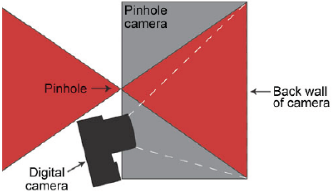
Cameras and Images

Pinhole Camera

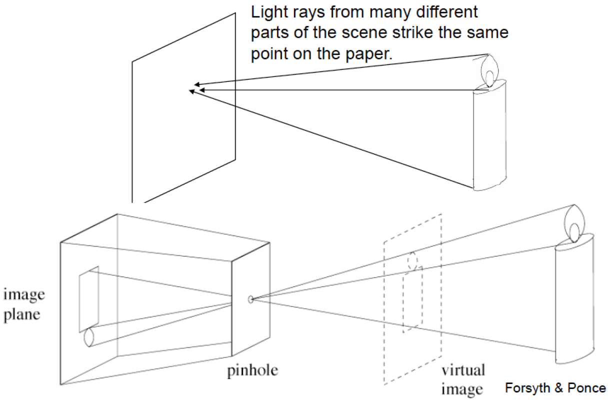
[Source: A. Torralba]



- Make your own camera
- http://www.foundphotography.com/PhotoThoughts/archives/2005/04/pinhole_camera_2.html

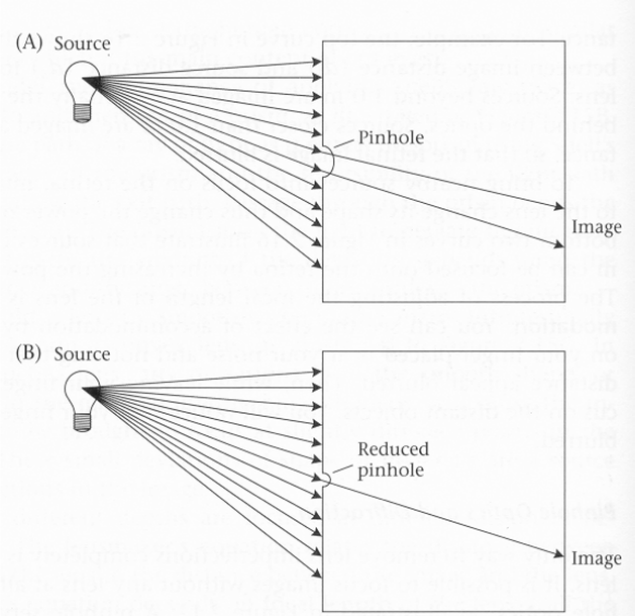
Pinhole Camera – How It Works

[Source: A. Torralba]



- The pinhole camera only allows rays from one point in the scene to strike each point of the paper.

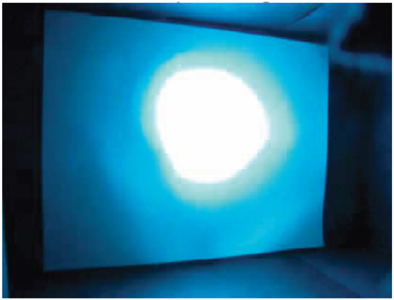
Pinhole Camera – How It Works



[Source: A. Torralba]

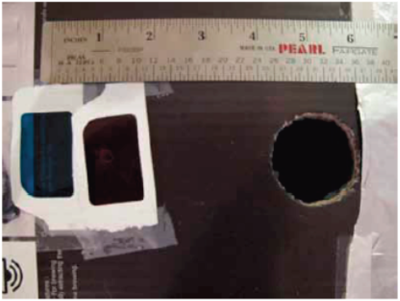
Pinhole Camera – Example

[Source: A. Torralba]



Pinhole Camera

[Source: A. Torralba]



- You can make it stereo

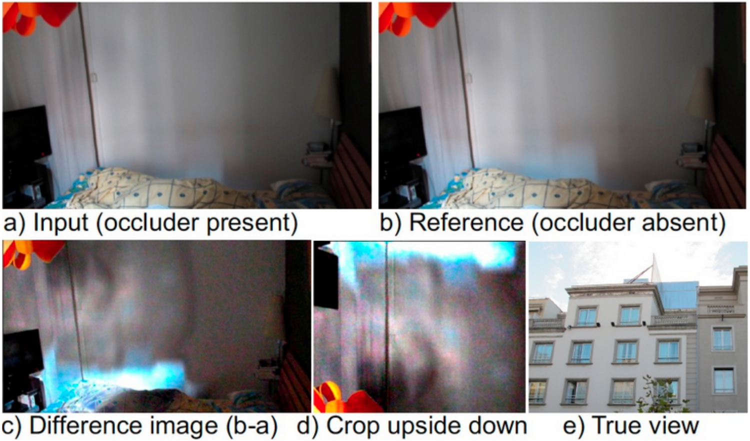
Pinhole Camera – Stereo Example

[Source: A. Torralba]

- Try it with 3D glasses!

Pinhole Camera

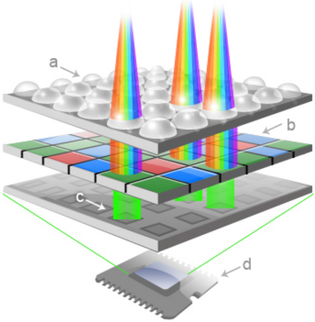
[Source: A. Torralba]



- Remember this example?
- In this case the window acts as a pinhole camera into the room

Digital Camera

[Adopted from S. Seitz]



- A digital camera replaces film with a sensor array
- Each cell in the array is a light-sensitive diode that converts photons to electrons
- http://electronics.howstuffworks.com/cameras-photography/digital/digital-camera.htm

Image Formation

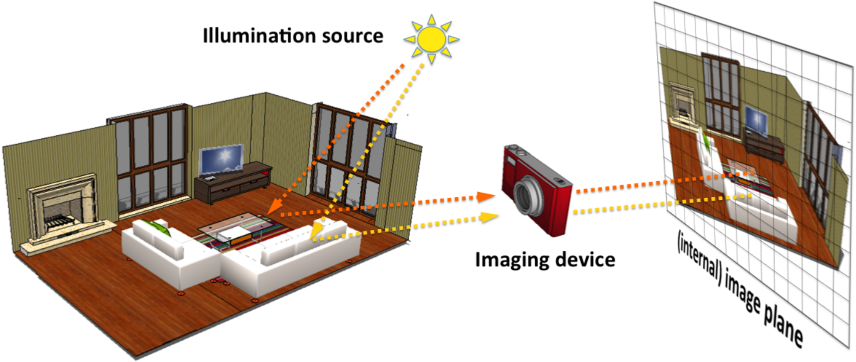
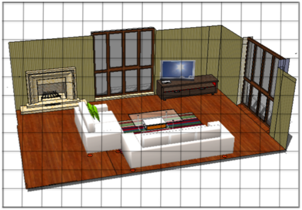


Image formation process producing a particular image depends on:

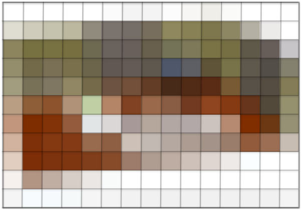
- lighting conditions
- scene geometry
- surface properties
- camera optics

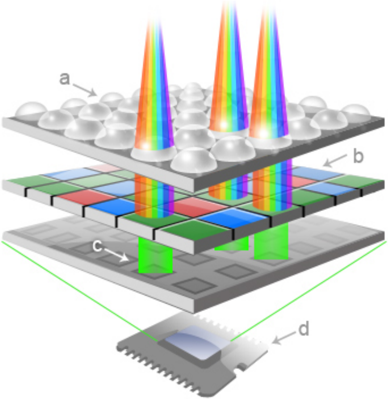
Digital Image

Continuous image projected to sensor array



Sampling and quantization





http://pho.to/media/images/digital/digital-sensors.jpg

- Sample the 2D space on a regular grid
- Quantize each sample (round to nearest integer)

