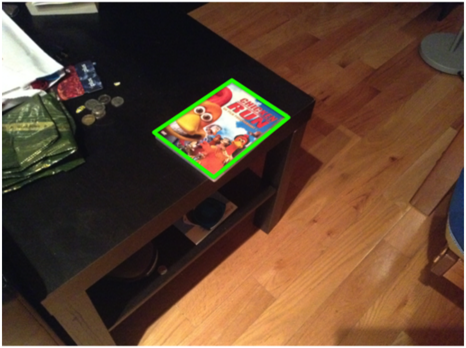
Matching Planar Objects In New Viewpoints ... And Much More – via Homography

What Transformation Happened To My DVD?

- Rectangle goes to a parallelogram

$T?$



Affine Transformations

Affine transformations are combinations of:

- Linear transformations, and
- Translations

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b & e \\ c & d & f \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Properties of affine transformations:

- Origin does not necessarily map to origin
- Lines map to lines
- Parallel lines remain parallel
- Ratios are preserved
- Closed under composition
- Rectangles go to parallelograms

[Source: N. Snavely, slide credit: R. Urtasun]

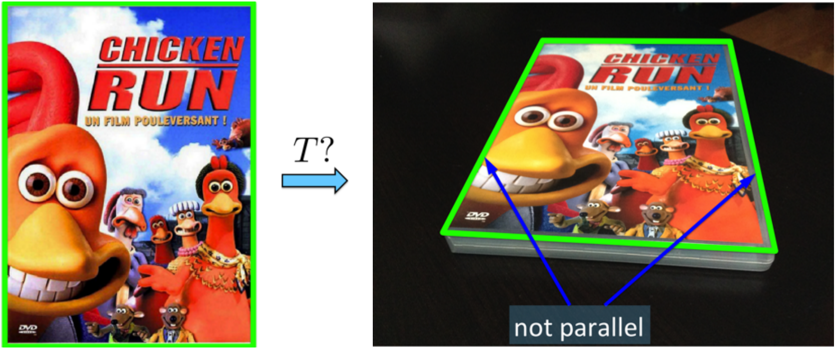
What Transformation Really Happened To My DVD?

- What about now?

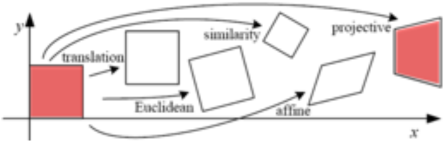
$T?$

What Transformation Really Happened To My DVD?

- Actually a rectangle goes to **quadrilateral**



2D Image Transformations



Transformation	Matrix	# DoF	Preserves	Icon
translation	$\begin{bmatrix} I \mid t \end{bmatrix}_{2\times 3}$	2	orientation	
rigid (Euclidean)	$\begin{bmatrix} R \mid t \end{bmatrix}_{2\times 3}$	3	lengths	
similarity	$\begin{bmatrix} sR \mid t \end{bmatrix}_{2\times 3}$	4	angles	
affine	$\begin{bmatrix} A \end{bmatrix}_{2\times 3}$	6	parallelism	
projective	$\begin{bmatrix} \tilde{H} \end{bmatrix}_{3\times 3}$	8	straight lines	

- These transformations are a nested set of groups
- Closed under composition and inverse is a member

[source: R. Szeliski]

Projective Transformations

- Homography:

$$w \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

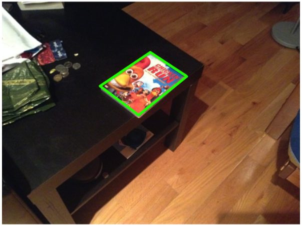
Properties:

- Origin does not necessarily map to origin
- Lines map to lines
- Parallel lines do not necessarily remain parallel
- Ratios are **not** preserved
- Closed under composition
- Rectangle goes to quadrilateral
- Affine transformation is a special case, where $g = h = 0$ and $i = 1$

[Source: N. Snavely, slide credit: R. Urtasun]

What Transformation Really Happened to My DVD?

$T?$



For **planar** objects:

- Viewpoint change for planar objects is a **homography**
- Affine transformation **approximates** viewpoint change for planar objects that are far away from camera

What Transformation Happened to My DVD?

- Why should I care about homography?
- Now that I care, how should I estimate it?
- I want to understand the geometry behind homography. That is, why aren't parallel lines mapped to parallel lines in oblique viewpoints? How did we get that equation for computing the homography?

Homography

- Why should I care about homography? **Let's answer this first**
- Now that I care, how should I estimate it?
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Homography

$T?$

- Why do we need homography? Can't we just assume that the transformation is affine? The approximation on the right looks pretty decent to me...
- That's right. If I want to detect (match) an object in a new viewpoint, an affine transformation is a relatively decent approximation

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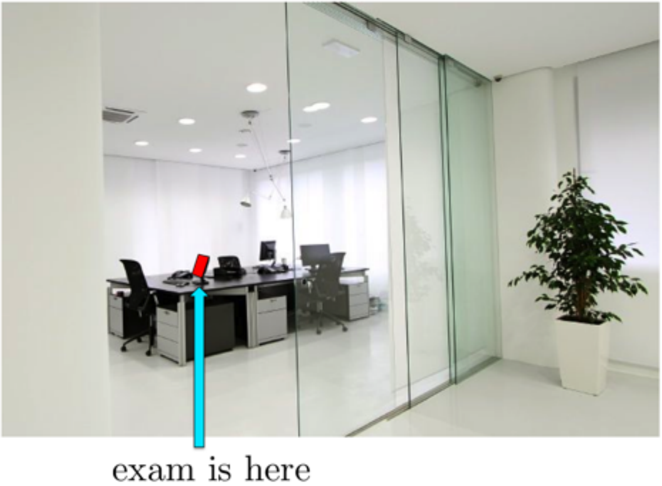
Application 1: a Little Bit of CSI

