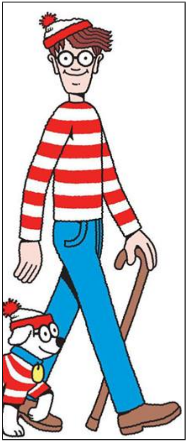
Edge Detection

Finding Waldo

- Let's revisit the problem of finding Waldo
- And let's take a simple example

image



template (filter)

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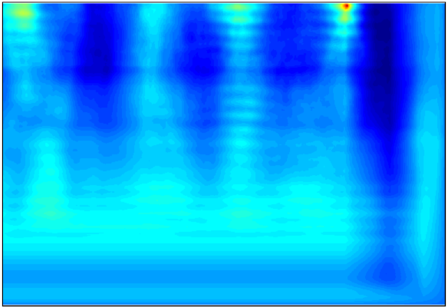
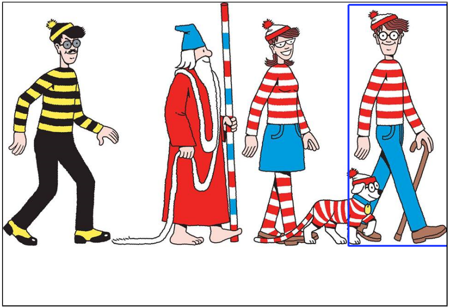


normalized cross-correlation



Waldo detection
(putting box around max response)

Finding Waldo

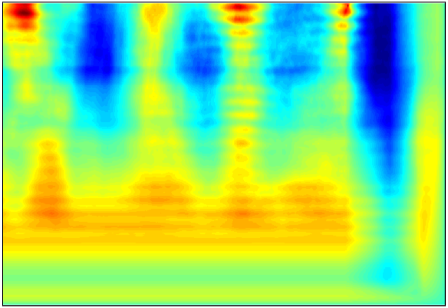
- Now imagine Waldo goes shopping
- ... but our filter **doesn't know that**

image

template (filter)

Finding Waldo

- Now imagine Waldo goes shopping (and the dog too)
- ... but our filter **doesn't know that**



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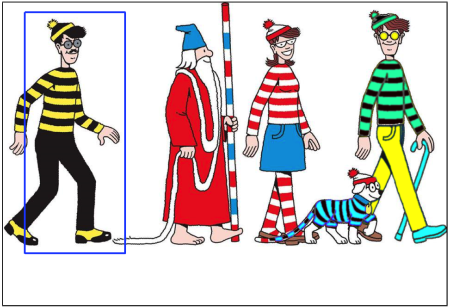

Waldo detection
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Finding Waldo (again)

- What can we do to find Waldo again?

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image

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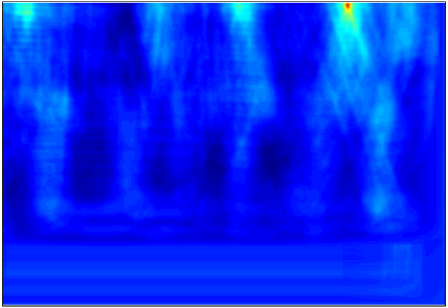
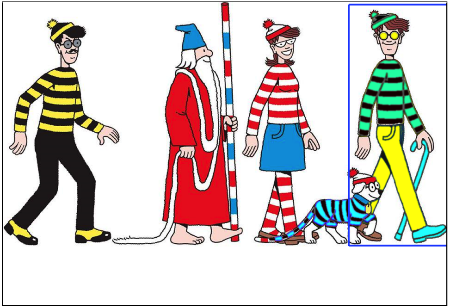


normalized cross-correlation

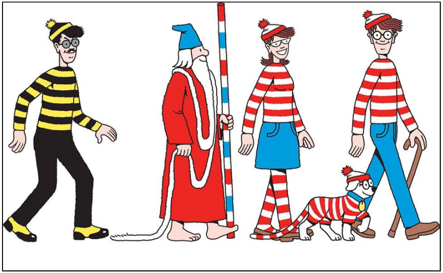
(using the edge maps)

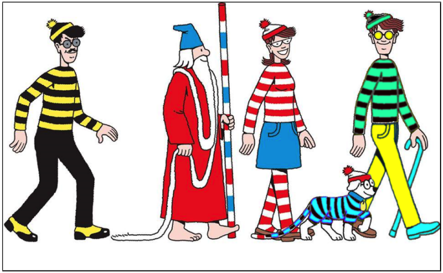


Waldo detection

(putting box around max response)

Waldo and Edges





Edge detection

- Map image to a set of **curves** or **line segments** or **contours**.
- More compact than pixels.
- Edges are invariant to changes in illumination
- Important for recognition

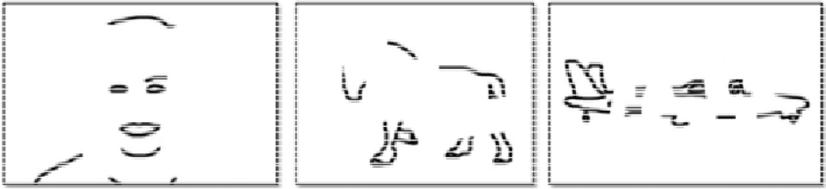


Figure: [Shotton et al. PAMI, 07]

[Source: K. Grauman]

Edge detection

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Figure: Parse basketball court (left) to figure out how far the guy is from net

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f0

Figure: How can a robot pick up or grasp objects?