Digital Image

- Image is a matrix with integer values
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255	255	255	255	255	255	255	255	255	255	255	255	255	255	250	248	249	242	241	248	249	254	255	255	255	255	255	255	255	255
255	255	255	255	255	255	255	255	255	255	255	255	252	248	241	238	232	220	222	231	240	245	251	255	255	255	255	255	255	255
255	255	255	255	255	255	255	255	255	255	253	238	237	240	235	228	215	210	217	227	239	242	243	243	247	255	255	255	255	255
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255	255	255	255	255	255	255	255	255	247	224	206	191	207	215	201	178	164	179	200	207	206	172	187	223	237	255	255	255	255
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255	255	255	255	255	255	255	255	233	206	170	110	121	151	174	186	174	151	170	183	175	161	91	65	112	211	238	255	255	255
255	255	255	255	255	255	255	252	202	151	32	39	64	135	170	179	167	148	163	177	174	159	64	42	32	123	222	251	255	255
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248	206	239	255	255	255	176	83	145	171	90	102	152	171	176	173	161	157	160	163	172	171	156	140	102	161	150	87	202	255
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255	255	232	217	243	225	162	125	81	153	173	173	188	191	173	162	148	139	143	154	166	182	192	182	173	182	115	79	141	242
255	255	233	221	231	183	142	106	71	136	185	174	198	184	168	150	126	119	119	134	156	175	197	203	179	182	110	75	140	245
255	252	208	218	217	163	149	94	71	116	187	188	186	155	148	125	107	103	100	111	134	154	163	199	195	161	100	81	117	225
255	244	189	213	214	171	141	114	76	84	158	206	189	140	136	116	101	99	94	109	120	138	150	197	223	136	98	80	105	222
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255	236	178	172	196	183	75	116	101	84	167	200	153	104	92	66	65	71	70	63	74	101	113	174	220	226	102	113	129	234
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203	152	144	197	224	174	156	105	71	69	177	249	255	247	209	166	145	133	128	143	179	236	255	255	255	255	244	186	212	255
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255	255	255	255	255	217	108	96	80	91	117	215	255	255	253	226	179	116	77	101	103	156	243	255	255	255	255	255	255	255
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255	255	255	255	255	255	255	248	245	251	255	255	255	255	255	255	255	255	254	253	255	255	255	255	255	255	255	255	255	255

Digital Image

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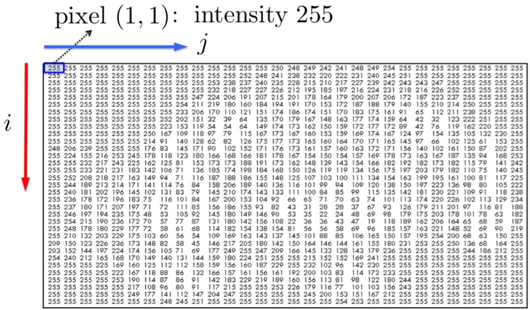
- $I(i, j)$ is called **intensity**



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255	255	255	255	255	255	255	255	255	255	255	255	255	255	250	248	249	242	241	248	249	254	255	255	255	255	255	255	255	255
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255	255	255	255	255	255	255	255	255	255	253	238	237	240	235	228	215	210	217	227	239	242	243	243	247	255	255	255	255	255
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255	255	255	255	255	255	255	255	233	206	170	110	121	151	174	186	174	151	170	183	175	161	91	65	112	211	238	255	255	255
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255	255	255	255	222	167	118	88	86	132	166	157	161	156	161	192	200	103	83	114	172	233	255	255	255	255	255	255	255	255
255	255	255	255	253	190	114	87	86	91	142	183	229	219	189	160	156	113	81	98	122	180	244	255	255	255	255	255	255	255
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255	255	255	255	255	255	255	248	245	251	255	255	255	255	255	255	255	255	254	253	255	255	255	255	255	255	255	255	255	255

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- Matrix I can be $m \times n$ (grayscale)
- or $m \times n \times 3$ (color)

255	255	255	255	255	255	255	255	255	255	255	255	255	255	250	248	249	242	241	248	249	254	255	255	255	255	255	255	255	255
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255	255	255	255	255	255	255	255	255	255	253	238	237	240	235	228	215	210	217	227	239	242	243	243	247	255	255	255	255	255
255	255	255	255	255	255	255	255	255	255	232	218	227	227	226	212	195	185	197	216	224	231	218	216	226	252	255	255	255	255
255	255	255	255	255	255	255	255	255	247	224	206	191	207	215	201	178	164	179	200	207	206	172	187	223	237	255	255	255	255
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255	255	255	255	255	255	255	255	233	206	170	110	121	151	174	186	174	151	170	183	175	161	91	65	112	211	238	255	255	255
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255	255	232	217	243	225	162	125	81	153	173	173	188	191	173	162	148	139	143	154	166	182	192	182	173	182	115	79	141	242
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255	252	208	218	217	163	149	94	71	116	187	188	186	155	148	125	107	103	100	111	134	154	163	199	195	161	100	81	117	225
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255	237	180	171	207	197	71	72	111	85	156	186	155	93	82	43	31	28	28	37	67	93	126	179	211	201	97	116	81	188
255	246	197	194	235	175	48	53	105	92	145	180	149	146	90	53	35	22	24	48	69	98	179	175	203	178	101	78	63	182
255	254	215	190	236	172	70	57	77	87	131	180	142	156	108	22	36	36	43	47	19	118	189	162	206	164	65	68	59	187
255	248	178	180	229	177	72	58	61	68	114	182	154	138	154	81	56	56	58	69	96	185	157	163	221	148	52	69	90	219
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203	152	144	197	224	174	156	105	71	69	177	249	255	247	209	166	145	133	128	143	179	236	255	255	255	255	244	186	212	255
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255	255	255	255	255	255	255	248	245	251	255	255	255	255	255	255	255	255	254	253	255	255	255	255	255	255	255	255	255	255

Digital Image

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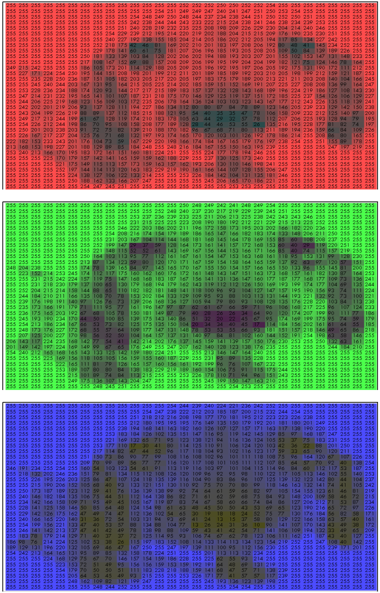
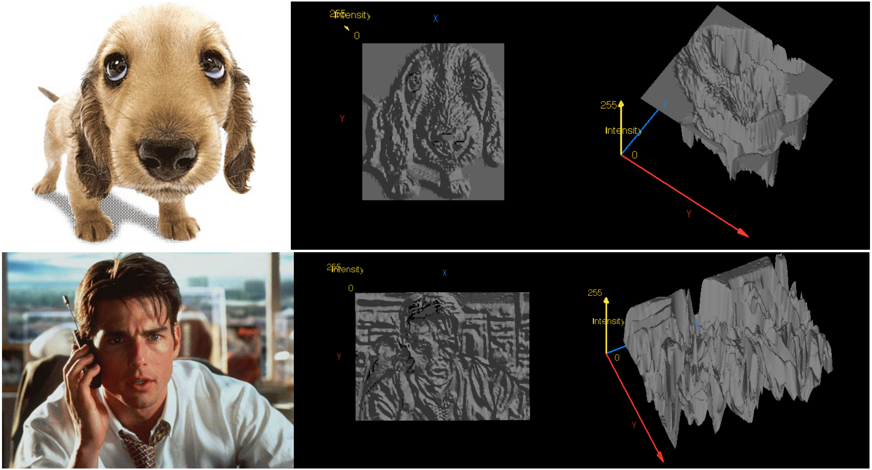


