

PLEASE HAND IN

UNIVERSITY OF TORONTO
Faculty of Arts and Science
DECEMBER EXAMINATIONS 2005

PLEASE HAND IN

CSC 487/2503H F
St. George Campus

Duration — 3 hours

Aids allowed: none

Student Number:

Family Name:

Given Name:

*Do **not** turn this page until you have received the signal to start.*
(In the meantime, please fill out the identification section above,
and read the instructions below.)

This examination consists of 6 questions on 12 pages (including this one). *When you receive the signal to start, please make sure that your copy of the examination is complete.* If you need more space for one of your solutions, use the reverse side of the page and *indicate clearly the part of your work that should be marked.*

Write your student number at the bottom of pages 2-12 of this test.

1: /14

2: /10

3: /10

4: /10

5: /10

6: /10

TOTAL: /64

Good Luck!

Question 1. [14 MARKS]

Give a short answer for each of the following questions.

Part (a) [2 MARKS] What is a homography between two images?

Part (b) [2 MARKS] Explain how the F matrix determines an epipolar line.

Part (c) [2 MARKS] What is the motion field, and how does it relate to optical flow?

Part (d) [2 MARKS] Give a summary of how to compute a Laplacian pyramid.

Part (e) [2 MARKS] Why is it important that images are not undersampled if you wish to accurately track the images of objects?

Question 2. [10 MARKS]

Consider estimating the translational component of optical flow given two images $I(\vec{x}, t)$ and $I(\vec{x}, t+1)$. You can assume that these images have been cropped so that we only need to estimate a constant displacement between these patches.

Part (a) [2 MARKS]

What is the brightness constancy constraint?

Part (b) [2 MARKS]

Given an initial guess $\vec{u}_0$ for the displacement, so $I(\vec{x} + \vec{u}_0, t+1) \approx I(\vec{x}, t)$, what are the linearized brightness constancy constraints that can be used to update the displacement? Derive a linear system of equations for the update, say $\vec{v}$.

question continued ... on next page.

Part (c) [2 MARKS]

How can a robust estimator $\rho(e)$ be applied to the linearized brightness constancy constraints? In particular, write out the objective function to be minimized by the displacement update $\vec{v}$.

Part (d) [2 MARKS]

Derive an iteratively reweighted least squares algorithm for minimizing your objective function in part (c) above.

Question 3. [10 MARKS]

Consider tracking a given pigeon in an image sequence, where several frames of the sequence are given below.

We are interested trying a simple form of tracking, which you think has a chance of performing reasonably well (but is not necessarily optimal).

Part (a) [2 MARKS]

What state space would you use, and why?

Part (b) [2 MARKS]

What would you use to model the dynamics. Explain.

Part (c) [2 MARKS]

What would you use for the likelihood function, and why?

Question continued on next page.

Part (d) [2 MARKS]

How would you maintain an estimate for the state parameters, along with their uncertainties? Explain.

Question 4. [10 MARKS]

Suppose we wish to use photometric stereo to determine a 3D model of your face. In order to determine the light source directions we have a sphere of radius R which is painted with white Lambertian (matte) paint. Moreover, we can approximate the camera projection as an orthographic projection, with the center of the sphere projecting to $(x, y) = (0, 0)$. Assume the center of the sphere is on the positive z-axis, and this is imaged from the plane at $z = 0$. The radius of the image of the sphere is simply R , so the image of the sphere maps to the disk in the image of radius R centered at the origin.

Part (a) [2 MARKS]

Assume we have a distant light source, with the light propagating uniformly in the direction $\vec{L} = (L_1, L_2, L_3)^T$ with $L_3 > 0$. Given an (orthographic) image of the sphere taken with that light source direction (and the geometry described above), what local property of the image brightness could you use to estimate the light source direction $\vec{L}$. Explain how you would estimate $\vec{L}$ from this local property.

Part (b) [2 MARKS]

Suppose the image brightness is contaminated by additive Gaussian image noise, say mean 0 and variance σ^2 , which is independently distributed noise at each pixel $\vec{x}$. Describe a better way to estimate the light source direction $\vec{L}$.

question continued ... on next page.

Part (c) [2 MARKS]

Given the light source directions (determined above), suppose you wish to image your face by placing your head where the Lambertian sphere was. To model the images obtained using different light source directions, let $\vec{n}(\vec{x})$ denote the surface normal at the point imaged to pixel $\vec{x}$. Describe in detail a mathematical model for the image brightnesses as a function of $\vec{n}(\vec{x})$ and one light source direction $\vec{L}$.

Part (d) [2 MARKS]

How many images with different light source directions do you need to use? Describe some of the sources of difficulty you might expect in getting an accurate surface model.

Question 5. [10 MARKS]

Suppose $\{\vec{x}_k\}_{k=1}^K$ is a set of image edgel positions. Consider estimating an image line, namely the set of image positions which satisfy $\vec{n}^T \vec{x} = c$ for some normal vector $\vec{n}$ and constant c .

Part (a) [2 MARKS]

Given a guess for $\vec{n}$ and c with $\|\vec{n}\|_2 = 1$, give a mathematical expression for the perpendicular error from a point $\vec{x}_k$ and the line?

Part (b) [2 MARKS]

Write out the least squares objective function for the best-fitting line to the given data $\{\vec{x}_k\}_{k=1}^K$.

Part (c) [2 MARKS]

Explain how to solve for the best fitting line according to this least squares condition.

Question continued ... on next page.

Part (d) [2 MARKS]

How can a robust estimator $\rho(e)$ be applied to the perpendicular error? In particular, write out the objective function to be minimized by the line parameters $\vec{n}$ and c .

Part (e) [2 MARKS]

Explain how RANSAC could be used to obtain a good initial guess for the line parameters $\vec{n}$ and c .

Question 6. [10 MARKS]

Suppose (t_k, y_k) are the y (vertical) components of the image of a bouncing ball. Assume orthographic projection. Between bounces we expect the observations to satisfy:

$$v_k = y_k - y_{k-1}, \quad \text{and} \quad v_{k+1} - v_k = -c + n_{k+1}, \quad (1)$$

where n_k is independently sampled Gaussian noise, with mean 0 and variance σ^2 . Here c is a known constant which describes the gravitational acceleration. Assume the bounces are instantaneous.

Part (a) [2 MARKS]

Suppose we wish to use a graph-based segmentation method to determine the time intervals (t_k, t_{k+1}) during which the ball bounces. To do this, consider a single chain of nodes with values v_k , for $k = 1, 2, \dots, K$. The only edges in this graph are between successive nodes. The affinity between v_{k+1} and v_k should be high when the change in velocities is consistent with gravitational acceleration (see above). What is a suitable affinity function?

Part (b) [2 MARKS]

Formulate the Ncut objective function for this problem.

Part (c) [2 MARKS]

Suppose we wish to find the best partitioning of the graph into two parts, according to the Ncut objective function. Since the graph is only one dimensional we can afford to try every possible cut. Specify the algorithm.

For scratch work.

Total Marks = 64

Student #:

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END OF EXAMINATION