- Tom Cruise is taking an exam on Monday

Application 1: a Little Bit of CSI

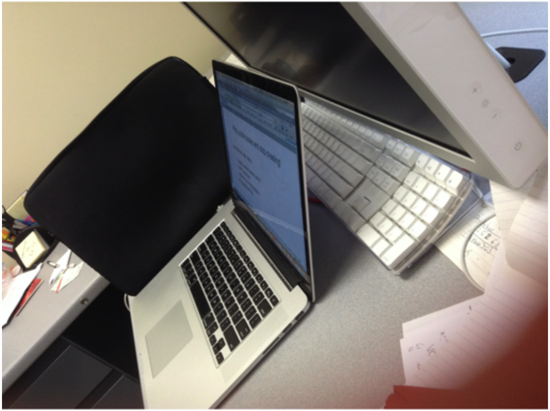


- The professor keeps the exams in this office

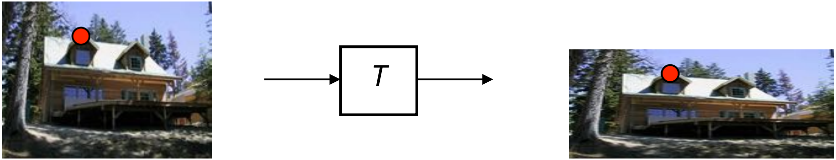
Application 1: a Little Bit of CSI

- He enters (without permission) and takes a picture of the laptop screen

Application 1: a Little Bit of CSI



- His picture turns out to not be from a viewpoint he was shooting for (it's difficult to take pictures while hanging)
- Can he still read the exam?



p = (x,y) **p’** = (x’,y’)

- Transformation T is a coordinate-changing machine:

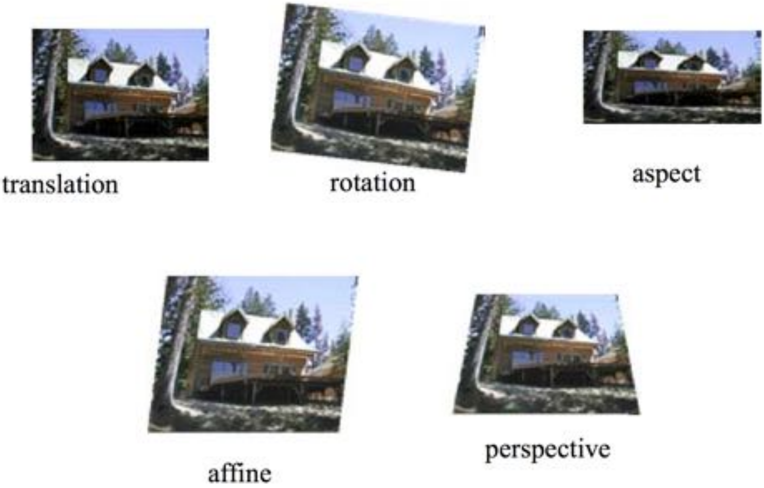
$$[x', y'] = T(x, y)$$

- What does it mean that T is global?
  - Is the same for any point p
  - Can be described by just a few numbers (parameters)

[Source: N. Snavely, slide credit: R. Urtasun]

Warping an Image with a Global Transformation

- Example of warping for different transformations:



Forward and Inverse Warping

- **Forward Warping**: Send each pixel $f(x)$ to its corresponding location $(x', y') = T(x, y)$ in $g(x', y')$

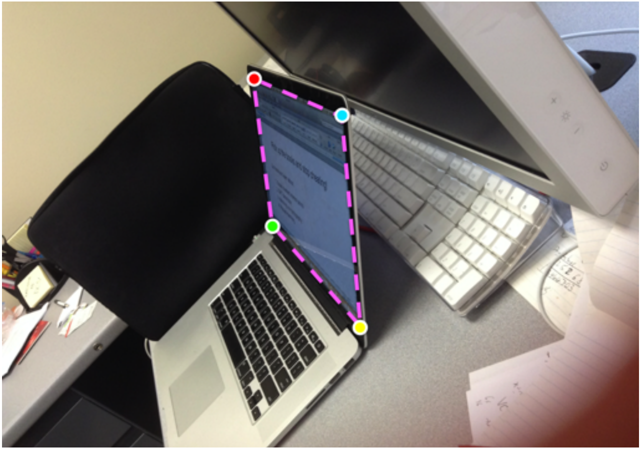
> **procedure** *forwardWarp*($f, h,$ **out** g):
>
> For every pixel x in $f(x)$
>
> 1. Compute the destination location $x' = h(x)$.
> 2. Copy the pixel $f(x)$ to $g(x')$.