Intensity



- We can think of a (grayscale) image as a function $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ giving the intensity at position (i, j)
- Intensity 0 is black and 255 is white

Image Transformations

- As with any function, we can apply operators to an image, e.g.:

$I(i,j)$ $\qquad$ $J(i,j) = I(i,j) + 50$

- We'll talk about special kinds of operators, **correlation** and **convolution** (linear filtering)

[Adapted from: N. Snavely]

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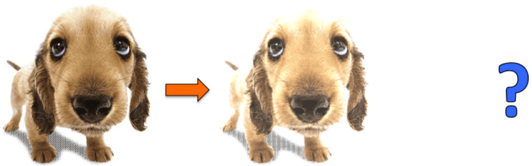

$I(i,j)$ $\quad$ $J(i,j) = I(i,j) + 50$ $\quad$ $J(i,j) = I(i,-j)$

- We'll talk about special kinds of operators, **correlation** and **convolution** (linear filtering)

[Adapted from: N. Snavely]

Image Transformations

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[Adapted from: N. Snavely]

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$I(i,j) \cdot (I(i,j) < 250)$

- We'll talk about special kinds of operators, **correlation** and **convolution** (linear filtering)

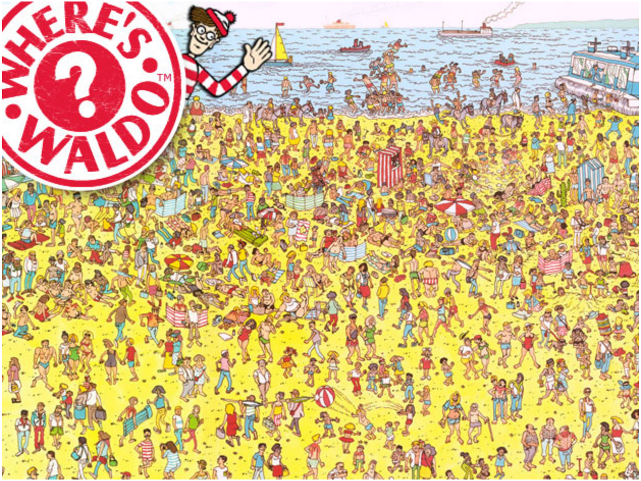
[Adapted from: N. Snavely]

Linear Filters

Reading: Szeliski book, Chapter 3.2

Motivation: Finding Waldo

- How can we find Waldo?



[Source: R. Urtasun]

Answer

- Slide and compare!
- In formal language: **filtering**

Motivation: Noise reduction

- Given a camera and a still scene, how can you reduce noise?

[Source: S. Seitz]

Image Filtering

- Modify the pixels in an image based on some function of a local neighborhood of each pixel
- In other words... Filtering

10	5	3
4	5	1
1	1	7

Local image data

Some function →

	7	

Modified image data

[Source: L. Zhang]

Applications of Filtering

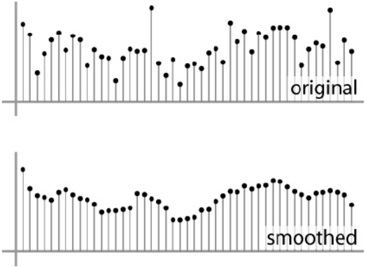
- Enhance an image, e.g., **denoise**.
- Detect patterns, e.g., **template matching**.
- Extract information, e.g., **texture, edges**.
- Filtering is used in Convolutional Neural Networks

Applications of Filtering

- Enhance an image, e.g., **denoise**. Let's talk about this first
- Detect patterns, e.g., **template matching**.
- Extract information, e.g., **texture, edges**.

Noise reduction

- Simplest thing: replace each pixel by the average of its neighbors.
- This assumes that neighboring pixels are similar, and the noise to be independent from pixel to pixel.



[Source: S. Marschner]

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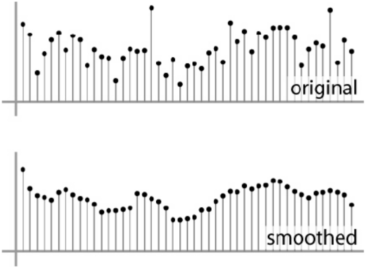
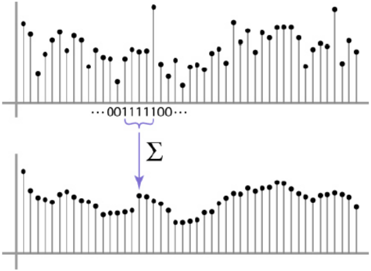


[Source: S. Marschner]

Noise reduction

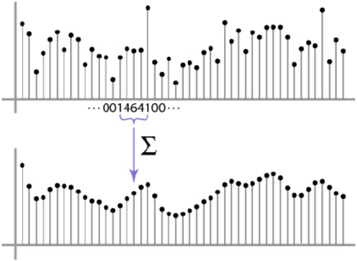
- Simplest thing: replace each pixel by the average of its neighbors
- This assumes that neighboring pixels are similar, and the noise to be independent from pixel to pixel.
- **Moving average** in 1D: [1, 1, 1, 1, 1]/5



[Source: S. Marschner]

Noise reduction

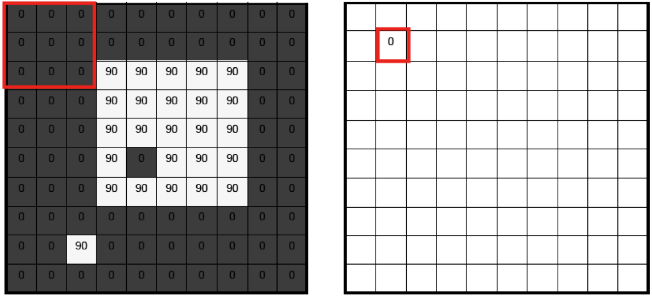
- Simplest thing: replace each pixel by the average of its neighbors
- This assumes that neighboring pixels are similar, and the noise to be independent from pixel to pixel.
- Non-uniform weights [1, 4, 6, 4, 1] / 16



[Source: S. Marschner]

Moving Average in 2D

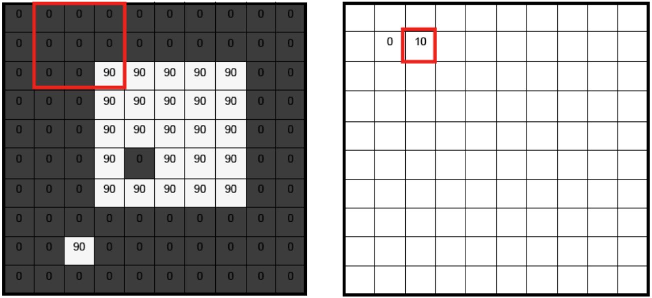
$I(i,j)$ $\qquad\qquad$ $G(i,j)$



[Source: S. Seitz]

