B), but do not cross the sides of the tube, **then** the radiant power of the light impinging on end A, due to these rays, is equal to the power impinging on B (due to the same rays).

Visual perception depends critically on wavelength. We use the term **spectral density** to capture the dependence of light density on wavelength λ (typically between 400 and 700nm).

Overview: Basic Radiometric Definitions

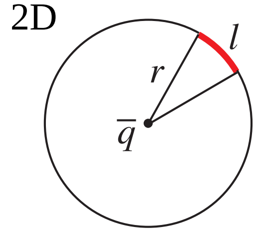
Flux (Power): Rate at which light is emitted from, or absorbed by, a (virtual) surface. Radiant power is specified in Watts ($1W = 1\frac{J}{s}$).

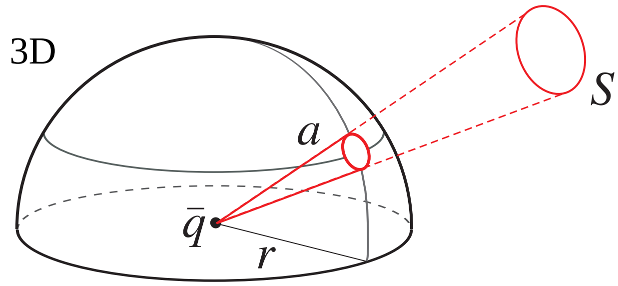
Irradiance: Measure of light arriving at a surface (real or virtual), as a function of surface position; i.e., as power per unit surface area, $\frac{W}{m^2}$. Irradiance spectral density is specified in units $\frac{W}{(nm)m^2}$. Irradiance is central to sensor (camera) measurements.

Radiance Measure of light as a function of direction and surface area, i.e., power per unit surface area per steradian, $\frac{W}{m^2 sr}$ (or $\frac{W}{(nm)m^2 sr}$). This allows us to describe the light emitted or reflected from a surface as a function of direction.

Steradian: Measure of solid angle (angular extent in 3D).

- 2D angular extent or a curve (in radians) is its projected length onto a circle of radius 1, i.e., l/r (1 half circle = π radians)
- Solid angle (of patch S w.r.t. point $\vec{q}$) is measured in steradians (denoted sr), defined as the ratio of projected area on a sphere to its squared radius, a/r^2. One hemisphere is 2π steradians.

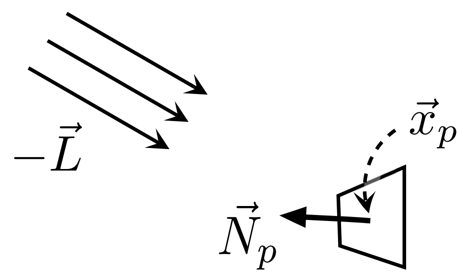




Irradiance for a Distant Point Light Source

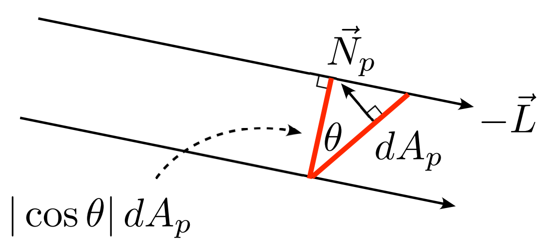
Let $I(\lambda, \vec{x}_p)$ denote the irradiance on a small patch (real or virtual) at position $\vec{x}_p$. We often write $I(\lambda, \vec{x}_p;\ \vec{N}_p)$ to express dependence of irradiance on surface orientation (i.e., the surface normal, $\vec{N}_p$).

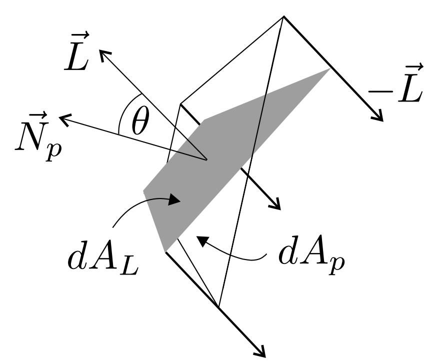
For a light source that is *far away*, in unit direction $\vec{L}$, with a *small angular extent* (eg, the sun), the light rays are approximately parallel:



A surface patch with normal $\vec{N}_p = \vec{L}$ will maximize the amount of light caught by the patch. Its irradiance is $I(\lambda, \vec{x}_p;\ \vec{L})$.