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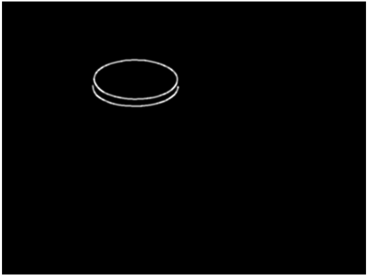
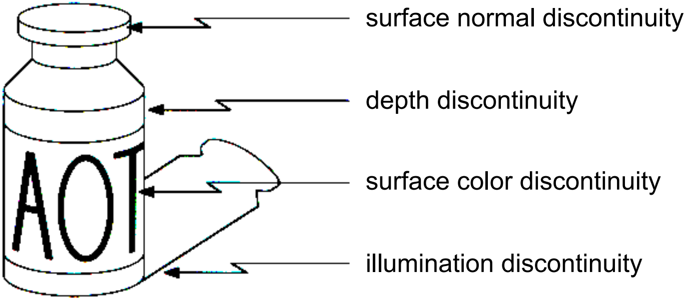


f0

Figure: How can a robot pick up or grasp objects?

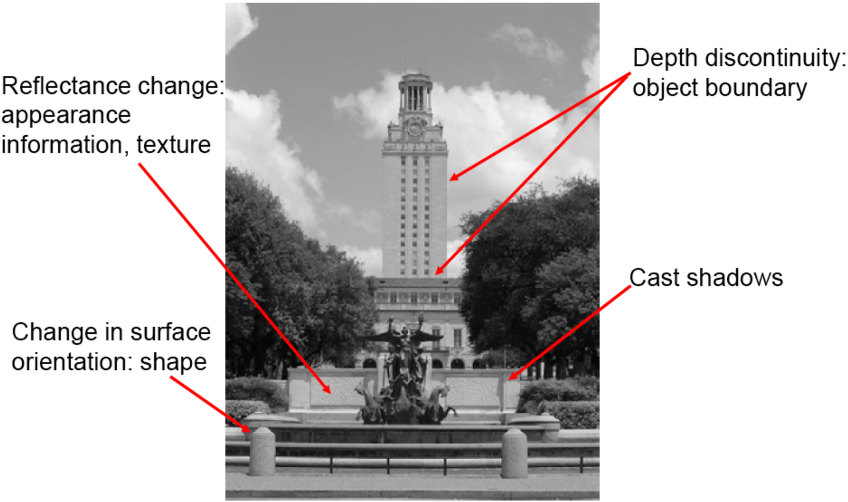
Origin of Edges

- Edges are caused by a variety of factors



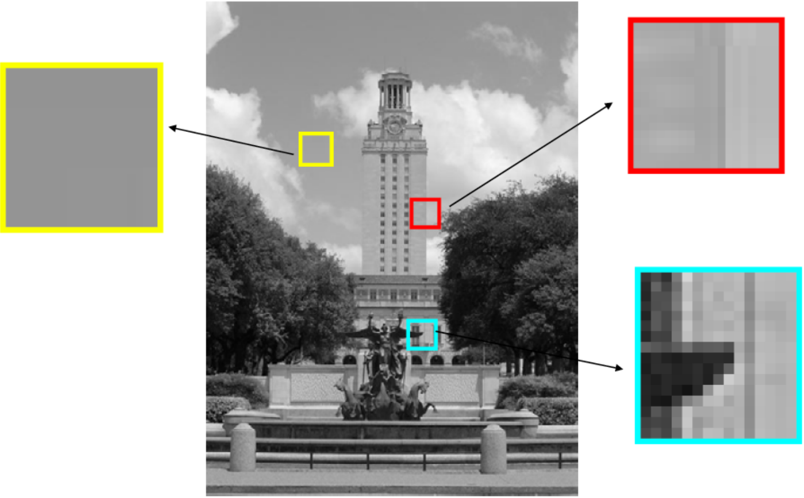
[Source: N. Snavely]

What Causes an Edge?



[Source: K. Grauman]

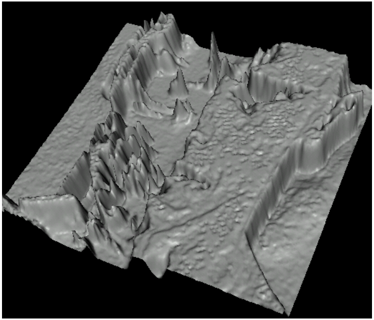
Looking More Locally...