Moving Average in 2D

$I(i,j)$ $G(i,j)$



[Source: S. Seitz]

Moving Average in 2D

$I(i,j)$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G(i,j)$

	0	10	20						

[Source: S. Seitz]

Moving Average in 2D

$I(i, j)$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G(i, j)$

	0	10	20	30					

[Source: S. Seitz]

Moving Average in 2D

$I(i,j)$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G(i,j)$

	0	10	20	30	30				

[Source: S. Seitz]

Moving Average in 2D

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0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G(i,j)$

	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	60	90	90	90	60	30	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
	10	20	30	30	30	30	20	10	
	10	10	10	0	0	0	0	0	

[Source: S. Seitz]

Linear Filtering: Correlation

- Involves weighted combinations of pixels in small neighborhoods:

$$G(i,j) = \frac{1}{(2k+1)^2} \sum_{u=-k}^{k} \sum_{v=-k}^{k} I(i+u, j+v)$$

- The output pixels value is determined as a weighted sum of input pixel values

$$G(i,j) = \sum_{u=-k}^{k} \sum_{v=-k}^{k} F(u,v) \cdot I(i+u, j+v)$$

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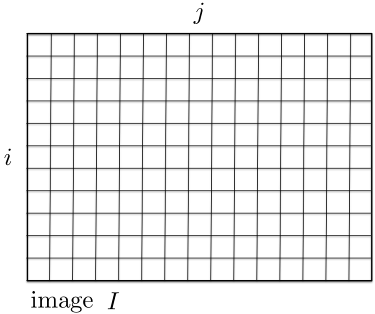
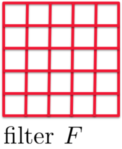
- It's really easy!





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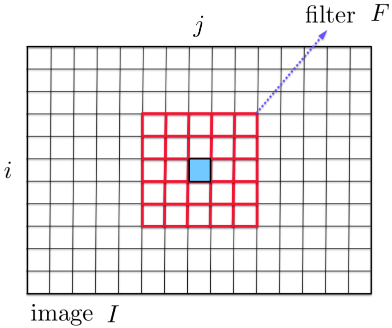

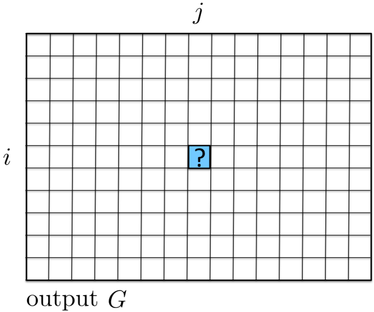

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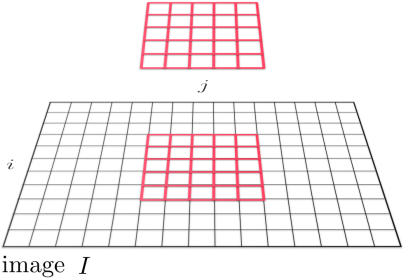

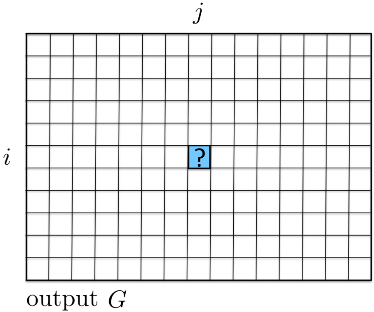

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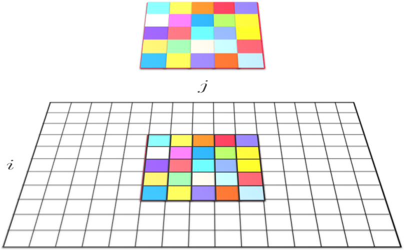
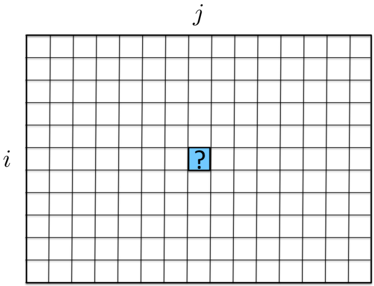


image I



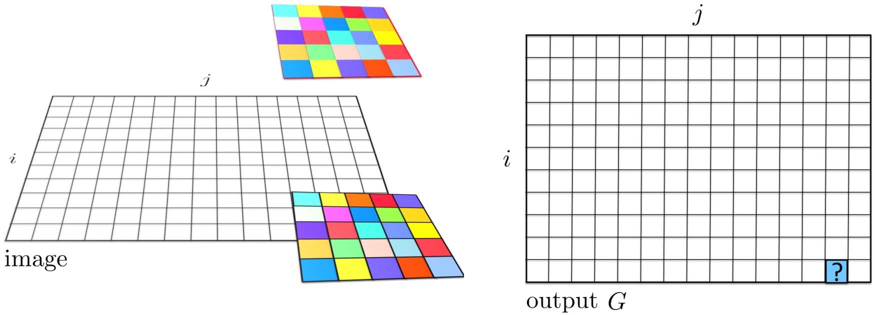
output G

$$G(i,j) = \sum_{u=-k}^{k} \sum_{v=-k}^{k} F(u,v) \cdot I(i+u, j+v)$$

$$G(i,j) = F(\square) \cdot I(\square) + F(\square) \cdot I(\square) + F(\square) \cdot I(\square) + \ldots + F(\square) \cdot I(\square)$$

Linear Filtering: Correlation

- What happens along the borders of the image?



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Boundary Effects

- What happens at the border of the image? What's the size of the output matrix?
- MATLAB: FILTER2(G, F, SHAPE)
- shape = "full" output size is sum of sizes of f and g
- shape = "same": output size is same as f
- shape = "valid": output size is difference of sizes of f and g

[Source: S. Lazebnik]

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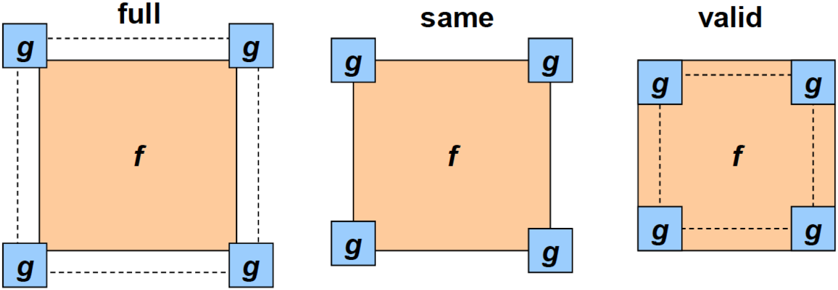
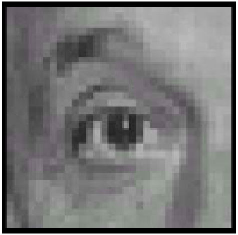


[Source: S. Lazebnik]

Filtering with Correlation: Example

- What's the result?



Original

0	0	0
0	1	0
0	0	0

?

[Source: D. Lowe]

Filtering with Correlation: Example

- What's the result?

Original

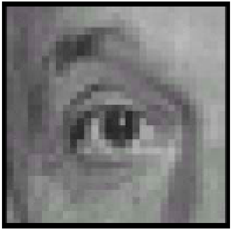
0	0	0
0	1	0
0	0	0

Filtered
(no change)

[Source: D. Lowe]

Filtering with Correlation: Example

- What's the result?



Original

0	0	0
0	0	1
0	0	0

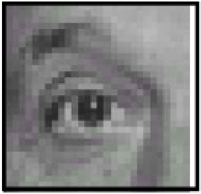
?

[Source: D. Lowe]

Filtering with Correlation: Example

- What's the result?