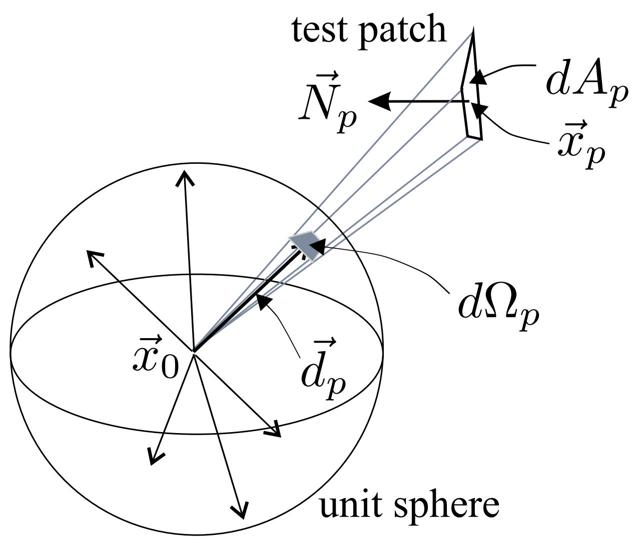
For a small patch of area dA_p, in general (i.e., $\vec{N}_p \neq \vec{L}$), irradiance is the power passing through the area of the patch projected onto a plane perpendicular $\vec{L}$, i.e., $dA_L = |\cos\theta|\, dA_p = |\vec{N}_p \cdot \vec{L}|\, dA_p$.





For opaque surfaces we require $\vec{N}_p \cdot \vec{L} \geq 0$ (otherwise it's in shadow). So, $I(\lambda, \vec{x}_p; \vec{N}_p) = \lfloor \vec{N}_p \cdot \vec{L} \rfloor I(\lambda, \vec{x}_p; \vec{L})$, where $\lfloor x \rfloor = \max(x, 0)$.

Irradiance for a Proximal Point Source



Radiant intensity $R_s(\lambda, \vec{d}_p)$ of a point source is the power, per unit wavelength per unit solid angle, caught by a patch in direction $\vec{d}_p$. Units: $\frac{W}{(nm)(sr)}$.

The solid angle, $d\Omega_p$, subtended by a small patch at position $\vec{x}_p$, decreases due to foreshortening, and distance to the source at $\vec{x}_0$; i.e.,

$$d\Omega_p \;=\; \frac{|\vec{N}_p \cdot \vec{L}|}{||\vec{x}_p - \vec{x}_0||^2}\, dA_p,$$

where $\vec{L} = \frac{\vec{x}_0 - \vec{x}_p}{||\vec{x}_0 - \vec{x}_p||}$ is the light source direction from $\vec{x}_p$; ie, $\vec{L} = -\vec{d}_p$.

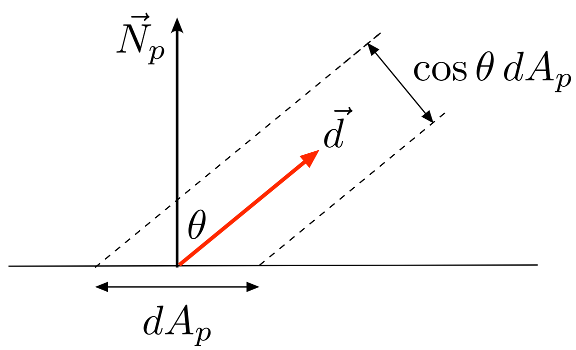
The irradiance on the test patch due to the proximal point light source is simply the amount of light (power) that reaches the patch divided by the area of the patch

$$I(\lambda, \vec{x}_p;\; \vec{N}_p) \;=\; \frac{R_s(\lambda, -\vec{L})\, d\Omega_p}{dA_p} \;=\; \frac{\lfloor \vec{N}_p \cdot \vec{L} \rfloor}{||\vec{x}_p - \vec{x}_0||^2}\, R_s(\lambda, -\vec{L})\;.$$