[Source: K. Grauman]

Images as Functions

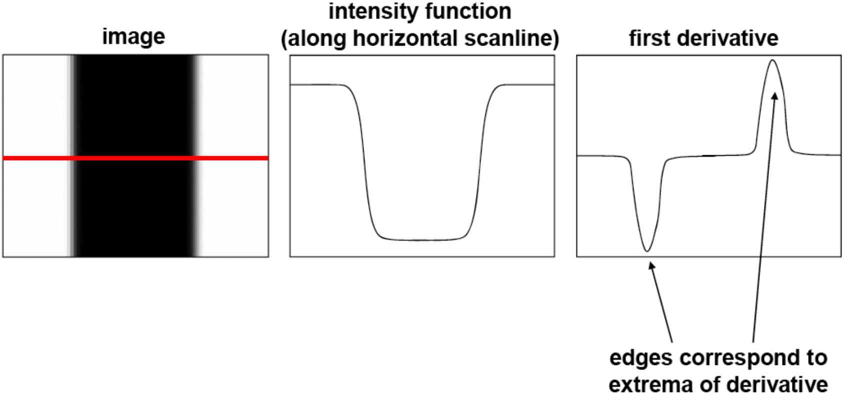
- Edges look like steep cliffs



[Source: N. Snavely]

Characterizing Edges

- An **edge** is a place of rapid change in the image intensity function.



[Source: S. Lazebnik]

How to Implement Derivatives with Convolution

How can we differentiate a digital image $f[x, y]$?

- If image f was continuous, then compute the partial derivative as

$$\frac{\partial f(x,y)}{\partial x} = \lim_{\epsilon \to 0} \frac{f(x+\epsilon, y) - f(x,y)}{\epsilon}$$

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$\frac{\partial f}{\partial x}$:

H_x

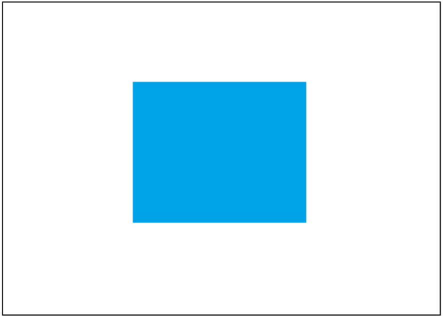
$\frac{\partial f}{\partial y}$:

H_y

[Source: S. Seitz]

Examples: Partial Derivatives of an Image

- How does the horizontal derivative using the filter $[-1, 1]$ look like?



Image

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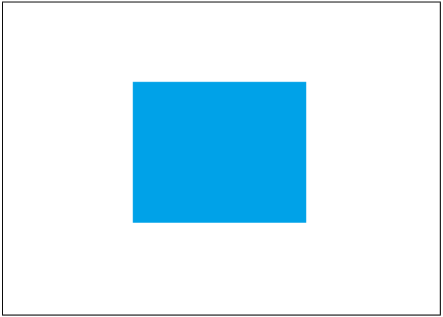
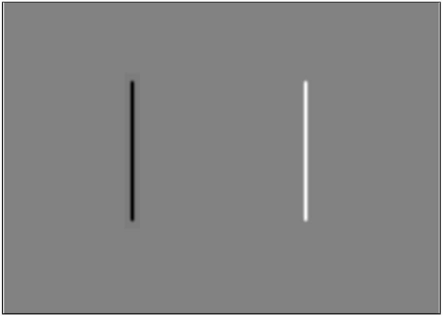


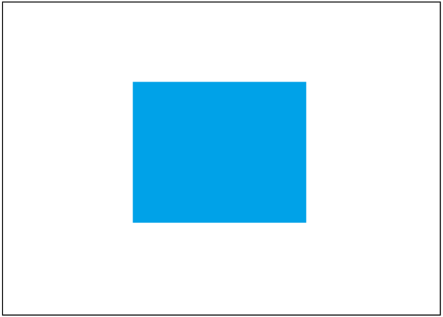
Image



$\frac{\partial f(x,y)}{\partial x}$ with $[-1, 1]$ and correlation

Examples: Partial Derivatives of an Image

- How about the vertical derivative using filter $[-1, 1]^T$?



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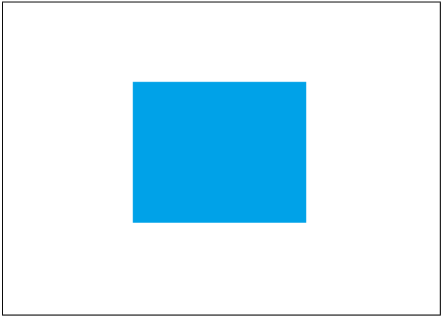

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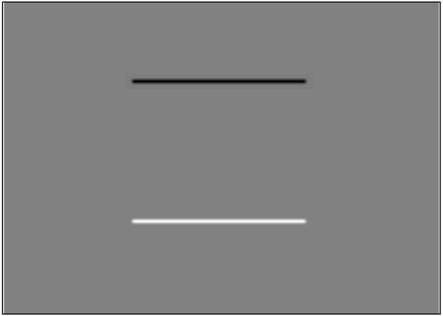

