

UNIVERSITY OF TORONTO
Faculty of Arts and Science
DECEMBER EXAMINATIONS 2008
CSC 487/2503H F
St. George Campus
Duration - 3 hours
Aids allowed: none
$\qquad$
Student Number:
Family Name: $\qquad$
Given Name: $\qquad$

Do not turn this page until you have received the signal to start. (In the meantime, please fill out the identification section above, and read the instructions below.)

This examination consists of 6 questions on 15 pages (including this one). When you receive the signal to start, please make sure that your copy of the examination is complete. If you need more space for one of your solutions, use the reverse side of the page and indicate clearly the part of your work that should be marked. Also, the last two pages are provided for scratch work.

Write your student number at the bottom of pages 2-15 of this test.
\# 1: $\qquad$ /15
\# 2:
$\qquad$ /15
\# 3: $\qquad$ /15
\# 4: $\qquad$ /15
\# 5 : $\qquad$ /15
\# 6 : $\qquad$ /15

TOTAL: $\qquad$ /90

## Good Luck!

## Question 1. [15 MARKS]

Give a short answer for each of the following questions.
Part (a) [3 marks] What is a BRDF and what is its units?

Part (b) [3 marks] State the Convolution Theorem.

Part (c) [3 MARKS] Name three current image segmentation algorithms.

Question continued on next page.

Part (d) [3 MARKS] Describe the Interpretation Tree Search for object recognition.

Part (e) [3 MARKS] Specify the intrinsic and extrinsic calibration matrices.

## Question 2. [15 MARKS]

Consider estimating the translational component of optical flow given two images $I(\vec{x}, t)$ and $I(\vec{x}, t+1)$. You can assume that these images have been cropped so that we only need to estimate a constant displacement between these images.

Part (a) [3 MARKS] What is the brightness constancy constraint?

Part (b) [3 marks] Given an initial guess $\vec{u}_{0}$ for the displacement, so $I\left(\vec{x}+\vec{u}_{0}, t+1\right) \approx I(\vec{x}, t)$, what are the linearized brightness constancy constraints that can be used to update the displacement? Derive a linear system of equations for the update, say $\vec{v}$.

Part (c) [3 marks] How can a robust estimator $\rho(e)$ be applied to the linearized brightness constancy constraints? In particular, write out the objective function to be minimized by the displacement update $\vec{v}$.

Part (d) [3 MARKS] Derive an iteratively reweighted least squares algorithm for minimizing your objective function in part (c) above.

Part (e) [3 marks] Given the converged estimate of the update $v$ from part (d) above corresponds to a translation of several pixels, what should you do next? Explain.

## Question 3. [15 MARKS]

Suppose you are given an ensemble of images of an object for which you wish to learn a view-based appearance model. Assume there are $N$ training images of the object to be detected, say $\left\{A_{j}(\overrightarrow{\mathbf{x}})\right\}_{j=1}^{N}$ and, in addition, $M$ training images of typical backgrounds $\left\{B_{j}(\overrightarrow{\mathbf{x}})\right\}_{j=1}^{M}$. These images are all of the same size, say $25 \times 20$ pixels.

Part (a) [5 marks] State and briefly explain what steps you might take to learn two view-based models using PCA, one of the object and another of the background.

Part (b) [5 marks] If a detector for the object is based on the PCA model derived in part (a) for the object alone (as you did in first part of Assignment 3), give some reasons why increasing the dimension of the PCA basis may not improve the detector's performance.

Question continued on next page.

Part (c) [5 marks] Given PCA models for both the object and the background, propose an alternative detector which might avoid at least one of the issues discussed in part (b). Be specific about what statistics are to be used, and what the detection criterion is. Explain how your detector might avoid one of the issues discussed in (b).

## Question 4. [15 MARKS]

Consider estimating the $F$-matrix from sets of corresponding points, $\left\{\vec{x}_{k}^{L}\right\}_{k=1}^{K}$ and $\left\{\vec{x}_{k}^{R}\right\}_{k=1}^{K}$, in the left and right images of a stereo pair. Here $\vec{x}_{k}^{L}$ is a 2 -vector specifying the pixel coordinates in the left image of the $k^{t h}$ scene point, and $\vec{x}_{k}^{R}$ provides the pixel coordinates for the same scene point in the right image.

Part (a) [2 MARKS] If there was no noise in the image points $\vec{x}_{k}^{L}$ and $\vec{x}_{k}^{R}$, write out an equation in terms of $F$ which relates these two image positions.

