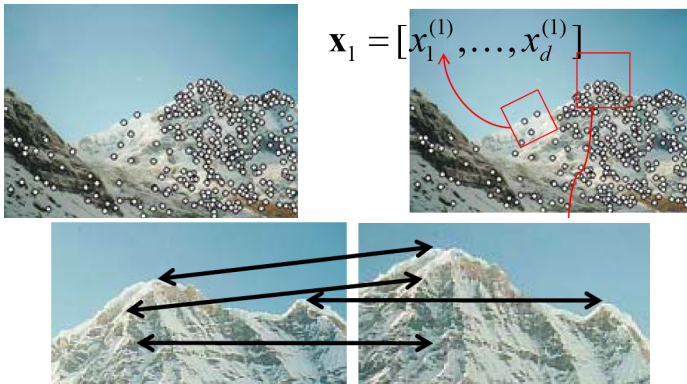Image Features:

Local Descriptors

Local Features

- **Detection**: Identify the interest points.
- **Description**: Extract a feature descriptor around each interest point.
- **Matching**: Determine correspondence between descriptors in two views.

$$\mathbf{x}_1 = [x_1^{(1)}, \dots, x_d^{(1)}]$$

[Source: K. Grauman]

e.g. scale,
translation,
rotation
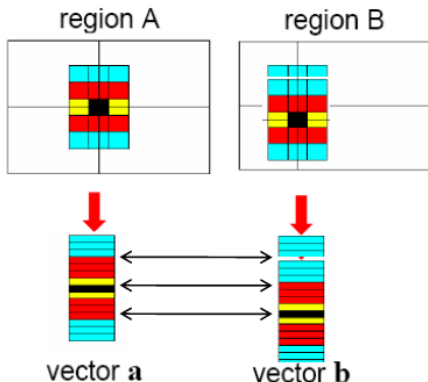
[Source: T. Tuytelaars]

Invariances

[Source: T. Tuytelaars]

What If We Just Took Pixels?

- The simplest way is to write down the list of intensities to form a feature vector, and normalize them (i.e., mean 0, variance 1).
- Why normalization?
- But this is very sensitive to even small shifts, rotations and any affine transformation.

Tones Of Better Options

- SIFT
- PCA-SIFT
- GLOH
- HOG
- SURF
- DAISY
- LBP
- Shape Contexts
- Color Histograms

Tones Of Better Options

- **SIFT**   **TODAY**
- PCA-SIFT
- GLOH
- HOG
- SURF
- DAISY
- LBP
- Shape Contexts
- Color Histograms

SIFT Descriptor [Lowe 2004]

- SIFT stands for Scale Invariant Feature Transform
- Invented by David Lowe, who also did DoG scale invariant interest points
- Actually in the same paper, which you should read:

> David G. Lowe
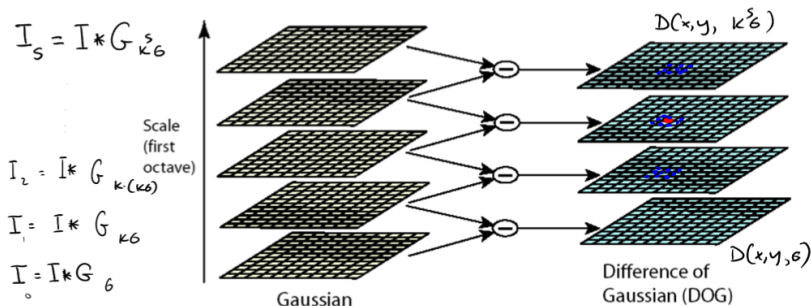> *Distinctive image features from scale-invariant keypoints*
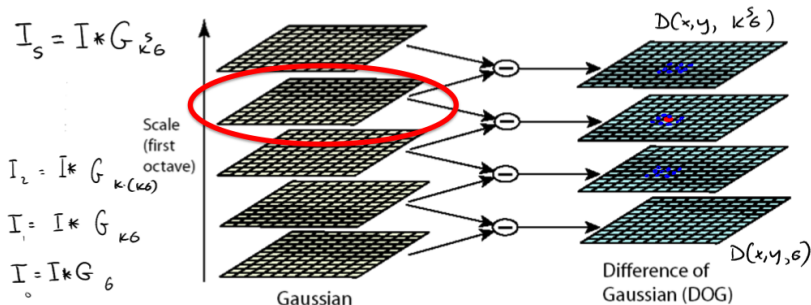> International Journal of Computer Vision, 2004
> Paper: http://www.cs.ubc.ca/~lowe/papers/ijcv04.pdf

(a) image gradients   (b) keypoint descriptor
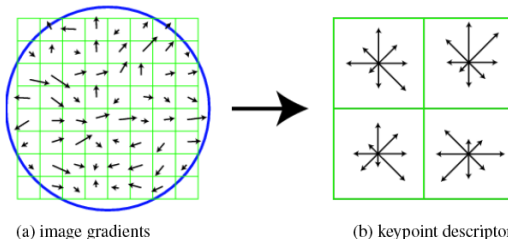
SIFT Descriptor

1. Our scale invariant interest point detector gives scale ρ for each keypoint

$$I_s = I * G_{\kappa^s_6}$$

$$I_2 = I * G_{\kappa \cdot (\kappa_6)}$$

$$I_1 = I * G_{\kappa_6}$$

$$I_\circ = I * G_6$$

Scale (first octave)

Gaussian

$$D(x, y, \kappa^s_6)$$

$$D(x, y, 6)$$

Difference of Gaussian (DOG)

[Adopted from: F. Flores-Mangas]

SIFT Descriptor

2. For each keypoint, we take the Gaussian-blurred image at corresponding scale ρ

$$I_s = I * G_{\kappa^s \sigma}$$

$$I_2 = I * G_{\kappa \cdot (\kappa \sigma)}$$

$$I_1 = I * G_{\kappa \sigma}$$

$$I_0 = I * G_\sigma$$

Scale (first octave)

Gaussian

$$D(x, y, \kappa^s \sigma)$$

$$D(x, y, \sigma)$$

Difference of Gaussian (DOG)

[Adopted from: F. Flores-Mangas]

SIFT Descriptor

3. Compute the gradient magnitude and orientation in neighborhood of each keypoint

$I * G_\rho$

Gaussian smoothed image at scale of keypoint

compute magnitude and orientation of gradients in neighborhood

16×16 pixel patch
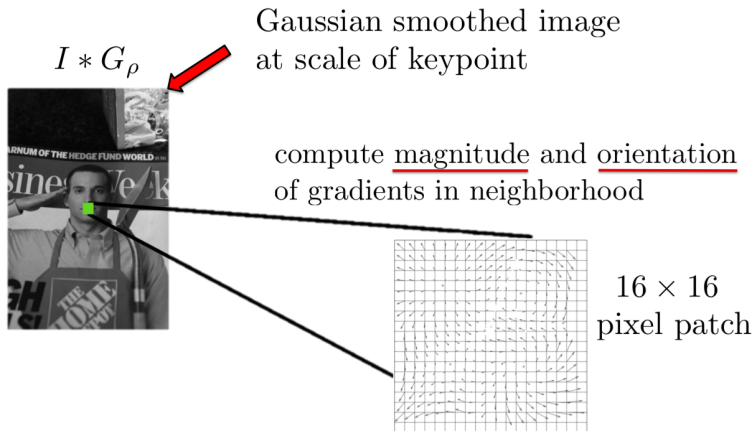
[Adopted from: F. Flores-Mangas]