0	0	0
0	0	1
0	0	0



[Source: D. Lowe]

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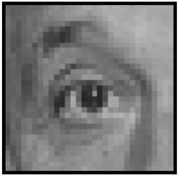


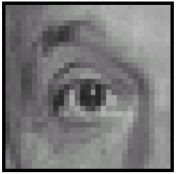
Original

$$* \left(\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \right) =$$

[Source: D. Lowe]

Filtering with Correlation: Example

- What's the result?



Original

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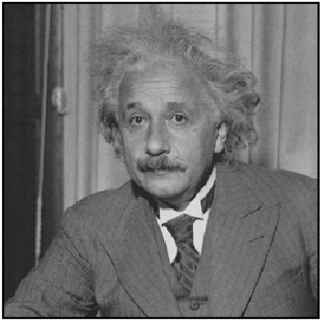
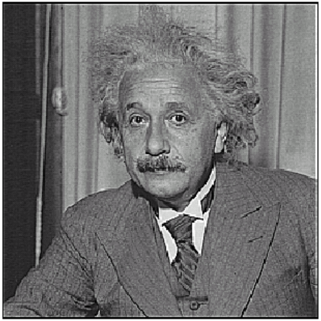
Sharpening filter
(accentuates edges)

[Source: D. Lowe]

Sharpening



before



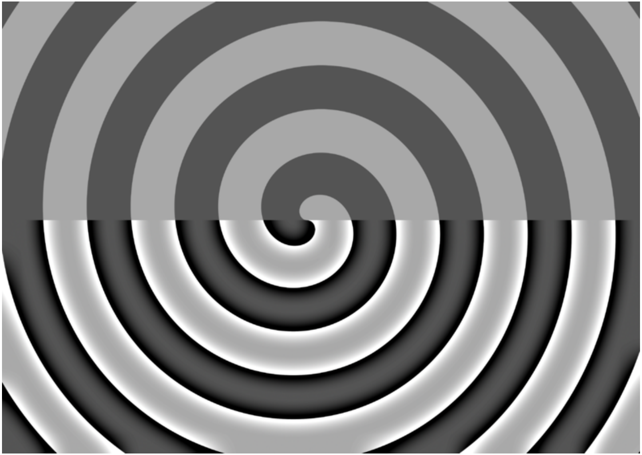
after

[Source: D. Lowe]

Sharpening

unfiltered

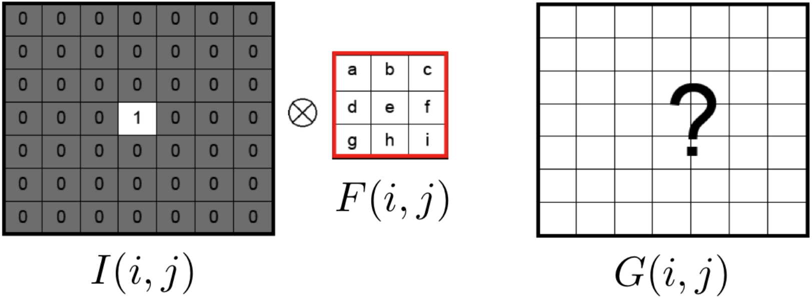
filtered



[Source: N. Snavely]

Example of Correlation

- What is the result of filtering the impulse signal (image) I with the arbitrary filter F?

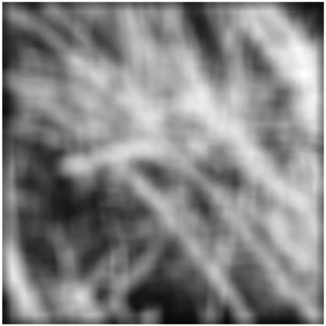


[Source: K. Grauman]

Smoothing by averaging

depicts box filter:
white = high value, black = low value

original



filtered

- What if the filter size was 5 x 5 instead of 3 x 3?

[Source: K. Graumann]

Gaussian filter

- What if we want nearest neighboring pixels to have the most influence on the output?
- Removes high-frequency components from the image (low-pass filter).

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

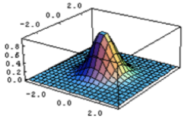
$$I(i,j)$$

$$\frac{1}{16}\boxed{\begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline\end{array}}$$

$$F(i,j)$$

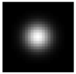
This kernel is an approximation of a 2d Gaussian function:

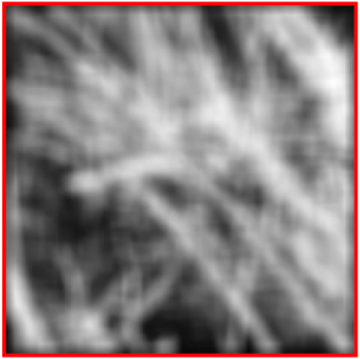
$$h(u,v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{\sigma^2}}$$



[Source: S. Seitz]

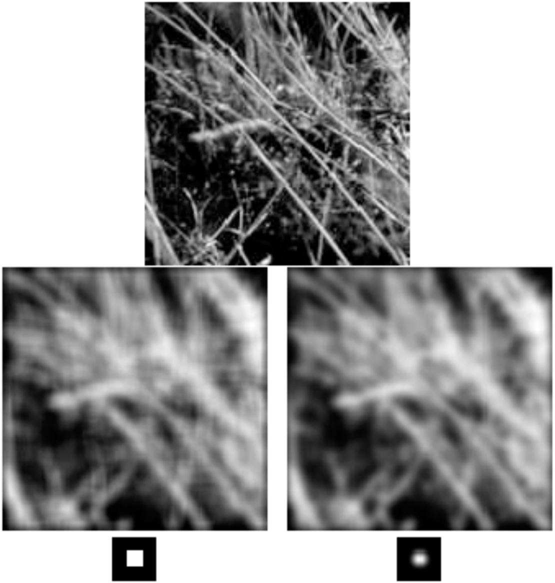
Smoothing with a Gaussian





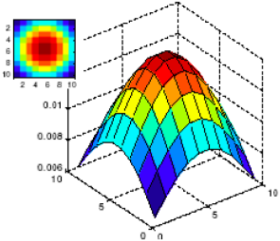
[Source: K. Grauman]

Mean vs Gaussian

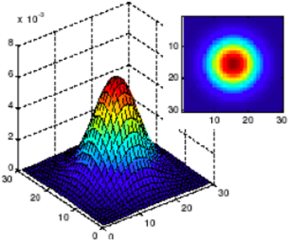


Gaussian filter: Parameters

- **Size of filter or mask**: Gaussian function has infinite support, but discrete filters use finite kernels.



σ = 5 with
10 x 10
kernel

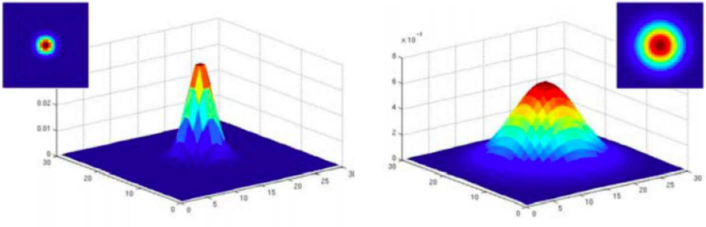


σ = 5 with
30 x 30
kernel

[Source: K. Grauman]

Gaussian filter: Parameters

- **Variance of the Gaussian**: determines extent of smoothing.