Radiance

Radiance parameterizes light as a function of the direction, as well as position and wavelength, denoted $R(\lambda, \vec{x}, \vec{d})$:

- $R(\lambda, \vec{x}, \vec{d})$ is the radiance of a (possibly virtual) surface patch in the neighborhood of $\vec{x}$ into directions about $\vec{d}$.
- Units: $\frac{W}{(nm)\, m^2 sr}$. I.e., power, per unit wavelength, per unit surface area, per unit solid angle.
- By convention, the surface area is measured perpendicular to the direction $\vec{d}$ (i.e., foreshortened area, $\cos\theta\, dA_p$).



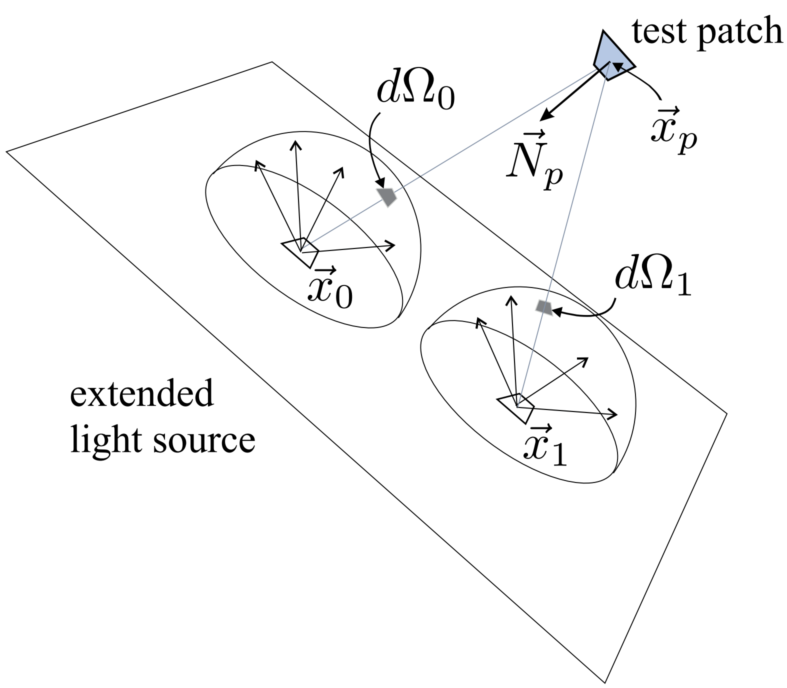
Radiance is often used to quantify

- surface reflection as a function of an *emittant* direction.
- the light emitted from an extended light source in the direction of a small surface patch.
- the *light field*: measure of the light in a 3D environment as a function of position and direction (and wavelength).

Irradiance for Extended Light Sources

Light combines *linearly*. So the contributions to irradiance at a camera sensor from different light sources are simply added (integrated).

For an extended light source, light is emitted as a function of position over the surface of the source. What's the irradiance at a test patch?