SIFT Descriptor

3. Compute the gradient magnitude and orientation in neighborhood of each keypoint

magnitude of gradient:

$$|\nabla I(x,y)| = \sqrt{\left(\frac{\partial(I(x,y) * G_\rho)}{\partial x}\right)^2 + \left(\frac{\partial(I(x,y) * G_\rho)}{\partial y}\right)^2}$$
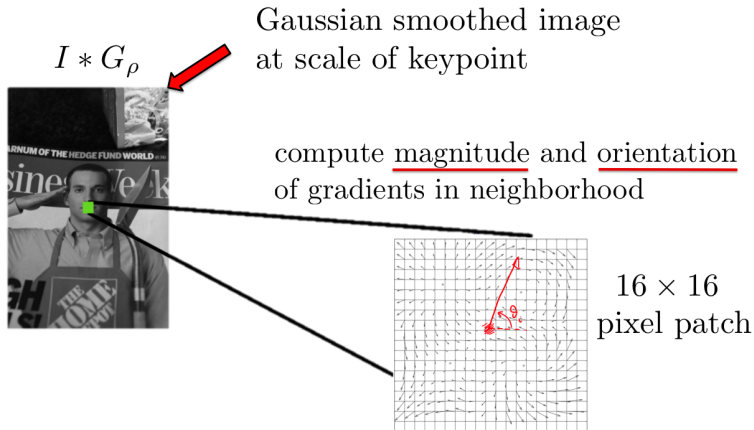
gradient orientation:

$$\theta(x,y) = \arctan\left(\frac{\partial I * G_\rho}{\partial y} / \frac{\partial I * G_\rho}{\partial x}\right)$$

(in case you forgot ;))

SIFT Descriptor

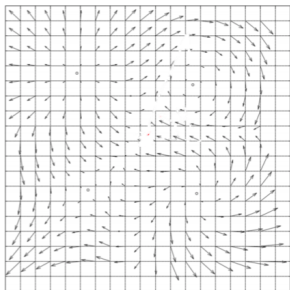④ Compute dominant orientation of each keypoint. How?

$I * G_\rho$

Gaussian smoothed image
at scale of keypoint

compute <u>magnitude</u> and <u>orientation</u>
of gradients in neighborhood

16×16
pixel patch
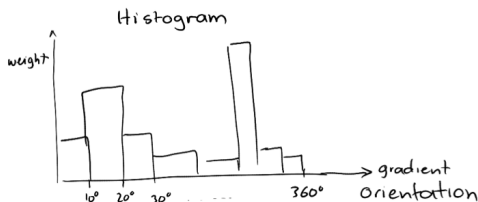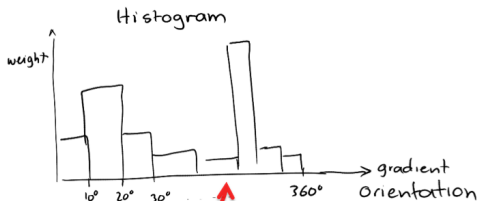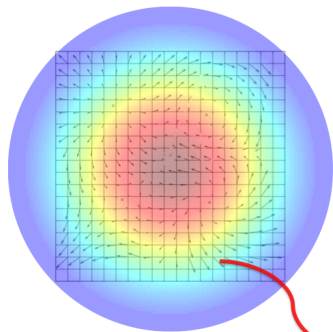
[Adopted from: F. Flores-Mangas]

SIFT Descriptor: Computing Dominant Orientation

- Compute a histogram of gradient orientations, each bin covers $10°$

16×16

compute histograms of orientations
by orientation increments of $10°$

[Adopted from: F. Flores-Mangas]

SIFT Descriptor: Computing Dominant Orientation

- Compute a histogram of gradient orientations, each bin covers $10°$
- Orientations closer to the keypoint center should contribute more

$G_{1.5\rho}$

weight influence of orientation
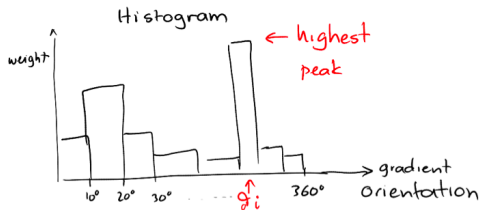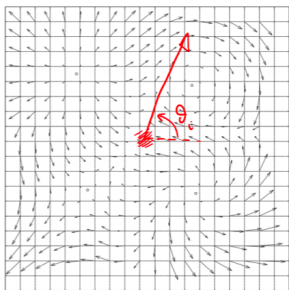based on distance from center

$$|\nabla I(x, y)| \cdot G_{1.5\rho}(d)$$

[Adopted from: F. Flores-Mangas]

SIFT Descriptor: Computing Dominant Orientation

- Compute a histogram of gradient orientations, each bin covers $10°$
- Orientations closer to the keypoint center should contribute more
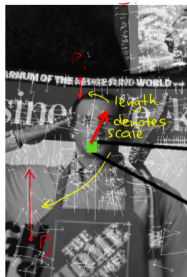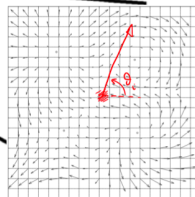- Orientation giving the peak in the histogram is the keypoint's orientation

16×16

[Adopted from: F. Flores-Mangas]

4. Compute dominant orientation

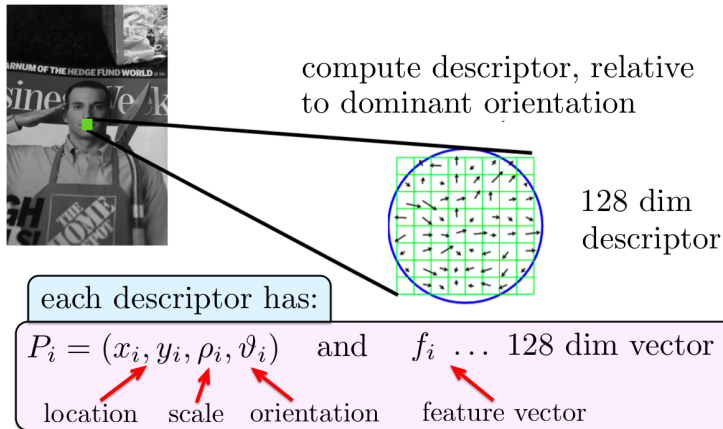compute magnitude and orientation
of gradients in neighborhood

16×16
pixel patch

[Adopted from: F. Flores-Mangas]

5. Compute a 128 dimensional descriptor: 4×4 grid, each cell is a histogram of 8 orientation bins relative to dominant orientation

compute descriptor, relative to dominant orientation

128 dim descriptor

each descriptor has:

$P_i = (x_i, y_i, \rho_i, \vartheta_i)$ and f_i ... 128 dim vector

location   scale   orientation   feature vector

[Adopted from: F. Flores-Mangas]

- Compute the orientations **relative** to the **dominant orientation**
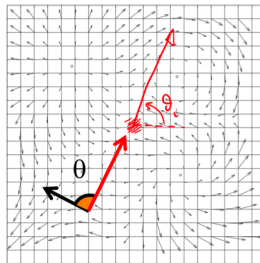
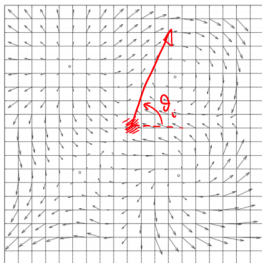16×16 patch
centered in (x_i, y_i)

[Adopted from: F. Flores-Mangas]

SIFT Descriptor: Computing the Feature Vector

- Compute the orientations **relative** to the **dominant orientation**

16×16 patch
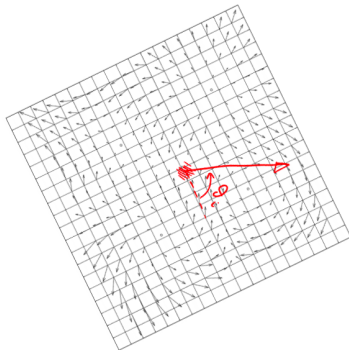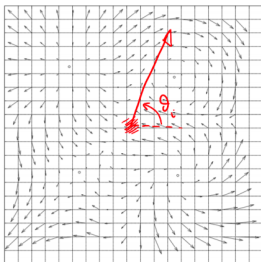centered in (x_i, y_i)