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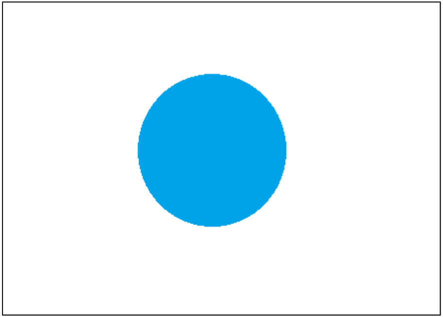

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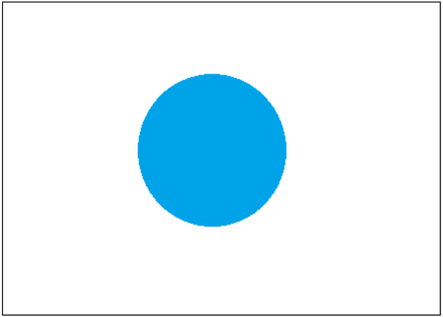

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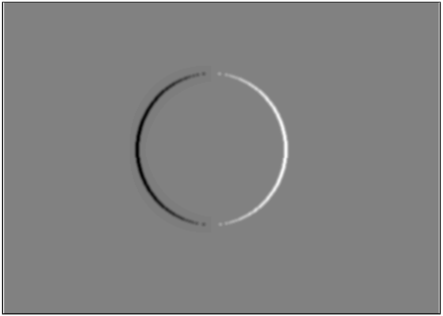

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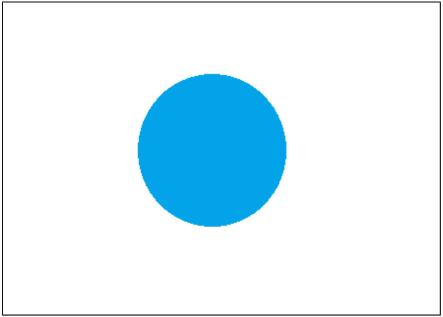

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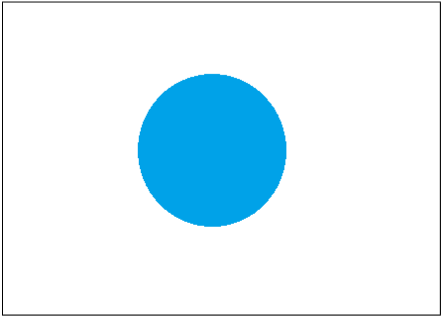

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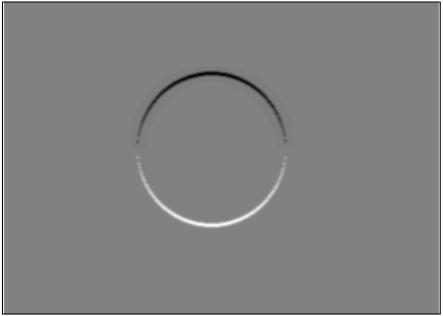
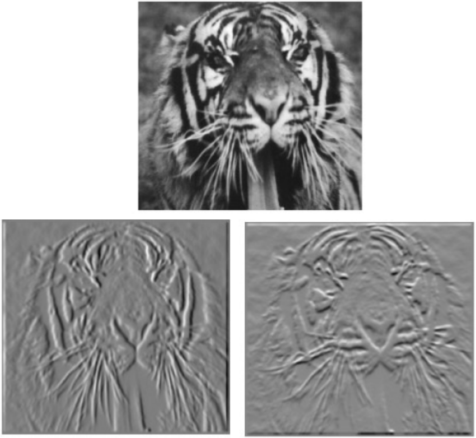


$\frac{\partial f(x,y)}{\partial y}$ with $[-1, 1]^T$ and correlation

Examples: Partial Derivatives of an Image

$$\frac{\partial f(x,y)}{\partial x}$$

$$\begin{bmatrix} -1 & 1 \end{bmatrix}$$



$$\frac{\partial f(x,y)}{\partial y}$$

$$\begin{bmatrix} -1 \\ 1 \end{bmatrix} \text{ or } \begin{bmatrix} 1 \\ -1 \end{bmatrix} ?$$

Figure: Using correlation filters

[Source: K. Grauman]

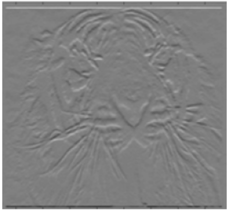
Finite Difference Filters

Prewitt: $M_x = \begin{array}{|c|c|c|}\hline -1 & 0 & 1 \\ \hline -1 & 0 & 1 \\ \hline -1 & 0 & 1 \\ \hline \end{array}$; $M_y = \begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 0 & 0 & 0 \\ \hline -1 & -1 & -1 \\ \hline \end{array}$

Sobel: $M_x = \begin{array}{|c|c|c|}\hline -1 & 0 & 1 \\ \hline -2 & 0 & 2 \\ \hline -1 & 0 & 1 \\ \hline \end{array}$; $\boxed{M_y = \begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline 0 & 0 & 0 \\ \hline -1 & -2 & -1 \\ \hline \end{array}}$