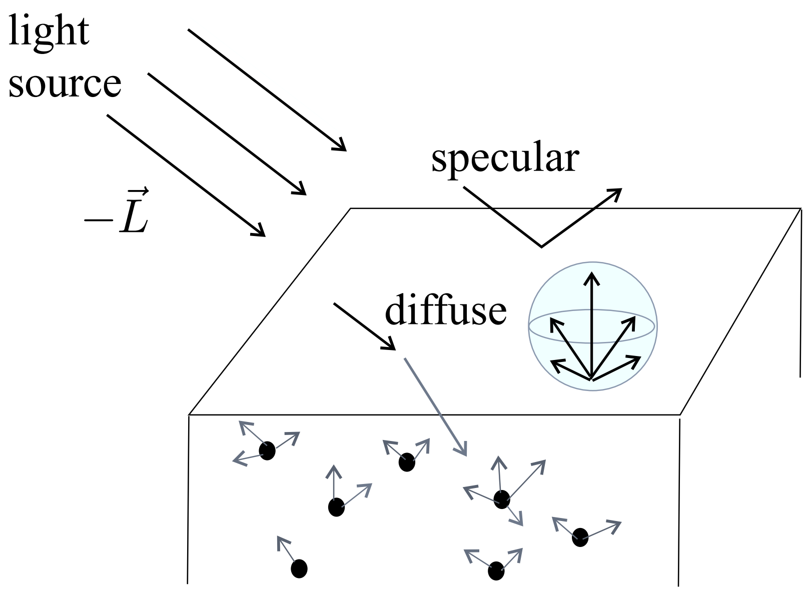
The irradiance on a small test patch at $\vec{x}_p$ is obtained by integrating contributions from infinitesimal patches over the entire light source.

- The contribution from the small light source patch at $\vec{x}$ is the light (radiance) emitted in the direction of the test patch $\vec{d}(\vec{x}_p, \vec{x})$:

  $$\frac{\lfloor \vec{N}_p \cdot \vec{L}(\vec{x}, \vec{x}_p) \rfloor}{||\vec{x} - \vec{x}_p||^2} R(\lambda,\, \vec{x},\, \vec{d}(\vec{x}_p, \vec{x}))\, d\vec{x} \; .$$

  This is radiance, times solid angle of the test patch, times areas on the light source, divided by the area of the patch.

Two Basic Types of Reflectance



- **Specular Reflectance**: Reflectance from the surface, primarily in the "mirror reflection" direction. The spectral distribution of the reflected light can be the same as the incident light.
- **Diffuse Reflectance**: Light is absorbed and re-emitted from the body, scattering in all directions. The spectral distribution of the reflected light depends on the pigmentation of the object.

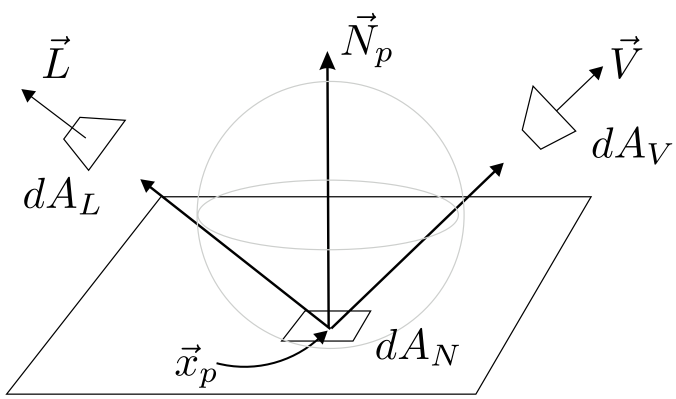
Bidirectional Reflectance Distribution Function

The Bidirectional Reflection Distribution Function **(BRDF)** is used to model a wide range of material reflectance properties.

The BRDF $r(\lambda, \vec{L}, \vec{V})$ captures the dependence of reflectance on the incident and emittant directions:

- $r(\lambda, \vec{L}, \vec{V})$ gives the proportion of the incident light, from direction $\vec{L}$, at wavelength λ, scattered in the viewing direction $\vec{V}$. ($\vec{L}$ and $\vec{V}$ are unit vectors)
- It is defined as the ratio of radiance to irradiance. (Units: $(sr)^{-1}$)

Suppose that irradiance on a surface patch due to light coming from direction $\vec{L}$ is $I(\lambda, \vec{x}_p; \vec{N}_p)$.