[Adopted from: F. Flores-Mangas]

SIFT Descriptor: Computing the Feature Vector

- Compute the orientations **relative** to the **dominant orientation**
- Form a 4 × 4 grid. For each grid cell compute a histogram of orientations for 8 orientation bins spaced apart by 45°

16×16 patch
centered in (x_i, y_i)

SIFT descriptor

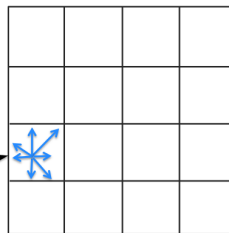compute histogram of orientations
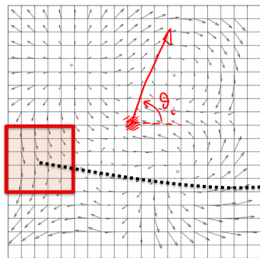this time 8 bins spaced by 45°

[Adopted from: F. Flores-Mangas]

SIFT Descriptor: Computing the Feature Vector

- Compute the orientations **relative** to the **dominant orientation**
- Form a 4 × 4 grid. For each grid cell compute a histogram of orientations for 8 orientation bins spaced apart by 45°

16×16 patch
centered in (x_i, y_i)

SIFT descriptor

again weigh contributions
this time: $|\nabla I(x, y)| \cdot G_{0.5\rho}$

[Adopted from: F. Flores-Mangas]
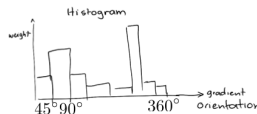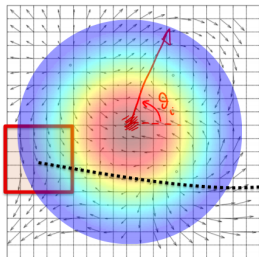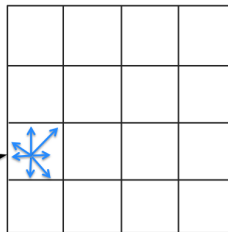
SIFT Descriptor: Computing the Feature Vector

- Compute the orientations **relative** to the **dominant orientation**
- Form a 4 × 4 grid. For each grid cell compute a histogram of orientations for 8 orientation bins spaced apart by 45°
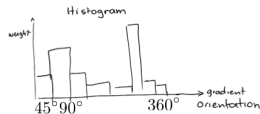- Form the 128 dimensional feature vector

16×16 patch
centered in (x_i, y_i)

SIFT descriptor

$f_i =$
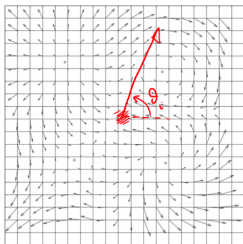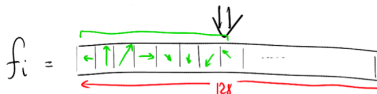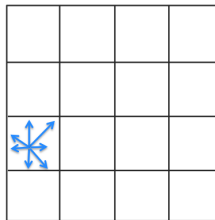
[Adopted from: F. Flores-Mangas]

SIFT Descriptor: Post-processing

- The resulting 128 non-negative values form a **raw version** of the SIFT descriptor vector.
- To reduce the **effects of contrast or gain** (additive variations are already removed by the gradient), the 128-D vector is normalized to unit length: $f_i = f_i / ||f_i||$

SIFT Descriptor: Post-processing

- The resulting 128 non-negative values form a **raw version** of the SIFT descriptor vector.

- To reduce the **effects of contrast or gain** (additive variations are already removed by the gradient), the 128-D vector is normalized to unit length: $f_i = f_i / ||f_i||$

- To further make the descriptor robust to other **photometric variations**, values are clipped to 0.2 and the resulting vector is once again renormalized to unit length.

SIFT Descriptor: Post-processing

- The resulting 128 non-negative values form a **raw version** of the SIFT descriptor vector.
- To reduce the **effects of contrast or gain** (additive variations are already removed by the gradient), the 128-D vector is normalized to unit length: $f_i = f_i / ||f_i||$
- To further make the descriptor robust to other **photometric variations**, values are clipped to 0.2 and the resulting vector is once again renormalized to unit length.
- Great engineering effort!

SIFT Descriptor: Post-processing

- The resulting 128 non-negative values form a **raw version** of the SIFT descriptor vector.
- To reduce the **effects of contrast or gain** (additive variations are already removed by the gradient), the 128-D vector is normalized to unit length: $f_i = f_i / ||f_i||$
- To further make the descriptor robust to other **photometric variations**, values are clipped to 0.2 and the resulting vector is once again renormalized to unit length.
- Great engineering effort!
- What is SIFT invariant to?

SIFT Descriptor: Post-processing

- The resulting 128 non-negative values form a **raw version** of the SIFT descriptor vector.
- To reduce the **effects of contrast or gain** (additive variations are already removed by the gradient), the 128-D vector is normalized to unit length: $f_i = f_i / ||f_i||$
- To further make the descriptor robust to other **photometric variations**, values are clipped to 0.2 and the resulting vector is once again renormalized to unit length.
- Great engineering effort!
- What is SIFT invariant to?

Properties of SIFT

Invariant to:

- Scale
- Rotation

Partially invariant to:

- Illumination changes (sometimes even day vs. night)
- Camera viewpoint (up to about 60 degrees of out-of-plane rotation)
- Occlusion, clutter (why?)

Also important:

- Fast and efficient – can run in real time
- Lots of code available

Examples

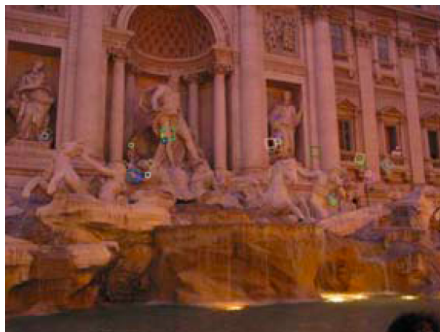Figure: Matching in day / night under viewpoint change
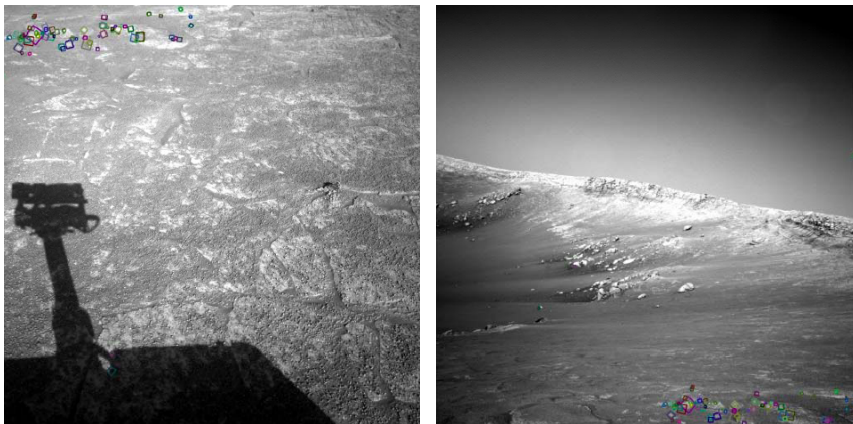
[Source: S. Seitz]

Figure: NASA Mars Rover images with SIFT feature matches

[Source: N. Snavely]

PCA-SIFT

- The dimensionality of SIFT is pretty high, i.e., 128D for each keypoint

- Reduce the dimensionality using linear dimensionality reduction

- In this case, principal component analysis (PCA)

- Use 10D or so descriptor

[Source: R. Urtasun]

Gradient location-orientation histogram (GLOH)