Roberts: $M_x = \begin{array}{|c|c|}\hline 0 & 1 \\ \hline -1 & 0 \\ \hline \end{array}$; $M_y = \begin{array}{|c|c|}\hline 1 & 0 \\ \hline 0 & -1 \\ \hline \end{array}$

```
>> My = fspecial('sobel');
>> outim = imfilter(double(im), My);
>> imagesc(outim);
>> colormap gray;
```



[Source: K. Grauman]

Image Gradient

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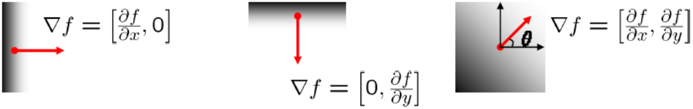

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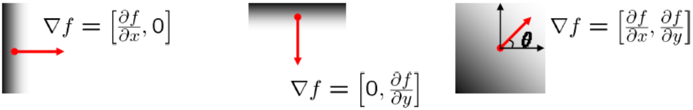

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$$\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} \Big/ \frac{\partial f}{\partial x}\right)$$

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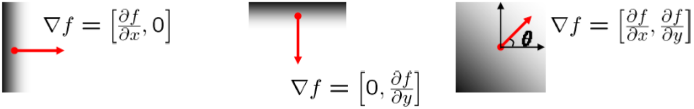


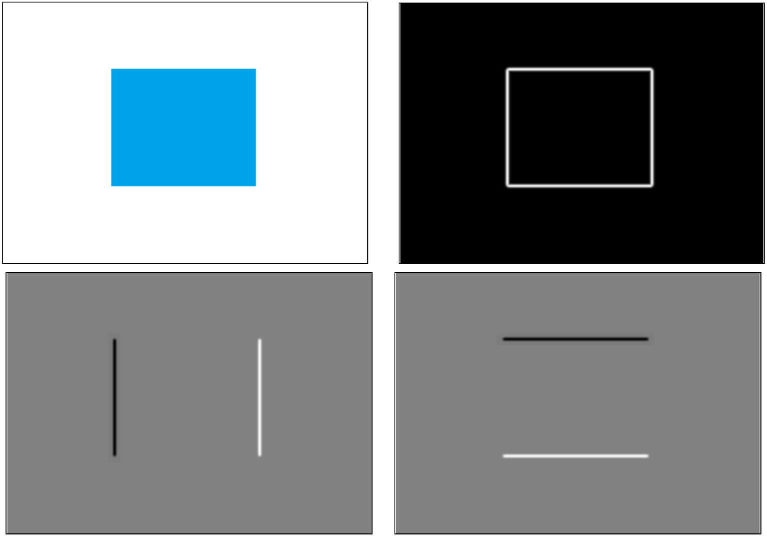
- The **gradient direction** (orientation of edge normal) is given by:

$$\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} \Big/ \frac{\partial f}{\partial x}\right)$$

- The **edge strength** is given by the magnitude $||\nabla f|| = \sqrt{(\frac{\partial f}{\partial x})^2 + (\frac{\partial f}{\partial y})^2}$

[Source: S. Seitz]

Example: Image Gradient



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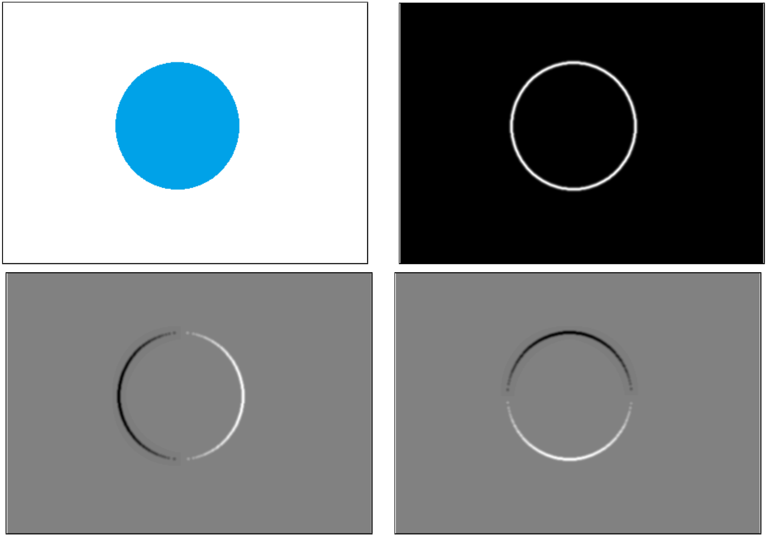

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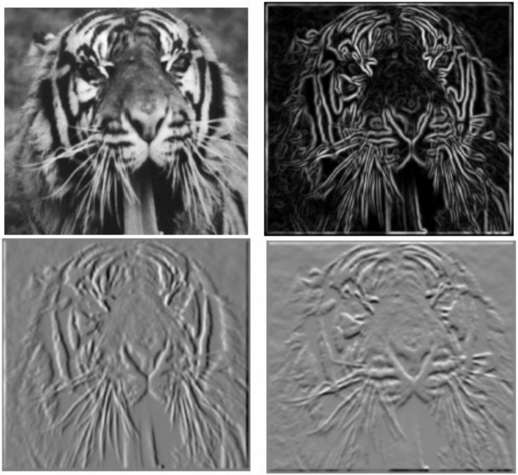
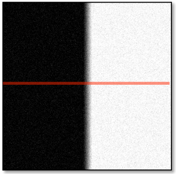