- **Inverse Warping:** Each pixel at destination is sampled from original image

> **procedure** *inverseWarp*($f, h,$ **out** g):
>
> For every pixel x' in $g(x')$
>
> 1. Compute the source location $x = \hat{h}(x')$
> 2. Resample $f(x)$ at location x and copy to $g(x')$

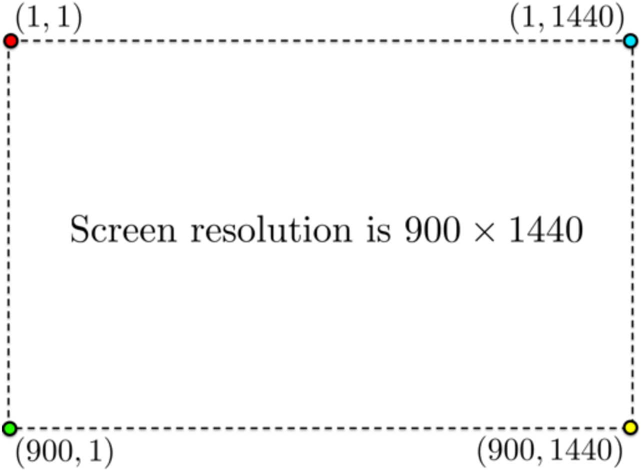
[source: R. Urtasun]

Application 1: a Little Bit of CSI



- We want to transform the picture (plane) inside these 4 points into a rectangle (laptop screen)

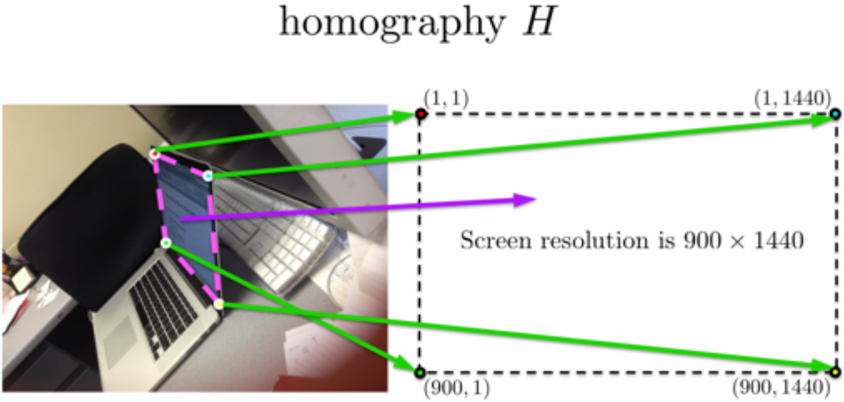
Application 1: a Little Bit of CSI



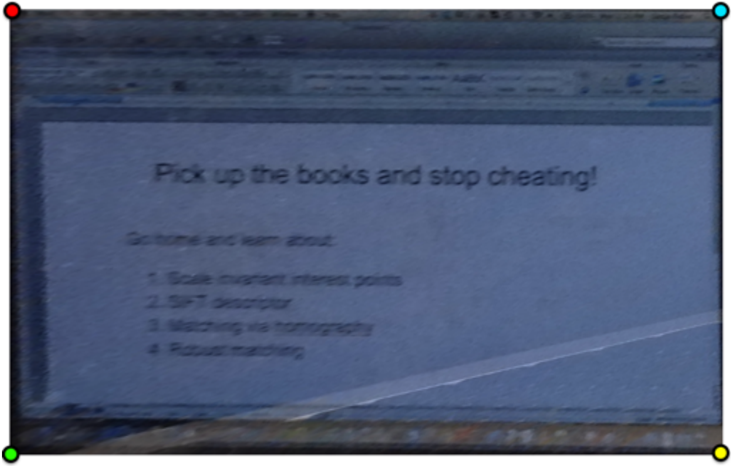
- We want it to look like this. How can we do this?

Application 1: a Little Bit of CSI

- A transformation that maps a projective plane (a quadrilateral) to another projective plane (another quadrilateral, in this case a rectangle) is a homography