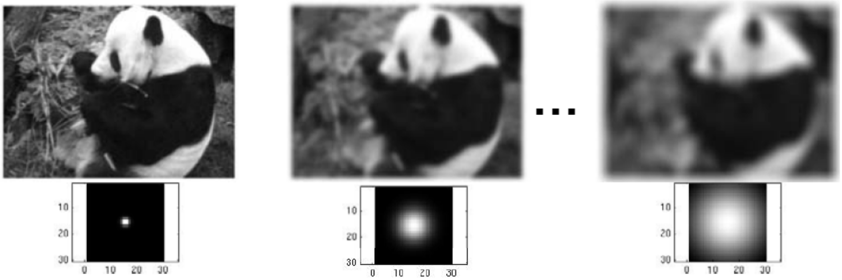


σ = 2 with 30 x 30 kernel

σ = 5 with 30 x 30 kernel

[Source: K. Grauman]

Gaussian filter: Parameters



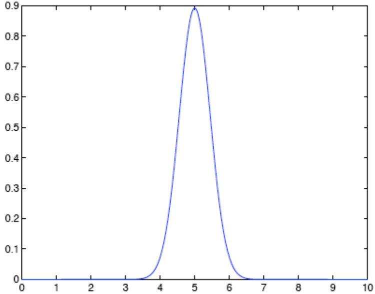

```
for sigma=1:3:10
   h = fspecial('gaussian', fsize, sigma);
   out = imfilter(im, h);
   imshow(out);
   pause;
end
```

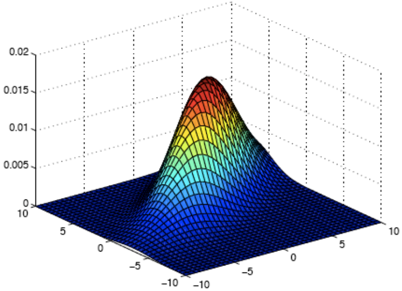
[Source: K. Grauman]

Is this the most general Gaussian?

- No, the most general form for $\mathbf{x} \in \Re^d$

$$\mathcal{N}(\mathbf{x};\, \mu, \Sigma) = \frac{1}{(2\pi)^{d/2}|\Sigma|^{1/2}} \exp\left(-\frac{1}{2}(\mathbf{x}-\mu)^T \Sigma^{-1}(\mathbf{x}-\mu)\right)$$





- But the simplified version is typically used for filtering.

Properties of the Smoothing

- All values are positive.
- They all sum to 1.

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Finding Waldo

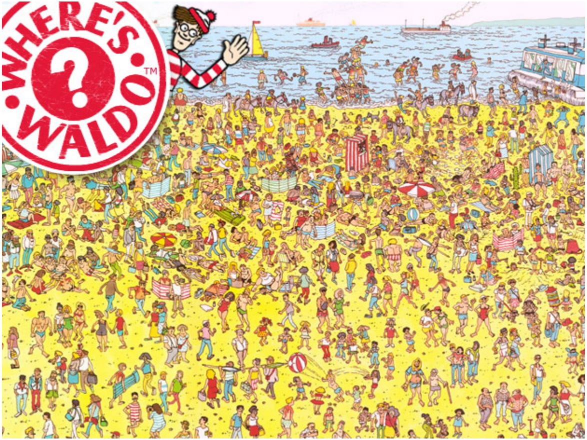

image I

- How can we use what we just learned about filtering to find Waldo?

Finding Waldo

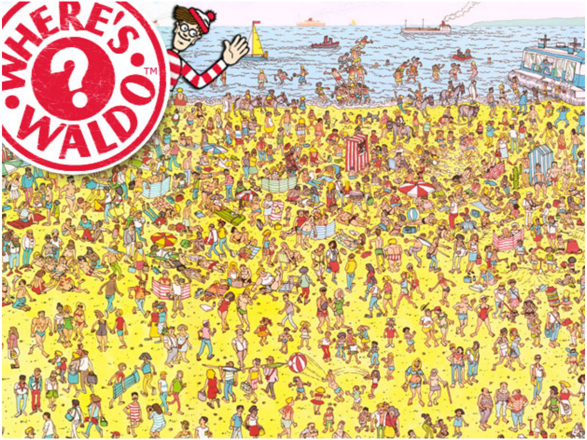

image I

filter F

- Is correlation a good choice?

A Slight Detour: Correlation in Matrix Form

- Remember correlation:

$$G(i,j) = \sum_{u=-k}^{k} \sum_{v=-k}^{k} F(u,v) \cdot I(i+u, j+v)$$

- Can we write that in a more compact form (with vectors)?

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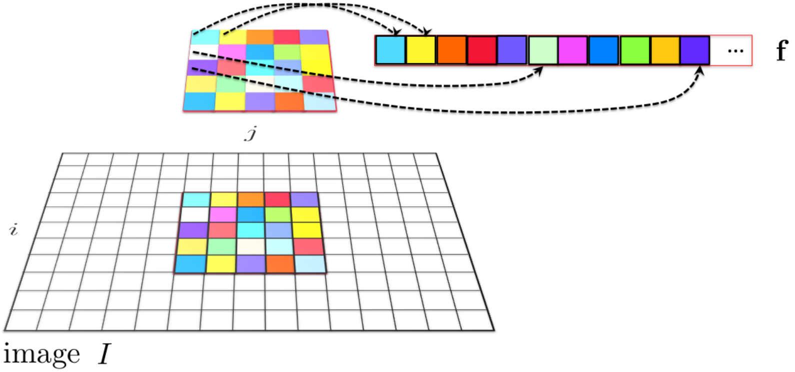

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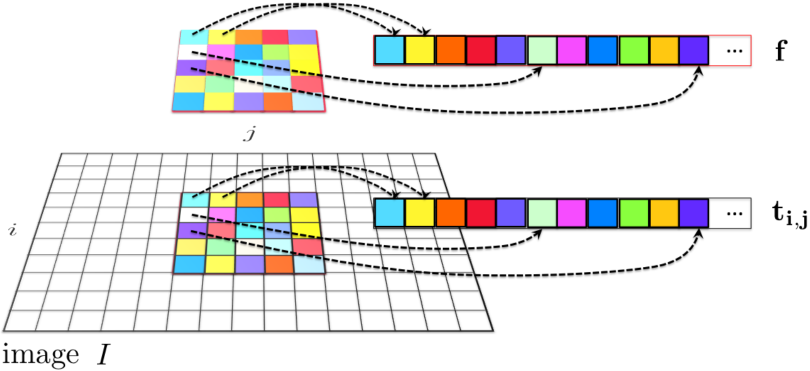

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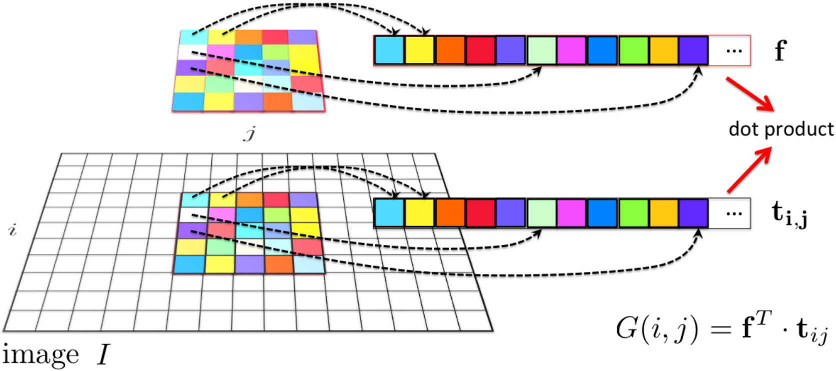

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- Can we write that in a more compact form (with vectors)?
- Define $\mathbf{f} = F(:)$, $T_{ij} = I(i-k : i+k, j-k : j+k)$, and $\mathbf{t}_{ij} = T_{ij}(:)$

$$G(i,j) = \mathbf{f}^T \cdot \mathbf{t}_{ij}$$

where $\cdot$ is a dot product

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- **Homework:** Can we write full correlation $G = F \otimes I$ in matrix form?