The corresponding **radiance** (of the reflected light) is given by:

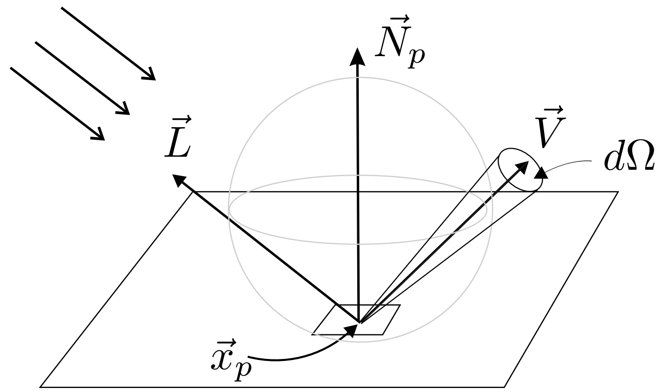
$$R(\lambda, \vec{x}_p, \vec{V}) \; = \; r(\lambda, \vec{L}, \vec{V}) \, I(\lambda, \vec{x}_p; \; \vec{N}_p)$$

Lambertian Reflectance

Diffusely reflecting objects appear similarly from all viewing directions. I.e., the radiance is not dependent on the viewing direction.

Lambertian Approximation: The BRDF is assumed to be invariant of both $\vec{L}$ and $\vec{V}$, that is $r(\lambda, \vec{L}, \vec{V}) = r(\lambda)$.

Consider a distant point light source from direction:



The irradiance on the surface (w.r.t. normal $\vec{N}_p$) is given by

$$I(\lambda, \vec{x}_p;\ \vec{N}_p) = \lfloor \vec{N}_p \cdot \vec{L} \rfloor I(\lambda, \vec{x}_p;\ \vec{L})$$

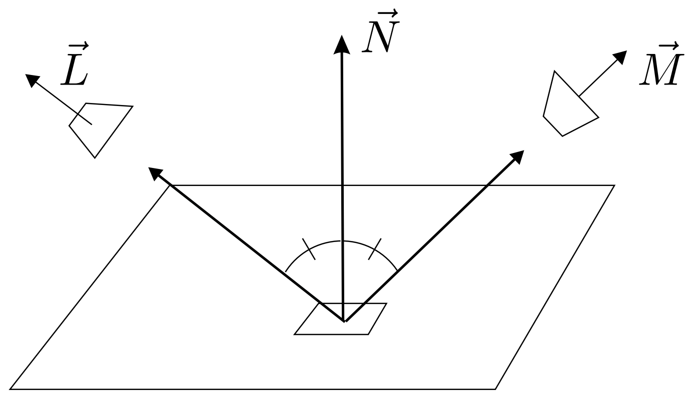
It follows that the radiance (*per unit area perpendicular to* $\vec{V}$), due to diffuse reflection from a Lambertian surface, is given by

$$R(\lambda, \vec{x}_p, \vec{V}) \ = \ r(\lambda) \lfloor \vec{N}_p \cdot \vec{L} \rfloor \, I(\lambda, \vec{x}_p;\ \vec{L})$$

Note that this does not depend on the viewing direction $\vec{V}$.

Specular Reflectance

Consider a distant point source with irradiance $I^i(\lambda; \vec{L})$. Here the superscript 'i' denotes the 'incident' irradiance.



Here $\vec{N}$ is the surface normal, $\vec{L}$ is the light-source direction, and $\vec{M}$ is the mirror reflection direction. These directions are related by

$$\vec{M} \;=\; -\vec{L} + 2\vec{N}\left[\vec{N} \cdot \vec{L}\right].$$

The reflected irradiance $I^r(\lambda, \vec{M})$ is given by

$$I^r(\lambda;\; \vec{M}) \;=\; F(\lambda, \vec{L})\, I^i(\lambda;\; \vec{L}).$$

Here the *Fresnel term*, $F(\lambda, \vec{L})$, dictates any change in the spectral distribution.

Scattering of the specular reflection around the mirror reflection direction, $\vec{M}$, can also be modelled (see the Phong model below).