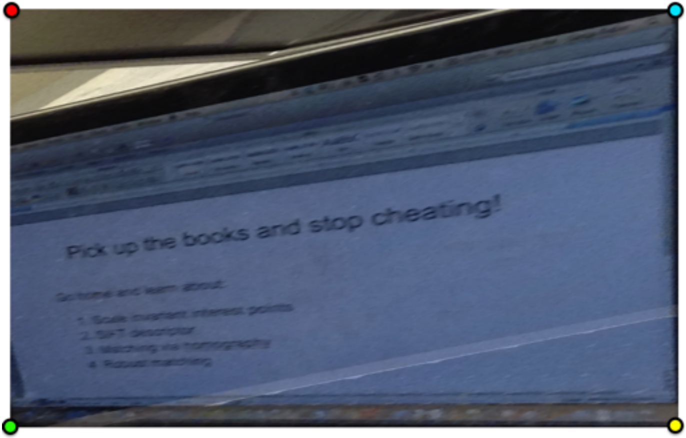


Application 1: a Little Bit of CSI



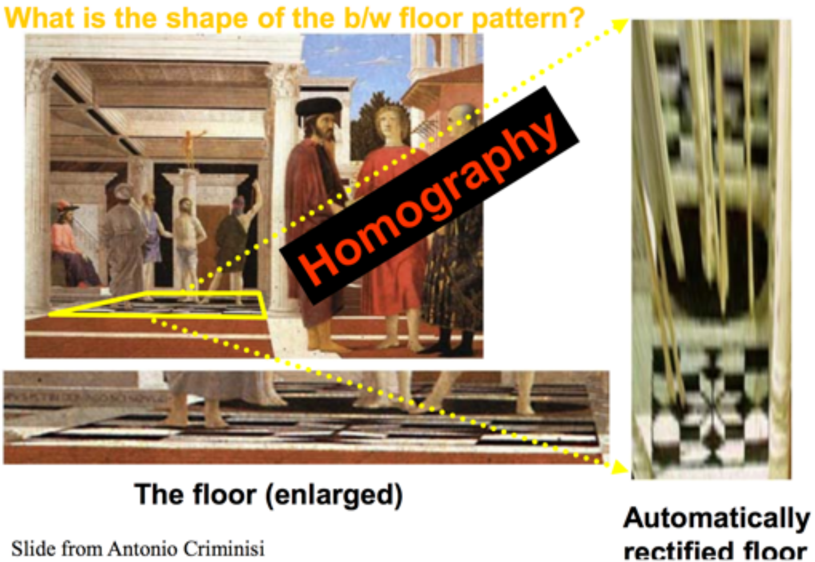
- If we compute the homography and warp the image according to it, we get this

Application 1: a Little Bit of CSI



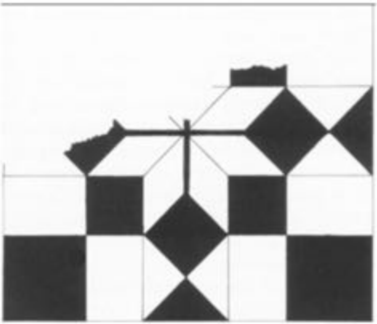
- If we used affine transformation instead, we'd get this. Would be even worse if our picture was taken closer to the laptop

Application 1: a Little More of CSI



Application 1: a Little More of CSI

Automatic rectification



From Martin Kemp *The Science of Art*
(manual reconstruction)

Slide from Antonio Criminisi

Application 1: a Little More of CSI

St. Lucy Altarpiece, D. Veneziano

Slide from Criminisi

What is the (complicated) shape of the floor pattern?

Automatically rectified floor

Automatic rectification

From Martin Kemp, *The Science of Art* *(manual reconstruction)*

Slide from Criminisi

Application 2: How Much do Soccer Players Run?

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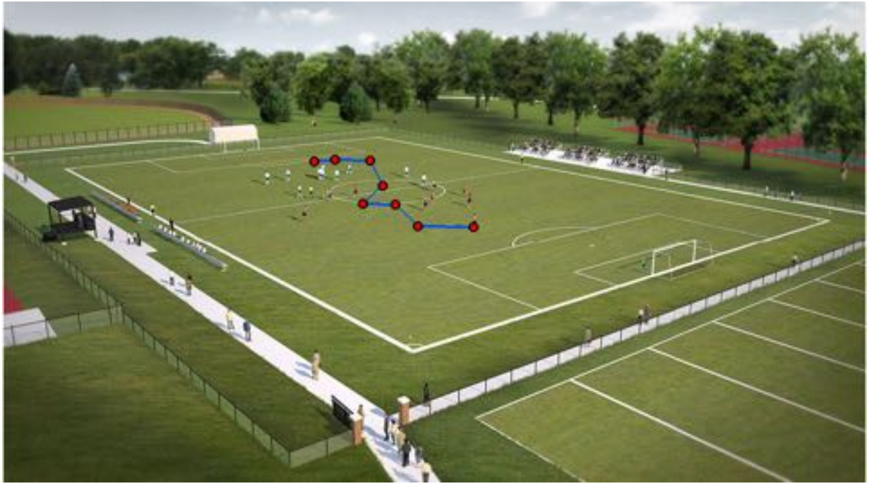
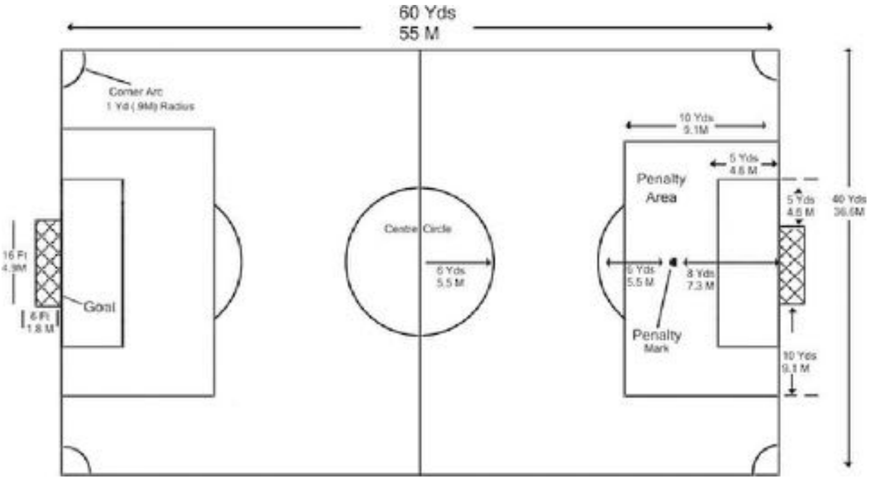


- How many meters did this player run?