- Developed by Mikolajczyk and Schmid (2005): variant of SIFT that uses a log-polar binning structure instead of the four quadrants.
- The spatial bins are 11, and 15, with eight angular bins (except for the central region), for a total of 17 spatial bins and 16 orientation bins.
- The 272D histogram is then projected onto a 128D descriptor using PCA trained on a large database.
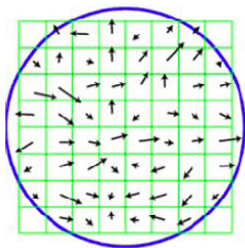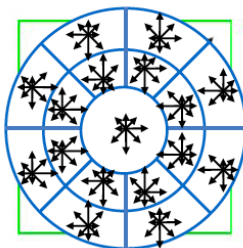
(a) image gradients   (b) keypoint descriptor

[Source: R. Szeliski]

Other Descriptors

- SURF
- DAISY
- LBP
- HOG
- Shape Contexts
- Color Histograms

Local Features

- **Detection**: Identify the interest points.
- **Description**: Extract feature descriptor around each interest point.
- **Matching**: Determine correspondence between descriptors in two views.

$$\mathbf{x}_1 = [x_1^{(1)}, \ldots, x_d^{(1)}]$$
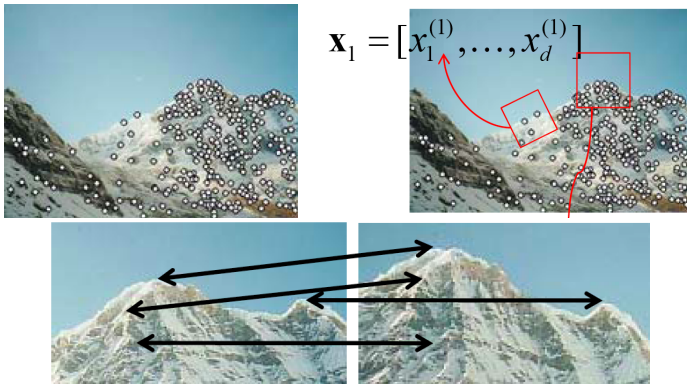
[Source: K. Grauman]

Image Features:

Matching the Local Descriptors

Matching the Local Descriptors

Once we have extracted keypoints and their descriptors, we want to match the features between pairs of images.

- Ideally a match is a correspondence between a local part of the object on one image to the same local part of the object in another image

- How should we compute a match?

Figure: Images from K. Grauman

Matching the Local Descriptors

Once we have extracted keypoints and their descriptors, we want to match the features between pairs of images.

- Ideally a match is a correspondence between a local part of the object on one image to the same local part of the object in another image
- How should we compute a match?

Figure: Images from K. Grauman

Matching the Local Descriptors

- Simple: **Compare them all**, compute Euclidean distance

Matching the Local Descriptors

- Simple: **Compare them all**, compute Euclidean distance

Matching the Local Descriptors

- Find closest match (min distance). How do we know if match is **reliable**?

$$\|f_1 - f'_1\|$$
$$\|f_1 - f'_2\|$$
$$\|f_1 - f'_3\|$$

min (closest match)

$$\|f_1 - f'_{k'}\|$$
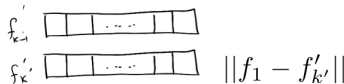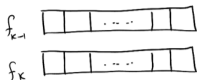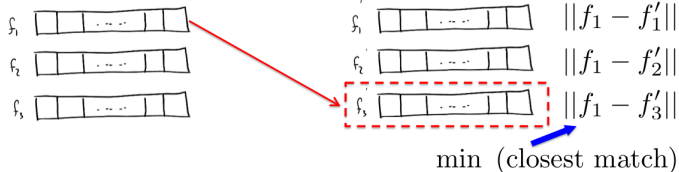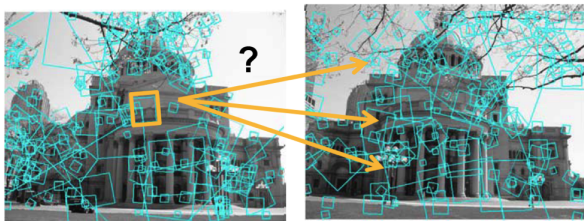
Matching the Local Descriptors

- Find also the second closest match. Match reliable if first distance "much" smaller than second distance

$$\min \text{ (closest match)}$$

second closest match
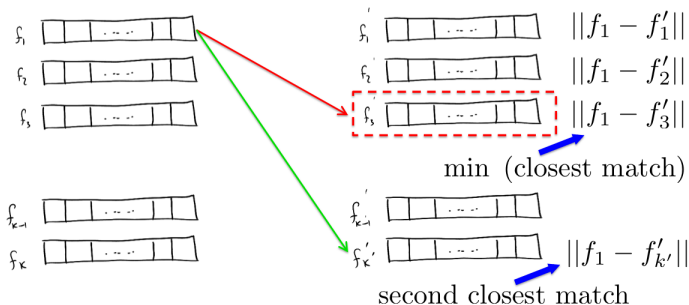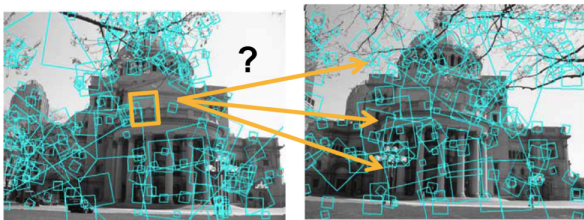
Matching the Local Descriptors

- Compute the ratio:

$$\phi_i = \frac{||f_i - f'^*_i||}{||f_i - f'^{**}_i||}$$

where f'^*_i is the closest and f'^{**}_i second closest match to f_i.
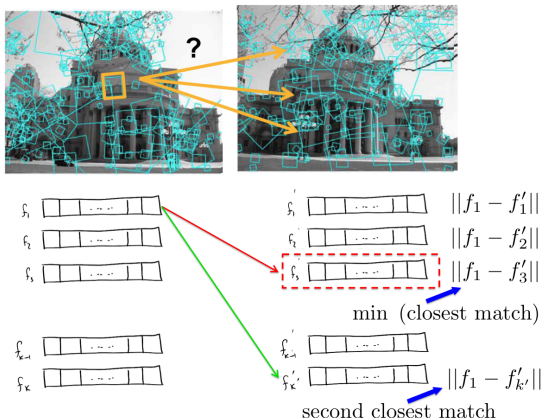
Which Threshold to Use?

- Setting the threshold too high results in too many false positives, i.e., incorrect matches being returned.

- Setting the threshold too low results in too many false negatives, i.e., too many correct matches being missed
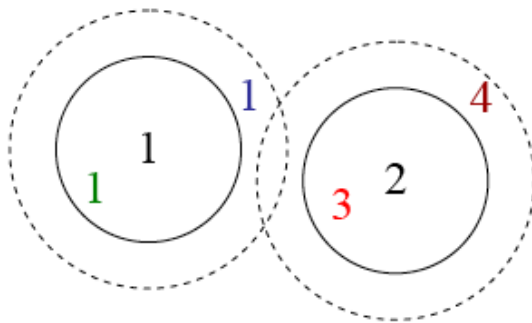
Figure: Images from R. Szeliski

Which Threshold to Use?

- Threshold ratio of nearest to 2nd nearest descriptor
- Typically: $\phi_i < 0.8$
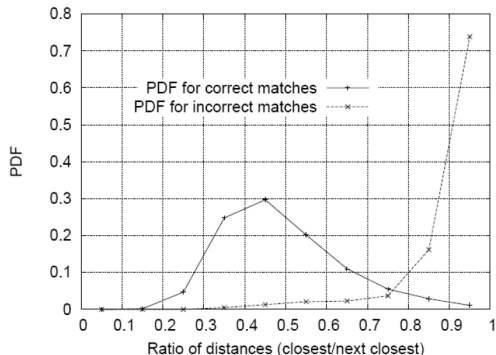
Figure: Images from D. Lowe

[Source: K. Grauman]

Applications of Local Invariant Features

- Wide baseline stereo
- Motion tracking
- Panorama stitching
- Mobile robot navigation
- 3D reconstruction
- Recognition
- Retrieval

[Source: K. Grauman]

[Source: T. Tuytelaars]

Schmid and Mohr 1997

Sivic and Zisserman, 2003

Rothganger et al. 2003

Lowe 2002

[Source: K. Grauman]

Figure: Images from J. Pilet

Now What

- Now we know how to extract scale and rotation invariant features
- We even know how to match features across images
- Can we use this to find Waldo in an even more sneaky scenario?