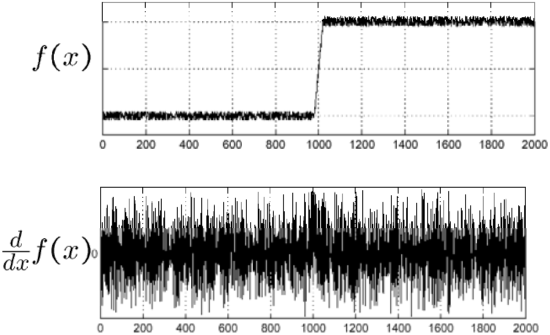
[Source: S. Lazebnik]

Effects of noise

- What if our image is noisy? What can we do?
- Consider a single row or column of the image.
- Plotting intensity as a function of position gives a signal.



Noisy input image



[Source: S. Seitz]

Effects of noise

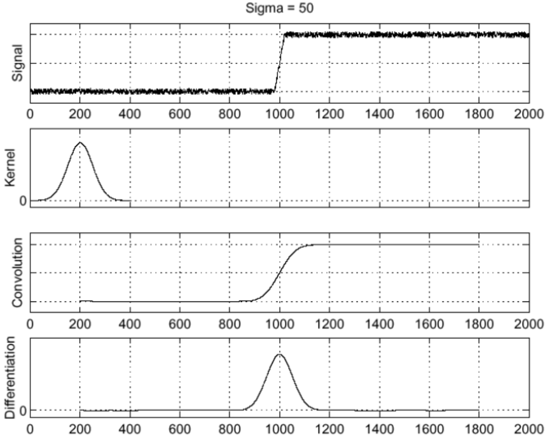
- Smooth first with h (e.g. Gaussian), and look for peaks in $\frac{\partial}{\partial x}(h * f)$.

f

h

$h \star f$

$\frac{\partial}{\partial x}(h \star f)$



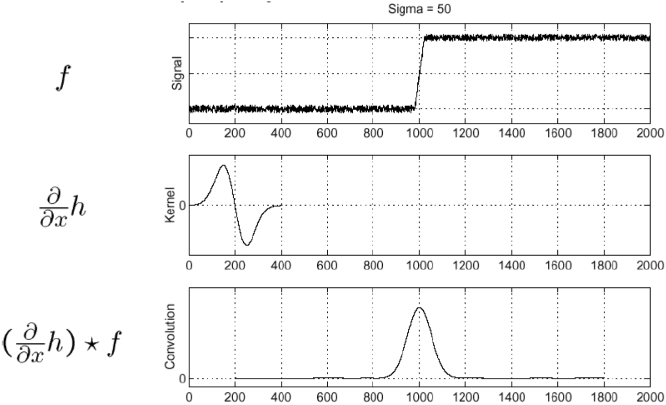
[Source: S. Seitz]

Derivative theorem of convolution

- Differentiation property of convolution

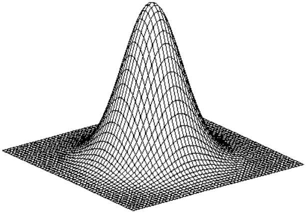
$$\frac{\partial}{\partial x}(h * f) = (\frac{\partial h}{\partial x}) * f = h * (\frac{\partial f}{\partial x})$$

- It saves one operation



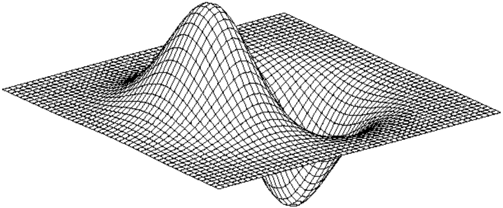
[Source: S. Seitz]

2D Edge Detection Filters



Gaussian

$h_\sigma(x, y) = \frac{1}{2\pi\sigma^2} \exp^{-\frac{u^2+v^2}{2\sigma^2}}$

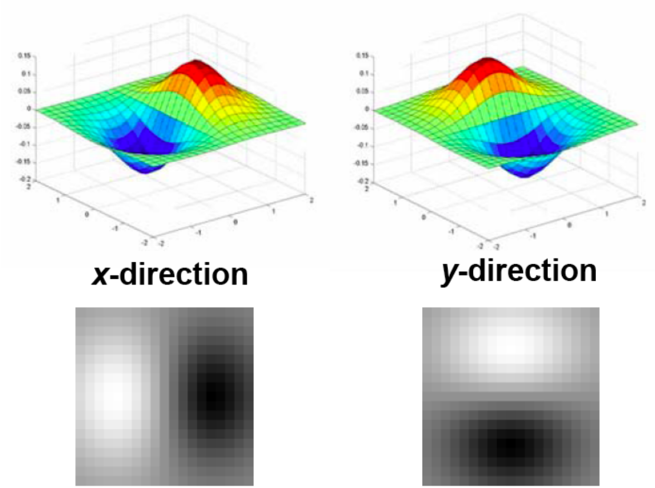


Derivative of Gaussian (x)