Application 2: How Much do Soccer Players Run?

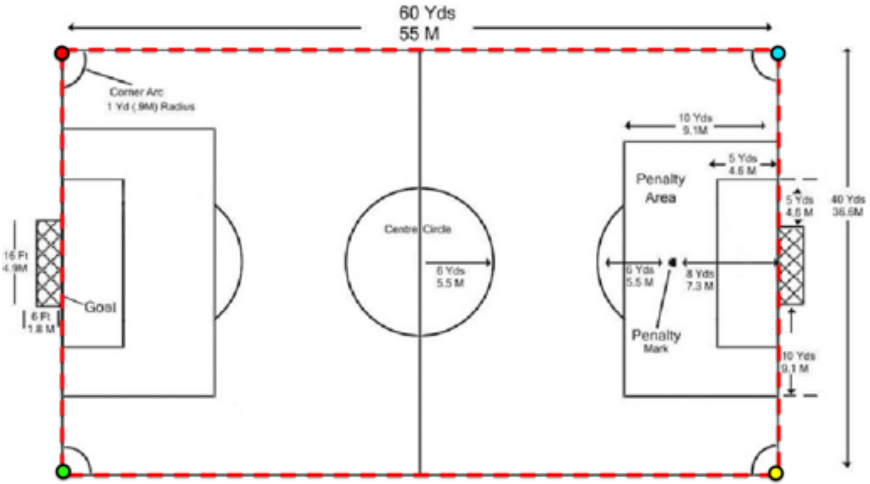


- Field is planar. We know its dimensions (look on Wikipedia).

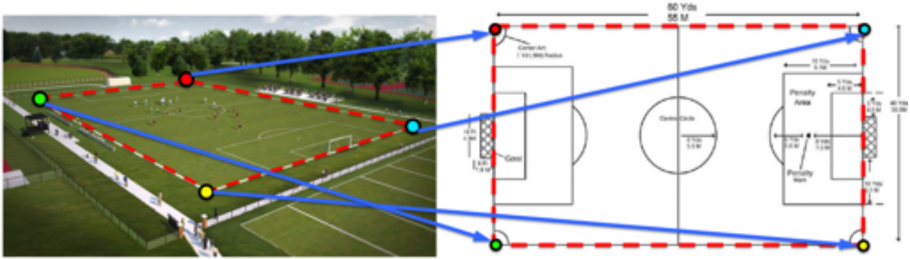
Application 2: How Much do Soccer Players Run?

- Let's take the 4 corner points of the field

Application 2: How Much do Soccer Players Run?

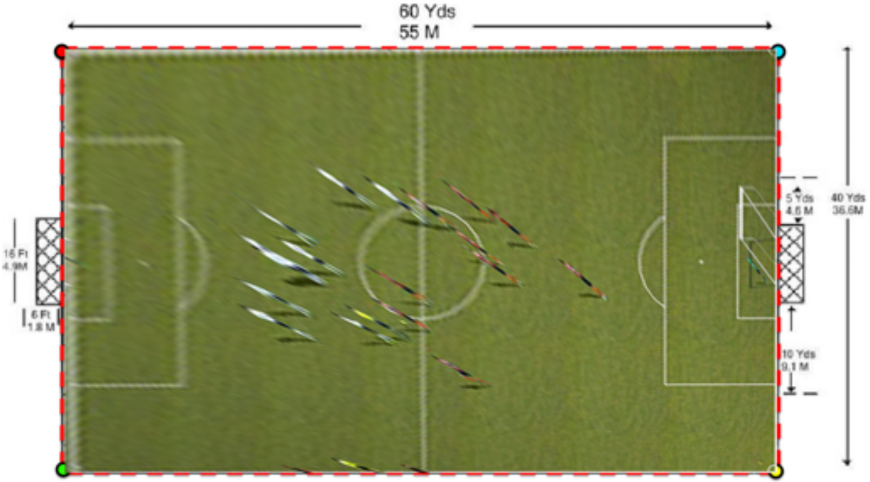


- We need to compute a homography that maps them to these 4 corners



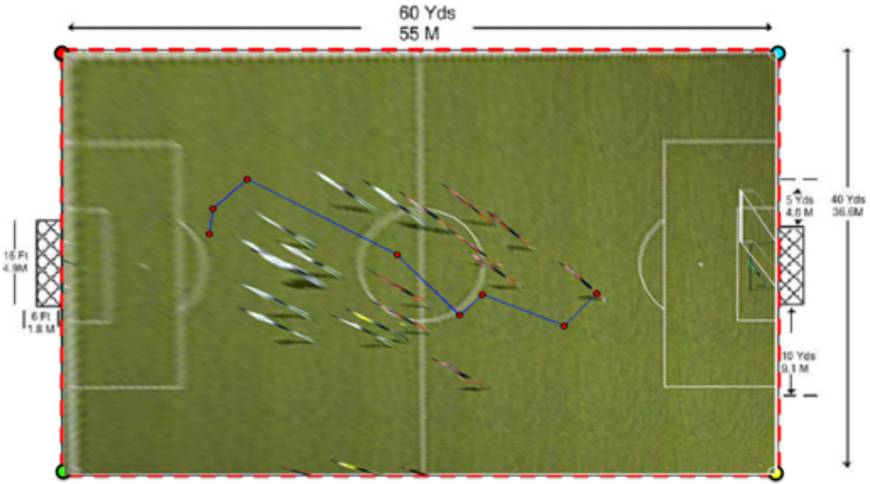
- We need to compute a homography that maps the 4 corners. Any other point from this plane (the field) also maps to the right with the same homography

Application 2: How Much do Soccer Players Run?