- Now we know how to extract scale and rotation invariant features
- We even know how to match features across images
- Can we use this to find Waldo in an even more sneaky scenario?

Waldo on the road

template

Now What

- Now we know how to extract scale and rotation invariant features
- We even know how to match features across images
- Can we use this to find Waldo in an even more sneaky scenario?

template

He comes closer... We know how to solve this

Now What

- Now we know how to extract scale and rotation invariant features
- We even know how to match features across images
- Can we use this to find Waldo in an even more sneaky scenario?

template

Someone takes a (weird) picture of him!

Find My DVD!

- More interesting: If we have DVD covers (e.g., from Amazon), can we match them to DVDs in real scenes?
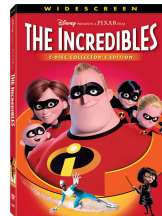
Matching Planar Objects In New Viewpoints

$T?$
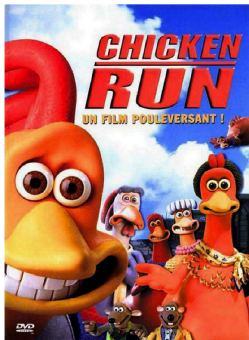
- Rectangle goes to a parallelogram (almost but not really, but let's believe that for now)

$T?$
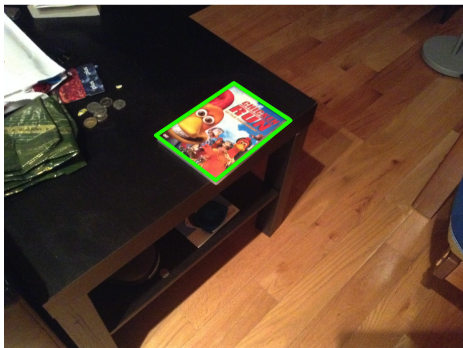
All 2D Linear Transformations

Linear transformations are combinations of É

- Scale,
- Rotation
- Shear
- Mirror

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

[Source: N. Snavely]

All 2D Linear Transformations

Properties of linear transformations:

- Origin maps to origin
- Lines map to lines

All 2D Linear Transformations

Properties of linear transformations:

- Origin maps to origin
- Lines map to lines
- Parallel lines remain parallel

All 2D Linear Transformations

Properties of linear transformations:

- Origin maps to origin
- Lines map to lines
- Parallel lines remain parallel
- Ratios are preserved

All 2D Linear Transformations

Properties of linear transformations:

- Origin maps to origin

- Lines map to lines

- Parallel lines remain parallel

- Ratios are preserved

- Closed under composition

All 2D Linear Transformations

Properties of linear transformations:

- Origin maps to origin

- Lines map to lines

- Parallel lines remain parallel

- Ratios are preserved

- Closed under composition

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} \begin{bmatrix} i & j \\ k & l \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

All 2D Linear Transformations

Properties of linear transformations:

- Origin maps to origin
- Lines map to lines
- Parallel lines remain parallel
- Ratios are preserved
- Closed under composition

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} \begin{bmatrix} i & j \\ k & l \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

What about the translation?

[Source: N. Snavely]

All 2D Linear Transformations

Properties of linear transformations:

- Origin maps to origin
- Lines map to lines
- Parallel lines remain parallel
- Ratios are preserved
- Closed under composition

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} \begin{bmatrix} i & j \\ k & l \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

What about the translation?

[Source: N. Snavely]

Affine Transformations

Affine transformations are combinations of:

- Linear transformations, and
- Translations

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} e \\ f \end{bmatrix}$$

same as:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b & e \\ c & d & f \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Affine Transformations

Affine transformations are combinations of:

- Linear transformations, and
- Translations

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} e \\ f \end{bmatrix}$$

same as:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b & e \\ c & d & f \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Affine Transformations

Affine transformations are combinations of:

- Linear transformations, and
- Translations

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b & e \\ c & d & f \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Properties of affine transformations:

- Origin does not necessarily map to origin
- Lines map to lines
- Parallel lines remain parallel
- Ratios are preserved
- Closed under composition
- Rectangles go to parallelograms

[Source: N. Snavely]

2D Image Tranformations
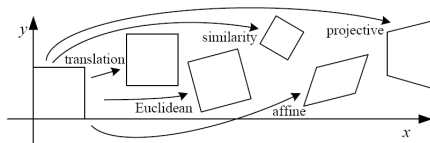
Transformation	Matrix	# DoF	Preserves	Icon
translation	$\left[\ \boldsymbol{I}\ \vert\ \boldsymbol{t}\ \right]_{2\times3}$	2	orientation	
rigid (Euclidean)	$\left[\ \boldsymbol{R}\ \vert\ \boldsymbol{t}\ \right]_{2\times3}$	3	lengths	
similarity	$\left[\ s\boldsymbol{R}\ \vert\ \boldsymbol{t}\ \right]_{2\times3}$	4	angles	
affine	$\left[\ \boldsymbol{A}\ \right]_{2\times3}$	6	parallelism	
projective	$\left[\ \tilde{\boldsymbol{H}}\ \right]_{3\times3}$	8	straight lines	

- These transformations are a nested set of groups
- Closed under composition and inverse is a member

What Transformation Happened to My DVD?

- Affine transformation approximates viewpoint changes for roughly **planar objects** and roughly **orthographic cameras** (more about these later in class)
- DVD went affine!
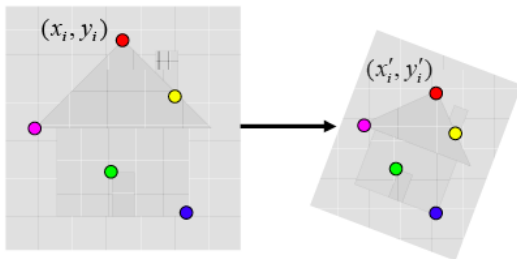
Computing the (Affine) Transformation

Given a set of matches between images I and J

- How can we compute the affine transformation A from I to J?
  - Find transform A that best "agrees" with the matches

Computing the (Affine) Transformation

Given a set of matches between images I and J

- How can we compute the affine transformation A from I to J?
- Find transform A that best "agrees" with the matches
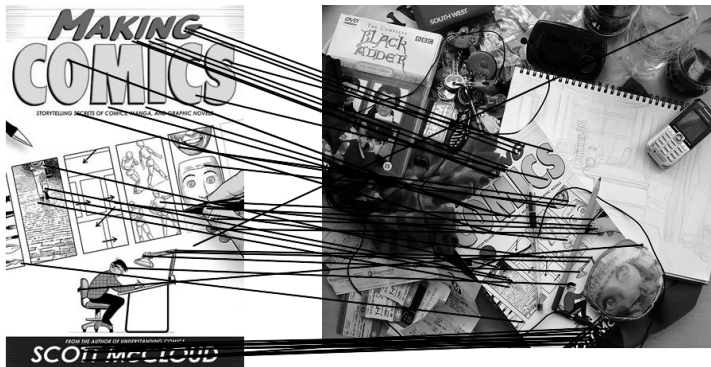
[Source: N. Snavely]

Computing the Affine Transformation
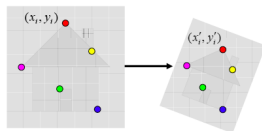
- Let (x_i, y_i) be a point on the reference (model) image, and (x'_i, y'_i) its match in the test image

- An affine transformation A maps (x_i, y_i) to (x'_i, y'_i):

$$\begin{bmatrix} x'_i \\ y'_i \end{bmatrix} = \begin{bmatrix} a & b & e \\ c & d & f \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}$$