$\frac{\partial}{\partial x} h_\sigma(u, v)$

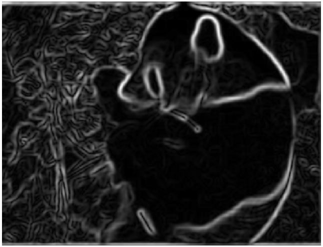
[Source: N. Snavely]

Derivative of Gaussians

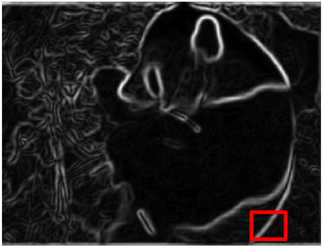


[Source: K. Grauman]

Example



- Applying the Gaussian derivatives to image



- Applying the Gaussian derivatives to image

Properties:

- Zero at a long distance from the edge
- Positive on both sides of the edge
- Highest value at some point in between, on the edge itself

Effect of σ on derivatives

The detected structures differ depending on the **Gaussian's scale parameter**:

- Larger values: detects edges of larger scale
- Smaller values: detects finer structures

σ = 1 pixel σ = 3 pixels

[Source: K. Grauman]

Locating Edges – Canny's Edge Detector

Let's take the most popular picture in computer vision: Lena

[Source: N. Snavely]

Locating Edges – Canny's Edge Detector

Figure: Canny's approach takes gradient magnitude

[Source: N. Snavely]

Locating Edges – Canny's Edge Detector

Figure: Thresholding

[Source: N. Snavely]

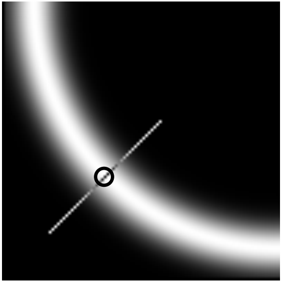
Locating Edges – Canny's Edge Detector

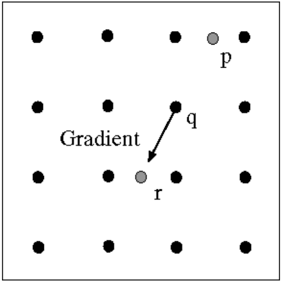
where is the edge?

Figure: Gradient magnitude

[Source: N. Snavely]

Non-Maxima Suppression





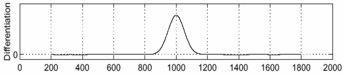


Figure: Gradient magnitude

- Check if pixel is local maximum along gradient direction
- If yes, take it

[Source: N. Snavely]

Finding Edges

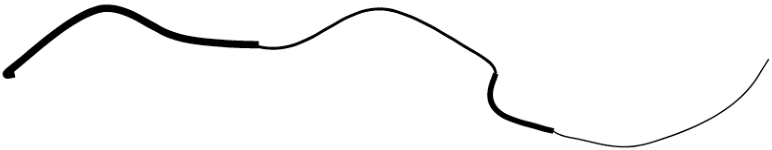
Problem: pixels along this edge didn't survive the thresholding

Figure: Problem with thresholding

[Source: K. Grauman]

Hysteresis thresholding

- Use a high threshold to start edge curves, and a low threshold to continue them



[Source: K. Grauman]

Hysteresis thresholding

original image

high threshold
(strong edges)

low threshold
(weak edges)

hysteresis threshold

[Source: L. Fei Fei]

Located Edges!

Figure: Thinning: Non-maxima suppression

[Source: N. Snavely]

Canny Edge Detector

Matlab: `edge(image,'canny')`

1. Filter image with derivative of Gaussian (horizontal and vertical directions)
2. Find magnitude and orientation of gradient
3. Non-maximum suppression
4. Linking and thresholding (hysteresis):
   - Define two thresholds: low and high
   - Use the high threshold to start edge curves and the low threshold to continue them

[Source: D. Lowe and L. Fei-Fei]

Canny Edge Detector

- large σ (in step 1) detects "large-scale" edges
- small σ detects fine edges

original   Canny with $\sigma = 1$ Canny with $\sigma = 2$

[Source: S. Seitz]

Canny Edge detector

- Still one of the most widely used edge detectors in computer vision
- J. Canny, A Computational Approach To Edge Detection, IEEE Trans. Pattern Analysis and Machine Intelligence, 8:679-714, 1986.
- Depends on several parameters: σ of the **blur** and the **thresholds**

[Adopted by: R. Urtasun]