- Nice. What happened to the players?

Application 2: How Much do Soccer Players Run?



- We can now also transform the player's trajectory → and we have it in meters!

Application 2: How Much do Soccer Players Run?

- If we used affine transformation... Our estimations of running would not be accurate!

Application 3: Panorama Stitching

Take a tripod, rotate camera and take pictures

[Source: Fernando Flores-Mangas]

Application 3: Panorama Stitching

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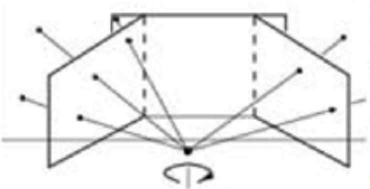
Application 3: Panorama Stitching



- Each pair of images is related by homography! **If we also moved the camera, this wouldn't be true** (next class) [Source: Fernando Flores-Mangas]

Application 3: Panorama Stitching

- To do panorama stitching, we need to:
  - Match points between pairs of images I and J
  - Compute a transformation between the between matches in I and J : a homography
  - Do it robustly (RANSAC)
  - Warp the first image to the second using the estimated homography
- Apart from the last point, this is exactly the same procedure as for the problem of matching planar objects across viewpoints
- So this should motivate the **why do I care part** of the homographies

Homography

- Why should I care about homography?
- Now that I care, how should I estimate it? **Let's do this now**
- I want to understand the geometry behind homography. That is, why aren't parallel lines mapped to parallel lines in oblique viewpoints? How did we get that equation for computing the homography?

Solving for Homographies

- Let (x_i, y_i) be a point on the reference (model) image, and (x_i', y_i') its match in the test image
- A homography H maps (x_i, y_i) to (x_i', y_i'):

$$\begin{bmatrix} ax_i' \\ ay_i' \\ a \end{bmatrix} = \begin{bmatrix} h_{00} & h_{01} & h_{02} \\ h_{10} & h_{11} & h_{12} \\ h_{20} & h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}$$

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- We can get rid of that a on the left:

$$\begin{aligned} x_i' &= \frac{h_{00}x_i + h_{01}y_i + h_{02}}{h_{20}x_i + h_{21}y_i + h_{22}} \\ y_i' &= \frac{h_{10}x_i + h_{11}y_i + h_{12}}{h_{20}x_i + h_{21}y_i + h_{22}} \end{aligned}$$

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- Hmmmm... Can I still rewrite this into a linear system in h?

[Source: R. Urtasun]

Solving for homographies

- From:

$$x_i' = \frac{h_{00}x_i + h_{01}y_i + h_{02}}{h_{20}x_i + h_{21}y_i + h_{22}}$$

$$y_i' = \frac{h_{10}x_i + h_{11}y_i + h_{12}}{h_{20}x_i + h_{21}y_i + h_{22}}$$

- We can easily get this:

$$\begin{aligned} x_i'(h_{20}x_i + h_{21}y_i + h_{22}) &= h_{00}x_i + h_{01}y_i + h_{02} \\ y_i'(h_{20}x_i + h_{21}y_i + h_{22}) &= h_{10}x_i + h_{11}y_i + h_{12} \end{aligned}$$

- Rewriting it a little:

$$\begin{aligned} h_{00}x_i + h_{01}y_i + h_{02} - x_i'(h_{20}x_i + h_{21}y_i + h_{22}) &= 0 \\ h_{10}x_i + h_{11}y_i + h_{12} - y_i'(h_{20}x_i + h_{21}y_i + h_{22}) &= 0 \end{aligned}$$

Solving for homographies

- We can re-write these equations:

$$
\begin{aligned}
h_{00}x_i + h_{01}y_i + h_{02} - x_i'(h_{20}x_i - h_{21}y_i - h_{22}) &= 0 \\
h_{10}x_i + h_{11}y_i + h_{12} - y_i'(h_{20}x_i - h_{21}y_i - h_{22}) &= 0
\end{aligned}
$$

- as a linear system!

$$
\begin{bmatrix}
x_i & y_i & 1 & 0 & 0 & 0 & -x_i'x_i & -x_i'y_i & -x_i' \\
0 & 0 & 0 & x_i & y_i & 1 & -y_i'x_i & -y_i'y_i & -y_i'
\end{bmatrix}
\begin{bmatrix}
h_{00} \\ h_{01} \\ h_{02} \\ h_{10} \\ h_{11} \\ h_{12} \\ h_{20} \\ h_{21} \\ h_{22}
\end{bmatrix}
=
\begin{bmatrix}
0 \\ 0
\end{bmatrix}
$$

[Source: R. Urtasun]