Computing the Affine Transformation
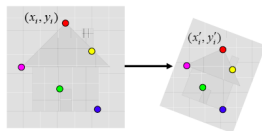
- Let (x_i, y_i) be a point on the reference (model) image, and (x_i', y_i') its match in the test image
- An affine transformation A maps (x_i, y_i) to (x_i', y_i'):

$$\begin{bmatrix} x_i' \\ y_i' \end{bmatrix} = \begin{bmatrix} a & b & e \\ c & d & f \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}$$

- We can rewrite this into a simple linear system:

$$\begin{bmatrix} x_i & y_i & 0 & 0 & 1 & 0 \\ 0 & 0 & x_i & y_i & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix} = \begin{bmatrix} x_i' \\ y_i' \end{bmatrix}$$

Computing the Affine Transformation

- But we have many matches:

$$
\underbrace{\begin{bmatrix} & & \vdots & & & \\ x_i & y_i & 0 & 0 & 1 & 0 \\ 0 & 0 & x_i & y_i & 0 & 1 \\ & & \vdots & & & \end{bmatrix}}_{\mathbf{P}} \underbrace{\begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix}}_{\mathbf{a}} = \underbrace{\begin{bmatrix} \vdots \\ x_i' \\ y_i' \\ \vdots \end{bmatrix}}_{\mathbf{P}'}
$$

- For each match we have two more equations

Computing the Affine Transformation

- But we have many matches:

$$
\underbrace{\begin{bmatrix} & & \vdots & & & \\ x_i & y_i & 0 & 0 & 1 & 0 \\ 0 & 0 & x_i & y_i & 0 & 1 \\ & & \vdots & & & \end{bmatrix}}_{\mathbf{P}} \underbrace{\begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix}}_{\mathbf{a}} = \underbrace{\begin{bmatrix} \vdots \\ x_i' \\ y_i' \\ \vdots \end{bmatrix}}_{\mathbf{P}'}
$$

- For each match we have two more equations
- How many matches do we need to compute A?

Computing the Affine Transformation

- But we have many matches:

$$\underbrace{\begin{bmatrix} & & \vdots & & & \\ x_i & y_i & 0 & 0 & 1 & 0 \\ 0 & 0 & x_i & y_i & 0 & 1 \\ & & \vdots & & & \end{bmatrix}}_{\mathbf{P}} \underbrace{\begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix}}_{\mathbf{a}} = \underbrace{\begin{bmatrix} \vdots \\ x_i' \\ y_i' \\ \vdots \end{bmatrix}}_{\mathbf{P}'}$$

- For each match we have two more equations
- How many matches do we need to compute A?

Computing the Affine Transformation

- But we have many matches:

$$\underbrace{\begin{bmatrix} & & \vdots & & & \\ x_i & y_i & 0 & 0 & 1 & 0 \\ 0 & 0 & x_i & y_i & 0 & 1 \\ & & \vdots & & & \end{bmatrix}}_{\mathbf{P}} \underbrace{\begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix}}_{\mathbf{a}} = \underbrace{\begin{bmatrix} \vdots \\ x_i' \\ y_i' \\ \vdots \end{bmatrix}}_{\mathbf{P}'}$$

- For each match we have two more equations
- How many matches do we need to compute A?
- 6 parameters $\rightarrow$ 3 matches
- But the more, the better (more reliable)
- How do we compute A?

Computing the Affine Transformation

$$\underbrace{\begin{bmatrix} & & \vdots & & & \\ x_i & y_i & 0 & 0 & 1 & 0 \\ 0 & 0 & x_i & y_i & 0 & 1 \\ & & \vdots & & & \end{bmatrix}}_{\mathbf{P}} \underbrace{\begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix}}_{\mathbf{a}} = \underbrace{\begin{bmatrix} \vdots \\ x_i' \\ y_i' \\ \vdots \end{bmatrix}}_{\mathbf{P}'}$$

- If we have 3 matches, then computing A is really easy:

$$\mathbf{a} = \mathbf{P}^{-1}\mathbf{P}'$$

Computing the Affine Transformation

$$\underbrace{\begin{bmatrix} & & \vdots & & & \\ x_i & y_i & 0 & 0 & 1 & 0 \\ 0 & 0 & x_i & y_i & 0 & 1 \\ & & \vdots & & & \end{bmatrix}}_{\mathbf{P}} \underbrace{\begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix}}_{\mathbf{a}} = \underbrace{\begin{bmatrix} \vdots \\ x_i' \\ y_i' \\ \vdots \end{bmatrix}}_{\mathbf{P}'}$$

- If we have 3 matches, then computing A is really easy:

$$\mathbf{a} = \mathbf{P}^{-1}\mathbf{P}'$$

- If we have more than 3, then we do **least-squares estimation**:

$$\min_{a,b,\cdots,f} ||\mathbf{P}\mathbf{a} - \mathbf{P}'||_2^2$$

Computing the Affine Transformation

$$\underbrace{\begin{bmatrix} & & \vdots & & & \\ x_i & y_i & 0 & 0 & 1 & 0 \\ 0 & 0 & x_i & y_i & 0 & 1 \\ & & \vdots & & & \end{bmatrix}}_{\mathbf{P}} \underbrace{\begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix}}_{\mathbf{a}} = \underbrace{\begin{bmatrix} \vdots \\ x_i' \\ y_i' \\ \vdots \end{bmatrix}}_{\mathbf{P}'}$$

- If we have 3 matches, then computing A is really easy:

$$\mathbf{a} = \mathbf{P}^{-1}\mathbf{P}'$$

- If we have more than 3, then we do **least-squares estimation**:

$$\min_{a,b,\cdots,f} ||\mathbf{P}\mathbf{a} - \mathbf{P}'||_2^2$$

- Which has a closed form solution:

$$\mathbf{a} = (\mathbf{P}^T\mathbf{P})^{-1}\mathbf{P}^T\mathbf{P}'$$

Image Alignment Algorithm: Affine Case

Given images I and J

1. Compute image features for I and J

2. Match features between I and J

3. Compute affine transformation A between I and J using least squares on the set of matches

Given images I and J

1. Compute image features for I and J

2. Match features between I and J

3. Compute affine transformation A between I and J using least squares on the set of matches

Is there a problem with this?

[Source: N. Snavely]

Robustness

outliers

inliers
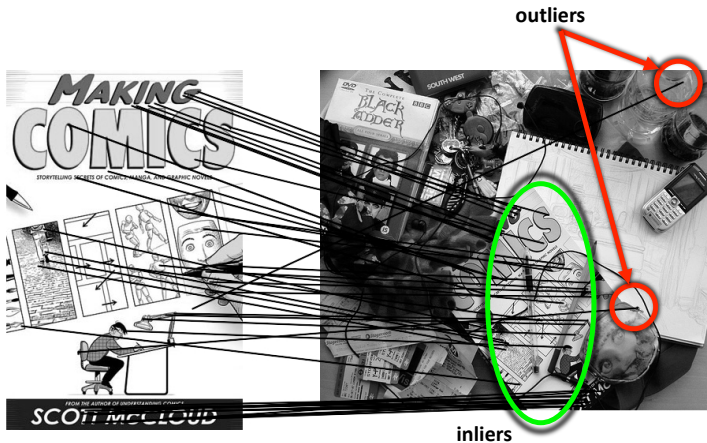
[Source: N. Snavely]

Simple Case

(This example is unrelated to the object matching example, but it nicely shows how to robustify estimation)

- Let's consider a simpler example ... Fit a line to the points below!
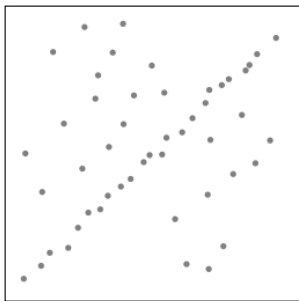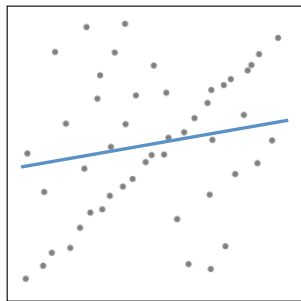
Problem: Fit a line to these datapoints   Least squares fit

Simple Case

(This example is unrelated to the object matching example, but it nicely shows how to robustify estimation)

- Let's consider a simpler example ... Fit a line to the points below!
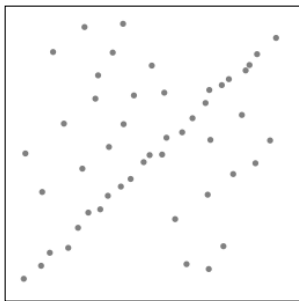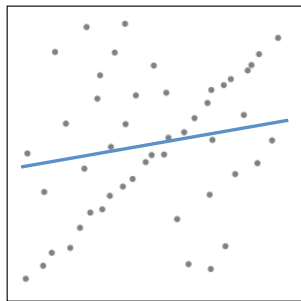
Problem: Fit a line to these datapoints   Least squares fit

- How can we fix this?