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- Finding Waldo: How could we ensure to get the best "score" (e.g. 1) for an image crop that looks exactly like our filter?
- **Normalized cross-correlation:**

$$G(i,j) = \frac{\mathbf{f}^T \cdot \mathbf{t}_{ij}}{||\mathbf{f}|| \cdot ||\mathbf{t_{ij}}||}$$

Back to Waldo

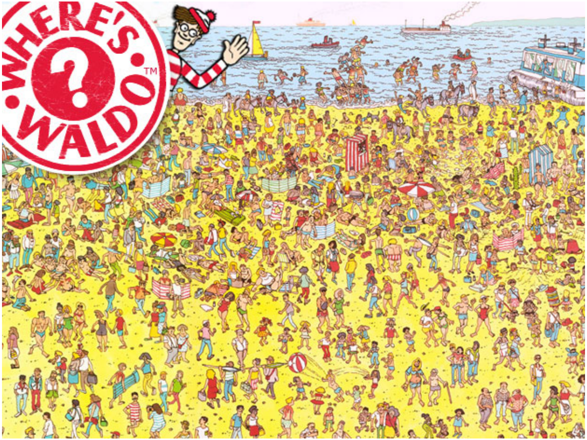
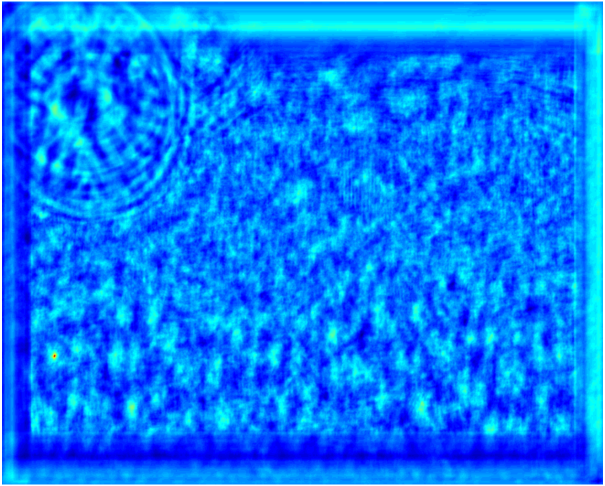


image I

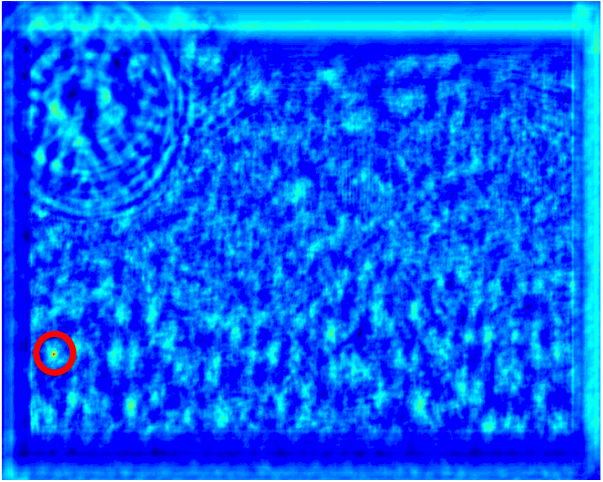
filter F

Back to Waldo



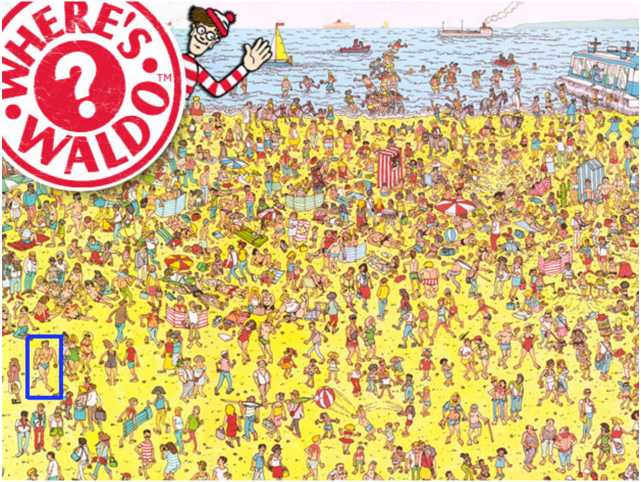
- Result of normalized cross-correlation

Back to Waldo



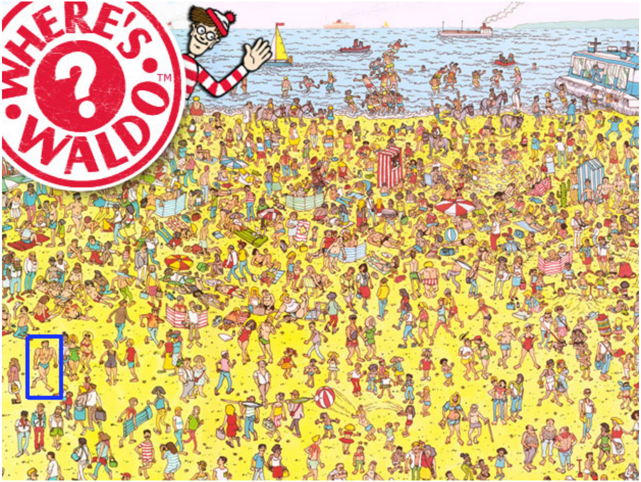
- Find the highest peak

Back to Waldo



And put a bounding box (rectangle the size of the template) at the point!

Back to Waldo



- **Homework**: Do it yourself! Code on class webpage. Don't cheat ;)

Convolution

- **Convolution** operator

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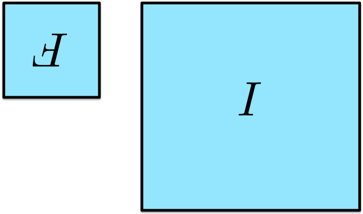
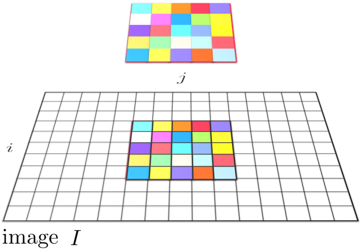
- **Equivalent** to flipping the filter in both dimensions (bottom to top, right to left) and apply correlation.

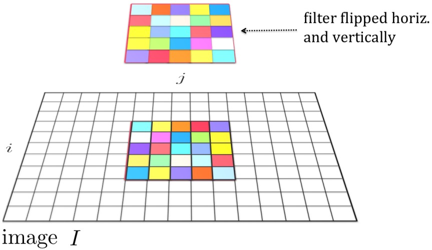


Correlation vs Convolution



Correlation

=



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Correlation vs Convolution

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- How will the outputs differ for:
$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$
- If the input is an impulse signal, how will the outputs differ? $\delta * I$ and $\delta \otimes I$?

"Optical" Convolution

- Camera Shake

Figure: Fergus, et al., SIGGRAPH 2006

- Blur in out-of-focus regions of an image.

Figure: Bokeh: `http://lullaby.homepage.dk/diy-camera/bokeh.html`
Click for more info

[Source: N. Snavely]

Properties of Convolution

$$
\begin{aligned}
\text{Commutative} &: f * g = g * f \\
\text{Associative} &: f * (g * h) = (f * g) * h \\
\text{Distributive} &: f * (g + h) = f * g + f * h \\
\text{Assoc. with scalar multiplier} &: \lambda \cdot (f * g) = (\lambda \cdot f) * h
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- **Homework:** Why is this good news?
- **Hint:** Think of complexity of convolution and Fourier Transform
- Both correlation and convolution are **linear shift-invariant (LSI) operators**: the effect of the operator is the same everywhere.