Part (b) [3 MARKS] In the presence of noise in the image positions of the corresponding points, write out an expression for the squared distance between $\vec{x}_{k}^{R}$ and the epipolar line specified by the corresponding point $\vec{x}_{k}^{L}$ (ignoring the noise in $\vec{x}_{k}^{L}$ ).

Part (c) [2 marks] How can a robust estimator $\rho(e)$ be applied to the epipolar distance constraints derived in part (b)? In particular, write out the objective function to be minimized to solve for $F$.

Question continued on next page.

Part (d) [4 marks] Suggest an iteratively reweighted least squares algorithm for minimizing your objective function in part (c) above. To do this, assume the denominator of the squared epipolar distance derived in part (c), and the robust weights, are both evaluated at the previous guess for $F$. Derive the equations up to the point where it is clear what type of equation you need to solve for the new $F$ on each iteration. State the general form of this equation for $F$. You do not need to simplify these equations for $F$ beyond the point which makes their general form clear.

Part (e) [4 MARKS] How would you get an intial guess for minimizing your objective function in part (d)? Be specific.

## Question 5. [15 MARKS]

Consider using colour histograms to track a pigeon in an enclosure using a camera mounted above the enclosure aimed directly downwards, as in Assignment 4. Let $\vec{h}(I, \vec{x}, \theta)$ denote the colour histogram in an elliptical region of the image $I$, with the region centered at pixel $\vec{x}$ and oriented according to the angle $\theta$. Assume $d\left(\vec{h}_{1}, \vec{h}_{2}\right) \geq 0$ is a histogram distance measure, with $d\left(\vec{h}_{1}, \vec{h}_{2}\right)=0$ only if histogram $\vec{h}_{1}$ is the same as $\vec{h}_{2}$.

Part (a) [5 MARKS] Suppose $\vec{h}_{0}$ is a colour histogram for the target pigeon. And suppose you are given two training sets of histogram distances, $D_{\text {on }}$ and $D_{o f f}$. Here $D_{\text {off }}$ is formed from distances $d\left(\vec{h}(I, \vec{x}, \theta), \vec{h}_{0}\right)$ for images $I$ of the background only, while $D_{o n}$ is a set of distances $d\left(\vec{h}(I, \vec{x}, \theta), \vec{h}_{0}\right)$, given that the target pigeon appears in image $I$ within the elliptical region specified by $\vec{x}$ and $\theta$. We expect that the distances in the set $D_{o n}$ are typically smaller than those in $D_{o f f}$. Given these sets of histogram distances, explain how you would develop a suitable likelihood function for tracking. Explain why you made this choice.

Part (b) [2 MARKS]
What would you use to model the dynamics? Explain.

Part (c) [4 marks] How would you maintain an estimate for the state parameters, along with their uncertainties? Explain.

Part (d) [2 marks] What sort of diagnostics would you use to identify when the tracker was working well (and not working well)?

Part (e) [2 maRks] Describe situations in which you would expect this to work well, and also other situations in which you would expect this tracker to have trouble.

## Question 6. [15 MARKS]

Let $I_{k}(\vec{x})$ be the monochromatic image of a Lambertian object illuminated by a distance light source in the direction $\vec{L}_{k}$, for $k=1, \ldots, K$. Assume the light sources are chosen so that there is no self shadowing on the object, and the object fills the whole image frame (i.e., we do not need a mask image to identify foreground and background pixels).

Part (a) [3 MARKS] Let $D$ be the $N \times K$ matrix formed by writing each image $I_{k}(\vec{x})$ as a column vector of brightnesses at all $N$ pixels, and setting this to be $k^{t h}$ column of $D$. For a general shaped Lambertian object, and $K=12$ light sources, what is the rank of $D$ ? Explain.

Part (b) [2 marks] Suppose the object is planar, and has been painted with different coloured Lambertian paints. What is the rank of $D$ ? Explain.

Part (c) [2 marks] Suppose the object is a long circular cylinder, and has been painted with a uniform grey Lambertian paint. Assume the image covers just a part of the side of the cylinder, with neither end visible. What is the rank of $D$ ? Explain.

Part (d) [5 marks] Suppose we have $K=12$ light sources, and a general Lambertian object as described in part (a). What information about the object normals $\vec{n}(\vec{x})$, the albedo $a(\vec{x})$, and the light source directions $\vec{L}_{k}, k=1, \ldots, K$ can you obtain by performing an SVD of the matrix $D$ ? Be as specific as you can about what remains unknown given this matrix decomposition.

Part (e) [3 marks] What additional information would be useful in order to solve for the remaining unknowns in part (d) above? Explain.

For scratch work.

For scratch work.