[Source: N. Snavely]

Simple Idea: RANSAC

- Take the minimal number of points to compute what we want. In the line example, two points (in our affine example, three matches)

Simple Idea: RANSAC

- Take the minimal number of points to compute what we want. In the line example, two points (in our affine example, three matches)

- By "take" we mean choose at random from all points

Simple Idea: RANSAC

- Take the minimal number of points to compute what we want. In the line example, two points (in our affine example, three matches)

- By "take" we mean choose at random from all points

- Fit a line to the selected pair of points

Simple Idea: RANSAC

- Take the minimal number of points to compute what we want. In the line example, two points (in our affine example, three matches)

- By "take" we mean choose at random from all points

- Fit a line to the selected pair of points

- Count the number of all points that "agree" with the line: We call the agreeing points **inliers**

Simple Idea: RANSAC

- Take the minimal number of points to compute what we want. In the line example, two points (in our affine example, three matches)

- By "take" we mean choose at random from all points

- Fit a line to the selected pair of points

- Count the number of all points that "agree" with the line: We call the agreeing points **inliers**

- "Agree" = within a small distance of the line

Simple Idea: RANSAC

- Take the minimal number of points to compute what we want. In the line example, two points (in our affine example, three matches)

- By "take" we mean choose at random from all points

- Fit a line to the selected pair of points

- Count the number of all points that "agree" with the line: We call the agreeing points **inliers**

- "Agree" = within a small distance of the line

- Repeat this many times, remember the number of inliers for each trial

- Among several trials, select the one with the largest number of inliers

This procedure is called **RA**ndom **SA**mple **C**onsensus

RANSAC for Line Fitting Example
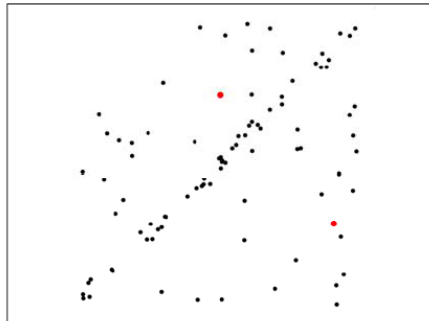
1. Randomly select minimal subset of points

[Source: R. Raguram]

RANSAC for Line Fitting Example

1. Randomly select minimal subset of points
2. Hypothesize a model
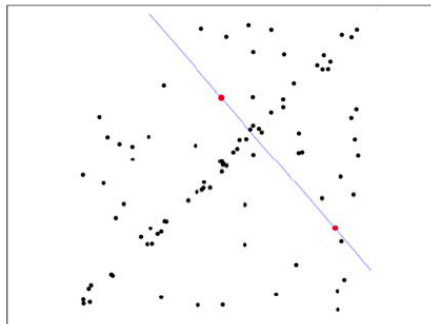
[Source: R. Raguram]

RANSAC for Line Fitting Example

1. Randomly select minimal subset of points

2. Hypothesize a model

3. Compute error function
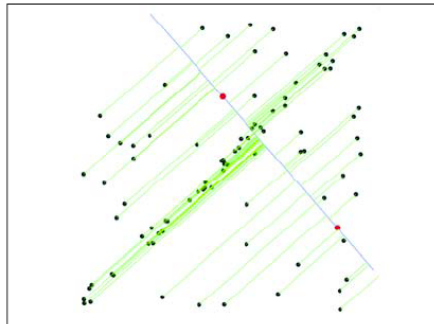
[Source: R. Raguram]

RANSAC for Line Fitting Example

1. Randomly select minimal subset of points
2. Hypothesize a model
3. Compute error function
4. Select points consistent with model

[Source: R. Raguram]

RANSAC for Line Fitting Example

1. Randomly select minimal subset of points

2. Hypothesize a model

3. Compute error function

4. Select points consistent with model

5. Repeat hypothesize and verify loop

[Source: R. Raguram]

RANSAC for Line Fitting Example

1. Randomly select minimal subset of points

2. Hypothesize a model

3. Compute error function

4. Select points consistent with model

5. Repeat hypothesize and verify loop

[Source: R. Raguram]

RANSAC for Line Fitting Example

1. Randomly select minimal subset of points

2. Hypothesize a model

3. Compute error function

4. Select points consistent with model

5. Repeat hypothesize and verify loop

6. Choose model with largest set of inliers

[Source: R. Raguram]

[Source: N. Snavely]

Select *one* match at random, count *inliers*

[Source: N. Snavely]

Select another match at random, count *inliers*
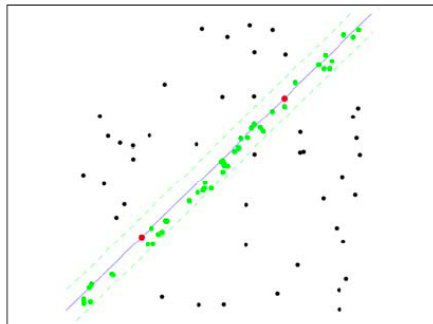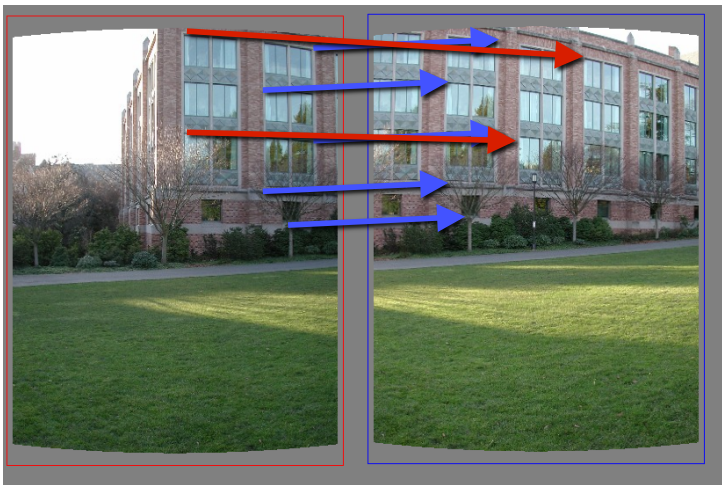
[Source: N. Snavely]

RAndom SAmple Consensus
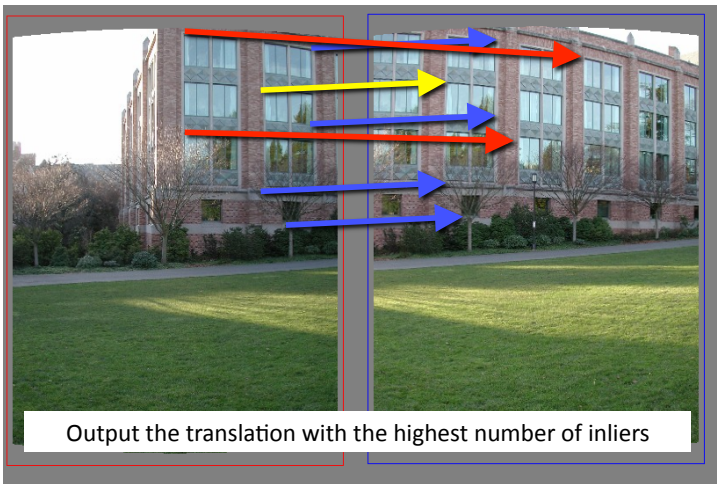
Output the translation with the highest number of inliers