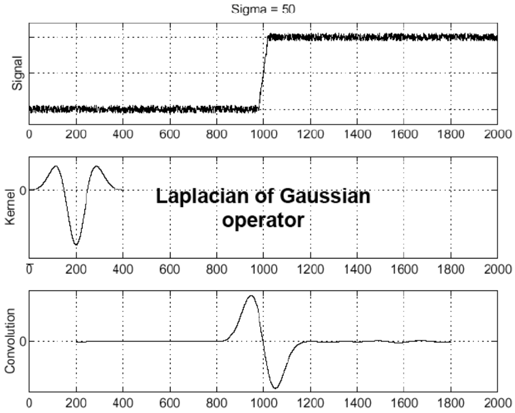
Another Way of Finding Edges: Laplacian of Gaussians

- Edge by detecting **zero-crossings** of bottom graph

f

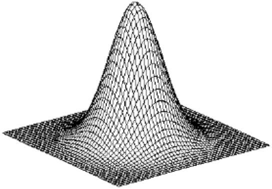
$\frac{\partial^2}{\partial x^2}h$

$(\frac{\partial^2}{\partial x^2}h) \star f$



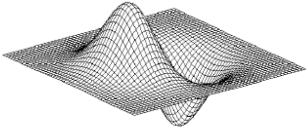
[Source: S. Seitz]

2D Edge Filtering



Gaussian

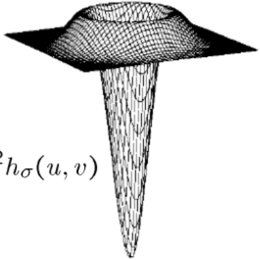
$$h_\sigma(u,v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{2\sigma^2}}$$



derivative of Gaussian

$$\frac{\partial}{\partial x} h_\sigma(u,v)$$

Laplacian of Gaussian



$$\nabla^2 h_\sigma(u,v)$$

with ∇^2 the Laplacian operator $\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$

[Source: S. Seitz]

Example

$\sigma = 1$ pixels $\sigma = 3$ pixels

- Applying the Laplacian operator to image

Example

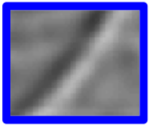
$\sigma = 1$ pixels

$\sigma = 3$ pixels

- Applying the Laplacian operator to image

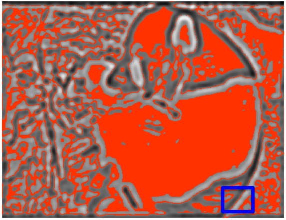
Properties:

- Zero at a long distance from the edge
- Positive on the lighter side of edge
- Negative on the darker side
- Zero at some point in between, on edge itself



Example

$\sigma = 1$ pixels

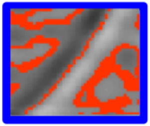


$\sigma = 3$ pixels

- Applying the Laplacian operator to image

Properties:

- Zero at a long distance from the edge
- Positive on the lighter side of edge
- Negative on the darker side
- Zero at some point in between, on edge itself



But Sanja, we are in 2022

This is "old-style" Computer Vision. We are now in the era of successful Machine Learning techniques.

Question: Can we use ML to do a better job at finding edges?

Summary – Stuff You Should Know

Not so good:

- **Horizontal image gradient**: Subtract intensity of left neighbor from pixel's intensity (filtering with $[-1, 1]$)
- **Vertical image gradient**: Subtract intensity of bottom neighbor from pixel's intensity (filtering with $[-1, 1]^T$)

Much better (more robust to noise):

- **Horizontal image gradient**: Apply derivative of Gaussian with respect to x to image (filtering!)
- **Vertical image gradient**: Apply derivative of Gaussian with respect to y to image
- **Magnitude of gradient**: compute the horizontal and vertical image gradients, square them, sum them, and $\sqrt{}$ the sum
- **Edges**: Locations in image where magnitude of gradient is high
- Phenomena that **causes** edges: rapid change in surface's normals, depth discontinuity, rapid changes in color, change in illumination

Summary – Stuff You Should Know

- **Properties of gradient's magnitude**:
  - Zero far away from edge
  - Positive on both sides of the edge
  - Highest value directly on the edge
  - Higher σ emphasizes larger structures
- **Canny's edge detector**:
  - Compute gradient's direction and magnitude
  - Non-maxima suppression
  - Thresholding at two levels and linking

Summary – Stuff You Should Know

Matlab functions:

- FSPECIAL: gives a few gradients filters (PREWITT, SOBEL, ROBERTS)
- SMOOTHGRADIENT: function to compute gradients with derivatives of Gaussians. Find it in Lecture's 3 code (check class webpage)
- EDGE: use EDGE(I,'CANNY') to detect edges with Canny's method, and EDGE(I,'LOG') for Laplacian method

Python functions (in skimage):

- SKIMAGE.FILTERS.(PREWITT/SOBEL/ROBERTS): gives a few gradients filters (PREWITT, SOBEL, ROBERTS)
- SCIPY.NDIMAGE.GAUSSIAN_FILTER(I, *order* = 1): compute image gradients with derivatives of Gaussians. Order 0 corresponds to convolution with a Gaussian kernel. A positive order implements convolution with a derivative of a Gaussian.
- SKIMAGE.FEATURE.CANNY: detect edges with Canny's method