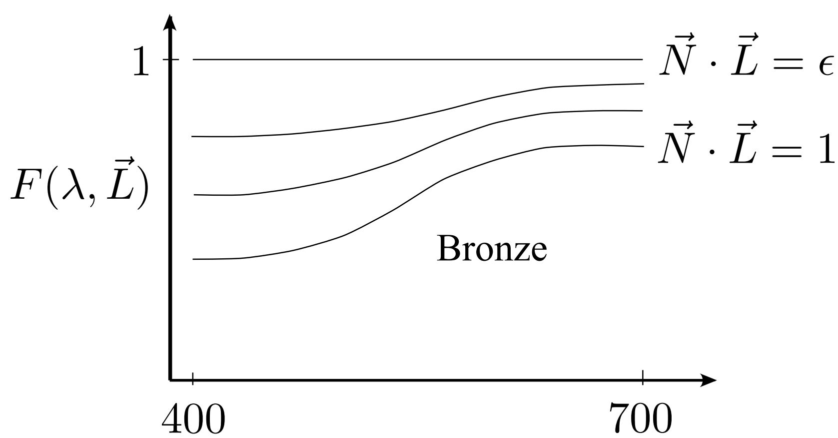
The 'Colour' of Highlights

For specular reflection the reflected irradiance $I^r(\lambda; \vec{M})$ is given by

$$I^r(\lambda; \vec{M}) \;=\; F(\lambda, \vec{L})\, I^i(\lambda; \vec{L}).$$

For plastics, $F(\lambda, \vec{L}) \approx f(\vec{L} \cdot \vec{N})$, so there is little spectral change between the incident and reflected light.

However, metals do show spectral changes. Moreover these changes depend on the incident angle. For example, for bronze:



Ref: Cook and Torrance, A reflectance model for computer graphics, ACM Trans. on Graphics, 1(1), Jan. 1982, pp. 7-24.

Phong Reflectance Model

The Phong model, commonly used in computer graphics, is an approximation to the reflected radiance of a surface. Suppose a distant light source from direction $\vec{L}$, with irradiance $I(\lambda) \equiv I(\lambda, \vec{x}; \vec{L})$.

The reflected radiance according to the Phong model, for surface point $\vec{x}_p$ with normal $\vec{N}_p$, per unit area perpendicular to the viewing direction $\vec{V}$, is

$$R(\lambda, \vec{x}_p, \vec{V}) \;=\; k_a\, r(\lambda) \;+\; k_d\, r(\lambda) \, \lfloor \vec{N} \cdot \vec{L} \rfloor \, I(\lambda) \;+\; k_s\, S(\lambda) (\vec{M} \cdot \vec{V})^{k_e}$$

where

- $r(\lambda)$ is the diffuse spectral reflectance distribution for the surface;
- $\vec{M}$ is the mirror reflection direction computed from $\vec{L}$ and $\vec{N}_p$;
- k_a, k_d, k_s are non-negative coefficients for the ambient, diffuse, and specular reflection terms, respectively;
- k_e is the spectral exponent, controlling the spread of the *off-axis* specular reflection (rougher surfaces modelled by smaller k_e);
- $S(\lambda)$ is the spectral distribution of the specular reflection. It is just $I(\lambda)$ for painted or plastic surfaces. For metals it can be approximated by some linear combination of $I(\lambda)$ and $r(\lambda)$.

See `phongDemo.m`.

See the Light

Our visual systems provide us with an interpretation of the scene.

It takes practice to actually pay attention to the *stimulus*, instead. That is, we may wish to "see the light" as opposed to the scene. (Note the reflectance illusions from the previous lecture illustrate some of the difficulties we might have.)

Once you can see the light, consider perception:

- How do we identify light sources (they aren't always the brightest objects in the image)?
- How do we identify highlights? Transparency? Haze?
- How do we identify metallic surfaces? Wax? Pearls?
- How do we perceive the pigmentation of a surface, $r(\lambda)$, separately from the illuminant $I(\lambda; \vec{L})$ and surface orientation $\vec{N}$, given only the reflected radiance R? Naively, this looks like, "Find r, I, and $\vec{N}$, given only the radiance $R = r\, I\, \lfloor \vec{N} \cdot \vec{L} \rfloor$." (See p. 10.)
- ... and so on.