[Source: N. Snavely]

RANSAC

- All the inliers will agree with each other on the translation vector; the (hopefully small) number of outliers will (hopefully) disagree with each other

RANSAC

- All the inliers will agree with each other on the translation vector; the (hopefully small) number of outliers will (hopefully) disagree with each other
- RANSAC only has guarantees if there are $< 50\%$ outliers

RANSAC

- All the inliers will agree with each other on the translation vector; the (hopefully small) number of outliers will (hopefully) disagree with each other
- RANSAC only has guarantees if there are $< 50\%$ outliers
- "All good matches are alike; every bad match is bad in its own way." – [Tolstoy via Alyosha Efros]

[Source: N. Snavely]

Affine Transformation

How?

Affine Transformation

How?

- Find matches across images I and J. This gives us points X_I in image I and X_J in J, where we know that the point X_I^k is a match with X_J^k

- Iterate:

    - Choose 3 pairs of matches randomly
    - Compute the affine transformation
    - Project all matched points X_I from I to J via the computed transformation. This gives us $\hat{X}_I$
    - Find how many matches are inliers, i.e., $||\hat{X}_I^k - X_J^k|| < thresh$.

- Choose the transformation with the most inliers

RANSAC

- **Inlier threshold** related to the amount of noise we expect in inliers

- **Inlier threshold** related to the amount of noise we expect in inliers
- Often model noise as Gaussian with some standard deviation (e.g., 3 pixels)

RANSAC

- **Inlier threshold** related to the amount of noise we expect in inliers
- Often model noise as Gaussian with some standard deviation (e.g., 3 pixels)
- **Number of rounds** related to the percentage of outliers we expect, and the probability of success we'd like to guarantee

RANSAC

- **Inlier threshold** related to the amount of noise we expect in inliers

- Often model noise as Gaussian with some standard deviation (e.g., 3 pixels)

- **Number of rounds** related to the percentage of outliers we expect, and the probability of success we'd like to guarantee

- Suppose there are 20% outliers, and we want to find the correct answer with 99% probability

RANSAC

- **Inlier threshold** related to the amount of noise we expect in inliers
- Often model noise as Gaussian with some standard deviation (e.g., 3 pixels)
- **Number of rounds** related to the percentage of outliers we expect, and the probability of success we'd like to guarantee
- Suppose there are 20% outliers, and we want to find the correct answer with 99% probability
- How many rounds do we need?

[Source: R. Urtasun]

How many rounds?

- Sufficient number of trials S must be tried.

How many rounds?

- Sufficient number of trials S must be tried.
- Let p be the probability that any given correspondence is valid and P be the total probability of success after S trials.

How many rounds?

- Sufficient number of trials S must be tried.
- Let p be the probability that any given correspondence is valid and P be the total probability of success after S trials.
- The likelihood in one trial that all k random samples are inliers is p^k

How many rounds?

- Sufficient number of trials S must be tried.
- Let p be the probability that any given correspondence is valid and P be the total probability of success after S trials.
- The likelihood in one trial that all k random samples are inliers is p^k
- The likelihood that S such trials will all fail is

$$1 - P = (1 - p^k)^S$$

How many rounds?

- Sufficient number of trials S must be tried.
- Let p be the probability that any given correspondence is valid and P be the total probability of success after S trials.
- The likelihood in one trial that all k random samples are inliers is p^k
- The likelihood that S such trials will all fail is

$$1 - P = (1 - p^k)^S$$

- The required minimum number of trials is

$$S = \frac{\log(1 - P)}{\log(1 - p^k)}$$

How many rounds?

- Sufficient number of trials S must be tried.
- Let p be the probability that any given correspondence is valid and P be the total probability of success after S trials.
- The likelihood in one trial that all k random samples are inliers is p^k
- The likelihood that S such trials will all fail is

$$1 - P = (1 - p^k)^S$$

- The required minimum number of trials is

$$S = \frac{\log(1 - P)}{\log(1 - p^k)}$$

- The number of trials grows quickly with the number of sample points used.

How many rounds?

- Sufficient number of trials S must be tried.

- Let p be the probability that any given correspondence is valid and P be the total probability of success after S trials.

- The likelihood in one trial that all k random samples are inliers is p^k

- The likelihood that S such trials will all fail is

$$1 - P = (1 - p^k)^S$$

- The required minimum number of trials is

$$S = \frac{\log(1 - P)}{\log(1 - p^k)}$$

- The number of trials grows quickly with the number of sample points used.

[Source: R. Urtasun]

RANSAC pros and cons

Pros

- Simple and general

RANSAC pros and cons

Pros

- Simple and general
- Applicable to many different problems

RANSAC pros and cons

Pros

- Simple and general
- Applicable to many different problems
- Often works well in practice

RANSAC pros and cons

Pros

- Simple and general
- Applicable to many different problems
- Often works well in practice

Cons

- Parameters to tune

RANSAC pros and cons

Pros

- Simple and general
- Applicable to many different problems
- Often works well in practice

Cons

- Parameters to tune
- Sometimes too many iterations are required

RANSAC pros and cons

Pros

- Simple and general
- Applicable to many different problems
- Often works well in practice

Cons

- Parameters to tune
- Sometimes too many iterations are required
- Can fail for extremely low inlier ratios
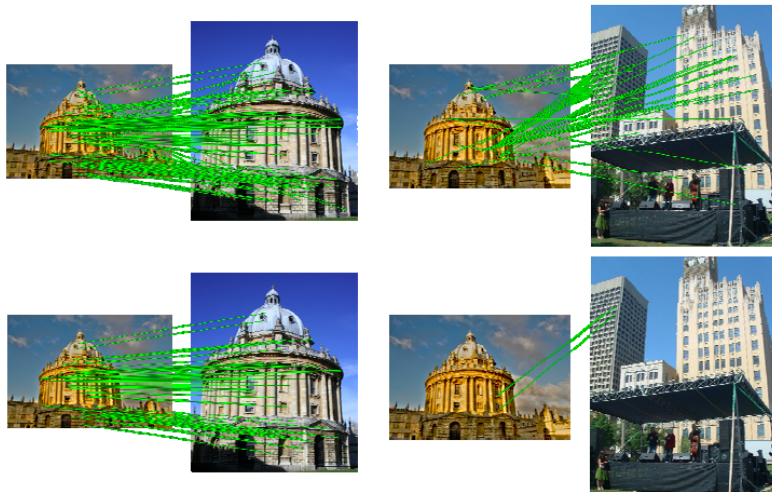
RANSAC pros and cons

Pros

- Simple and general
- Applicable to many different problems
- Often works well in practice

Cons

- Parameters to tune
- Sometimes too many iterations are required
- Can fail for extremely low inlier ratios
- We can often do better than brute-force sampling

[Source: N. Snavely, slide credit: R. Urtasun]

Ransac Verification

[Source: K. Grauman, slide credit: R. Urtasun]

Summary – Stuff You Need To Know

To match image I and J under affine transformation:

- Compute scale and rotation invariant keypoints in both images

- Compute a (rotation invariant) feature vector in each keypoint (e.g., SIFT)

- Match all features in I to all features in J

- For each feature in reference image I find closest match in J

- If ratio between closest and second closest match is < 0.8, keep match

- Do RANSAC to compute affine transformation A:
    - Select 3 matches at random
    - Compute A
    - Compute the number of inliers
    - Repeat
    - Find A that gave the most inliers