Gaussian Filter

- Convolving twice with Gaussian kernel of width σ is the same as convolving once with kernel of width $\sigma\sqrt{2}$

- We don't need to filter twice, just once with a bigger kernel

[Source: K. Grauman]

Separable Filters: Speed-up Trick!

- The process of performing a convolution requires K^2 operations per pixel, where K is the size (width or height) of the convolution filter.

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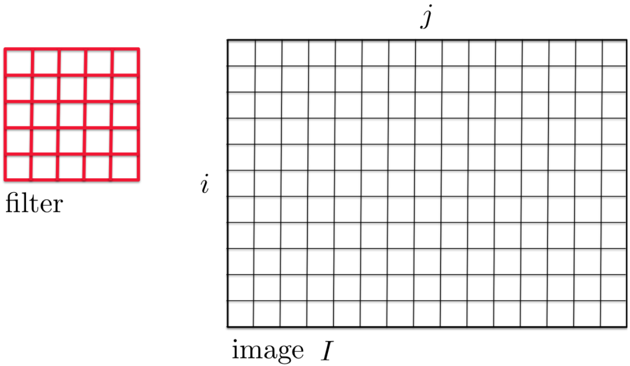
- In many cases (**not all!**), this operation can be speed up by first performing a 1D horizontal convolution followed by a 1D vertical convolution, **requiring only $2K$ operations**.
- If this is possible, then the convolution filter is called **separable**.
- And it is the outer product of two filters:

$$\mathbf{F} = \mathbf{v}\,\mathbf{h}^T$$

- **Homework:** Think **why** in the case of separable filters 2D convolution is the same as two 1D convolutions

[Source: R. Urtasun]

How it Works



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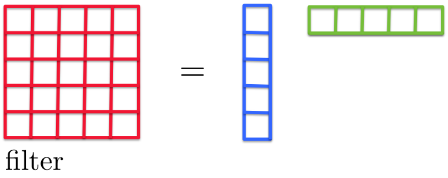

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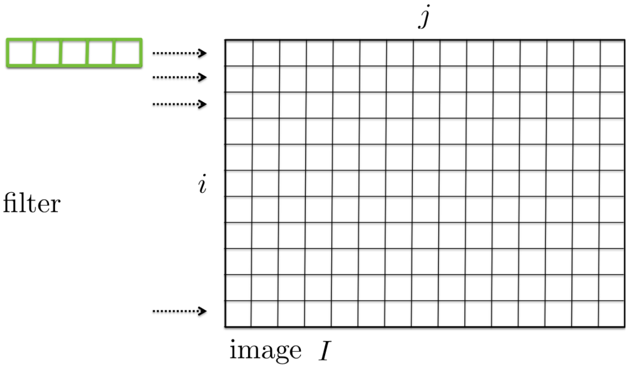

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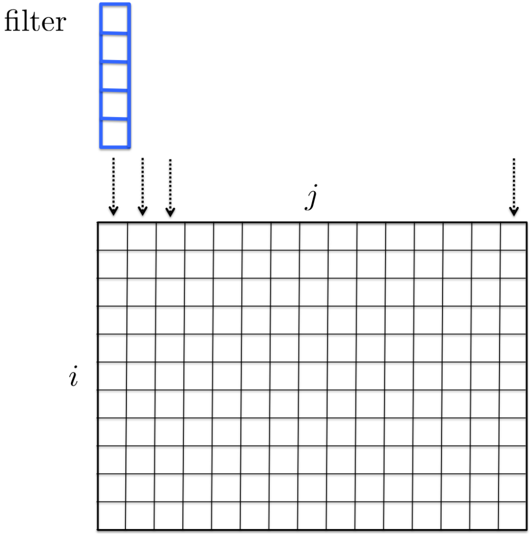
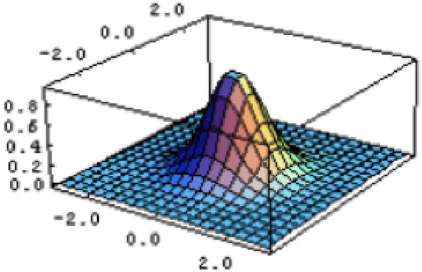


output of horizontal convolution

Separable Filters: Gaussian filters

- One famous separable filter we already know:

$$\text{Gaussian} \;:\; f(x,y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{\sigma^2}}$$



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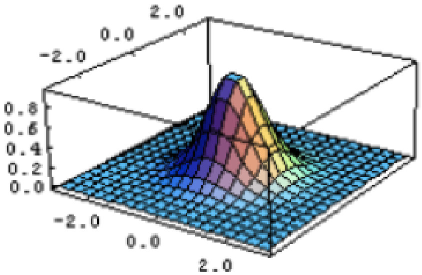
$$= \left(\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{\sigma^2}}\right) \cdot \left(\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{y^2}{\sigma^2}}\right)$$



Let's play a game...

Is this separable? If yes, what's the separable version?

$$\frac{1}{K^2}\begin{array}{|c|c|c|c|}\hline 1 & 1 & \cdots & 1 \\ \hline 1 & 1 & \cdots & 1 \\ \hline \vdots & \vdots & 1 & \vdots \\ \hline 1 & 1 & \cdots & 1 \\ \hline \end{array}$$

[Source: R. Urtasun]

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What does this filter do?

[Source: R. Urtasun]

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What does this filter do?

[Source: R. Urtasun]

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[Source: R. Urtasun]

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$$\frac{1}{2}\begin{array}{|c|c|c|}\hline -1 & 0 & 1 \\ \hline \end{array}$$

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[Source: R. Urtasun]

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with $\Sigma = \mathrm{diag}(\sigma_i)$.

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- Matlab: $[\mathrm{U,S,V}] = \textsc{svd}(\mathrm{F});$
- $\sqrt{\sigma_1}\mathbf{u}_1$ and $\sqrt{\sigma_1}\mathbf{v}_1^T$ are the vertical and horizontal filter.

[Source: R. Urtasun]

Summary – Stuff You Should Know

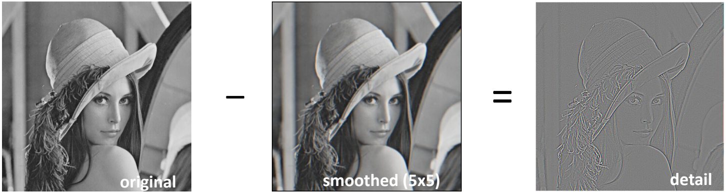
- **Correlation**: Slide a filter across image and compare (via dot product)
- **Convolution**: Flip the filter to the right and down and do correlation
- **Smooth** image with a Gaussian kernel: bigger σ means more blurring
- **Some** filters (like Gaussian) are **separable**: you can filter faster. First apply 1D convolution to each row, followed by another 1D conv. to each column
- Applying first a Gaussian filter with σ_1 and then another Gaussian with σ_2 is the same as applying one Gaussian filter with $\sigma = \sqrt{\sigma_1^2 + \sigma_2^2}$

Matlab functions:

- IMFILTER: can do both correlation and convolution
- CORR2, FILTER2: correlation, NORMXCORR2 normalized correlation
- CONV2: does convolution
- FSPECIAL: creates special filters including a Gaussian

Edges

- What does blurring take away?



[Source: S. Lazebnik]

Next time:

Edge Detection