Solving for homographies

- Taking all our matches into account:

$$\underbrace{\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x'_1 x_1 & -x'_1 y_1 & -x'_1 \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -y'_1 x_1 & -y'_1 y_1 & -y'_1 \\ & & & & \vdots & & & & \\ x_n & y_n & 1 & 0 & 0 & 0 & -x'_n x_n & -x'_n y_n & -x'_n \\ 0 & 0 & 0 & x_n & y_n & 1 & -y'_n x_n & -y'_n y_n & -y'_n \end{bmatrix}}_{\substack{\mathbf{A} \\ 2n \times 9}} \underbrace{\begin{bmatrix} h_{00} \\ h_{01} \\ h_{02} \\ h_{10} \\ h_{11} \\ h_{12} \\ h_{20} \\ h_{21} \\ h_{22} \end{bmatrix}}_{\substack{\mathbf{h} \\ 9}} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}}_{\substack{\mathbf{0} \\ 2n}}$$

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- How many matches do I need to estimate H?
- This defines a least squares problem:

$$\min_{\mathbf{h}} ||\mathbf{Ah}||_2^2$$

Solving for homographies

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$$\underbrace{\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x'_1x_1 & -x'_1y_1 & -x'_1 \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -y'_1x_1 & -y'_1y_1 & -y'_1 \\ & & & & \vdots & & & & \\ x_n & y_n & 1 & 0 & 0 & 0 & -x'_nx_n & -x'_ny_n & -x'_n \\ 0 & 0 & 0 & x_n & y_n & 1 & -y'_nx_n & -y'_ny_n & -y'_n \end{bmatrix}}_{\substack{\mathbf{A} \\ 2n \times 9}} \underbrace{\begin{bmatrix} h_{00} \\ h_{01} \\ h_{02} \\ h_{10} \\ h_{11} \\ h_{12} \\ h_{20} \\ h_{21} \\ h_{22} \end{bmatrix}}_{\substack{\mathbf{h} \\ 9}} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}}_{\substack{\mathbf{0} \\ 2n}}$$

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- Since **h** is only defined up to scale, solve for unit vector

Solving for homographies

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$$\underbrace{\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x'_1x_1 & -x'_1y_1 & -x'_1 \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -y'_1x_1 & -y'_1y_1 & -y'_1 \\ & & & & \vdots & & & & \\ x_n & y_n & 1 & 0 & 0 & 0 & -x'_nx_n & -x'_ny_n & -x'_n \\ 0 & 0 & 0 & x_n & y_n & 1 & -y'_nx_n & -y'_ny_n & -y'_n \end{bmatrix}}_{\substack{\mathbf{A} \\ 2n \times 9}} \underbrace{\begin{bmatrix} h_{00} \\ h_{01} \\ h_{02} \\ h_{10} \\ h_{11} \\ h_{12} \\ h_{20} \\ h_{21} \\ h_{22} \end{bmatrix}}_{\substack{\mathbf{h} \\ 9}} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}}_{\substack{\mathbf{0} \\ 2n}}$$

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- Solution: $\hat{\mathbf{h}} =$ eigenvector of $\mathbf{A}^T\mathbf{A}$ with smallest eigenvalue

Solving for homographies

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- Works with 4 or more points

[Source: R. Urtasun]

Image Alignment Algorithm: Homography

Given images I and J

1. Compute image features for I and J
2. Match features between I and J
3. Compute **homography** transformation A between I and J (with RANSAC)

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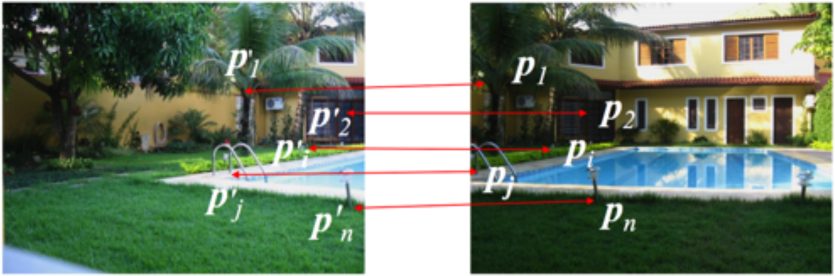
[Source: N. Snavely]

Panorama Stitching: Example 1



- Compute the matches

[Source: R. Queiroz Feitosa]

Panorama Stitching: Example 1

- Estimate the homography and warp

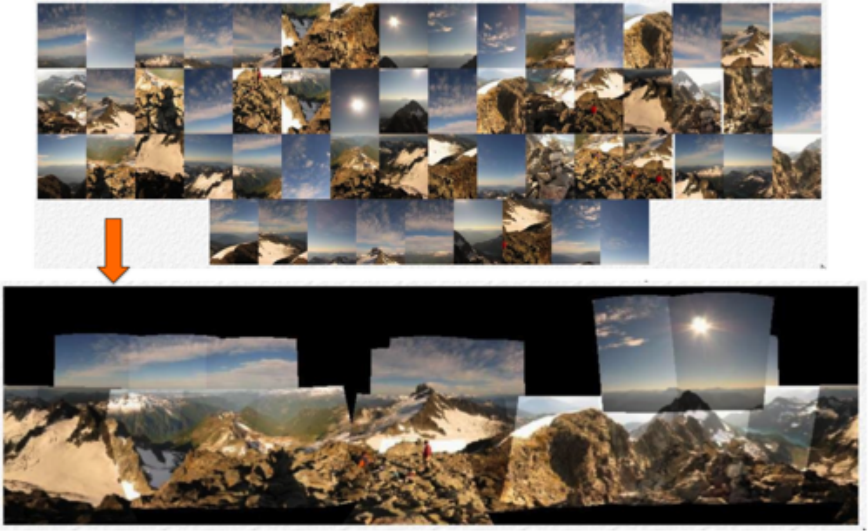
[Source: R. Queiroz Feitosa]

Panorama Stitching: Example 1

- Stitch

[Source: R. Queiroz Feitosa]

Panorama Stitching: Example 2

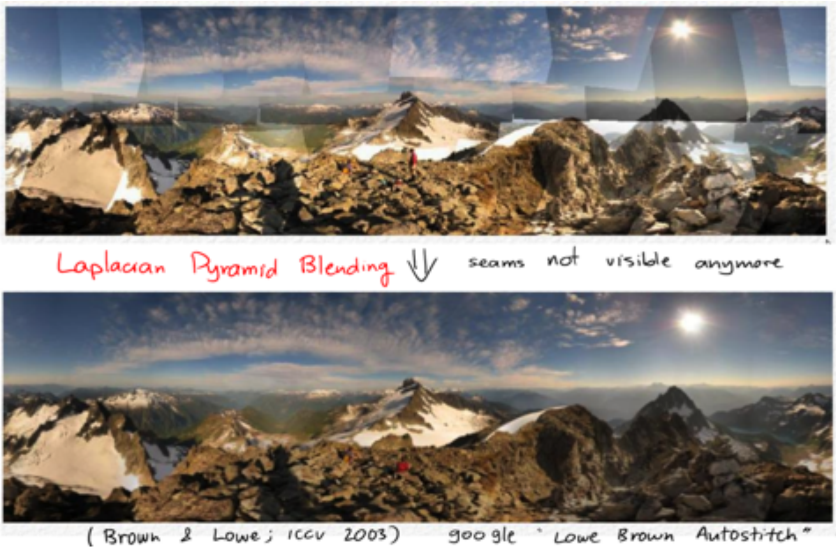


[Source: Fernando Flores-Mangas]

Panorama Stitching: Example 2

[Source: Fernando Flores-Mangas]

Panorama Stitching: Example 2



[Source: Fernando Flores-Mangas]

Summary – Stuff You Need To Know

- A homography is a mapping between projective planes
- You need at least 4 correspondences (matches) to compute it

Matlab functions:

- TFORM = MAKETFORM('AFFINE',[X1,Y1],[X2,Y2]); % Computes affine transformation between points $[x_1, y_1]$ and $[x_2, y_2]$. Needs 3 pairs of matches (x_1, y_1, x_2, y_2 have three rows)
- TFORM = MAKETFORM('PROJECTIVE',[X1,Y1],[X2,Y2]); % Computes homography between points $[x_1, y_1]$ and $[x_2, y_2]$. Needs 4 pairs of matches
- IMW = IMTRANSFORM(IM, TFORM, 'BICUBIC','FILL', 0); % Warps the image according to transformation

Birdseye View on What We Learned So Far

Problem	**Detection**	**Description**	**Matching**
Find Planar Distinctive Objects	Scale Invariant Interest Points	Local feature: SIFT	All features to all features + Affine / Homography
Panorama Stitching	Scale Invariant Interest Points	Local feature: SIFT	All features to all features + Homography

Exercise: How Dangerous is This Street?

- Can I walk here during the night? Can we tell this from an image?

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Exercise: How Dangerous is This Street?

- It's Chicago...

http://www.neighborhoodscout.com/il/chicago/crime/

Exercise: How Dangerous is This Street?

- It's Chicago... Can I walk here during the day?

Exercise: How Dangerous is This Street?

- Idea: Match image to Google's StreetView images of Chicago!

Exercise: How Dangerous is This Street?

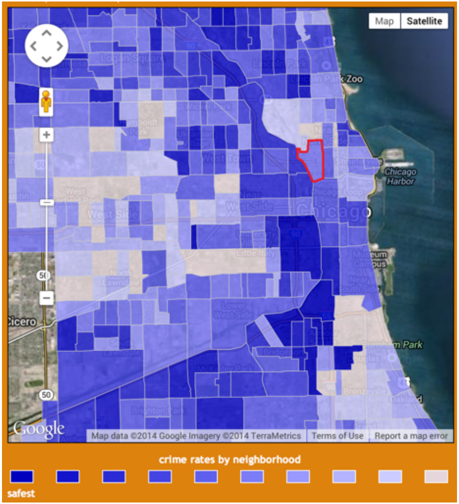
- Our match to StreetView

Exercise: How Dangerous is This Street?

- Lookup the GPS location...

Exercise: How Dangerous is This Street?

- Lookup the crime map for that GPS location



http://www.neighborhoodscout.com/il/chicago/crime/

Exercise: How Dangerous is This Street?

- Lookup the crime map for that GPS location

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Lesson of the Execise

- We're in 2022...

Think not (only) what you can do with one image, but what **lots and lots** of images can do for you

- We're in 2022...

Think not (only) what you can do with one image, but what **lots and lots** of images can do for you

- Would our current matching method work with lots of data?

Big Data

- So far we matched a known object in a new viewpoint
- What if we have to match an object to **LOTS** of images? Or **LOTS** of objects to one image?
- Please read this and we will discuss:

Josef Sivic, Andrew Zisserman

Video Google: A Text Retrieval Approach to Object Matching in Videos

ICCV 2003

Paper link: `http://www.robots.ox.ac.uk/~vgg/publications/papers/sivic03.pdf`

Next Time:
Camera Models