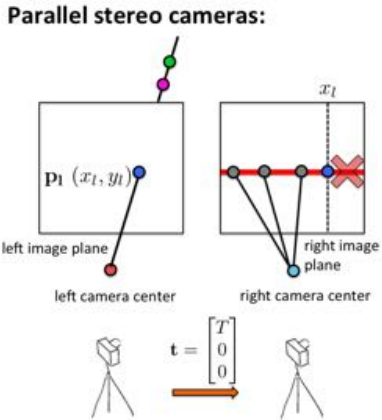
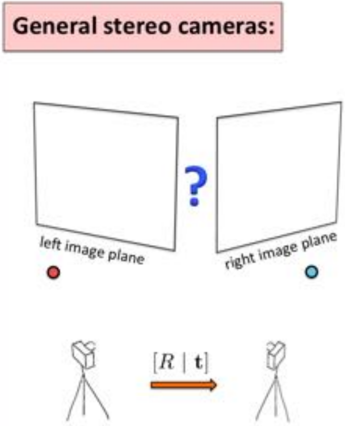
Stereo

Epipolar Geometry for General Cameras

Epipolar geometry

- Case with two cameras with parallel optical axes
- General case $\leftarrow$ **Now this**





Epipolar Geometry

- If I can always mount two cameras parallel to each other, why do I need to learn math for the general case?

Epipolar Geometry

- Let's say that you want to reconstruct a CN tower in 3D

Epipolar Geometry

- Let's say that you want to reconstruct a CN tower in 3D
- One out of endless possibilities of why you would do that:
  - You can print it with a 3D printer to get a nice pocket or not-so-pocket edition (better than those that are sold in Chinatown)
  - Give it to your mum for Christmas (say it's a present from CSC420)

Epipolar Geometry

- Let's say that you want to reconstruct a CN tower in 3D
- You obviously can't get a good 360 shot of the CN tower with just parallel cameras. Particularly not the top of the CN tower which is very high up.

Epipolar Geometry

- Let's say that you want to reconstruct a CN tower in 3D
- You obviously can't get a good 360 shot of the CN tower with just parallel cameras. Particularly not the top of the tower.
- But you can download great images of the tower from the web without even needing to leave the house.

Epipolar Geometry

- But these images are not taken from parallel cameras...

Photosynth

- You could even do part of Venice...

Figure: `https://www.youtube.com/watch?v=HrgHFDPJHXo`

Noah Snavely, Steven M. Seitz, Richard Szeliski, "Photo tourism: Exploring photo collections in 3D", SIGGRAPH 2006, `https://photosynth.net/`

World Cup 2014 – High Tech 3D

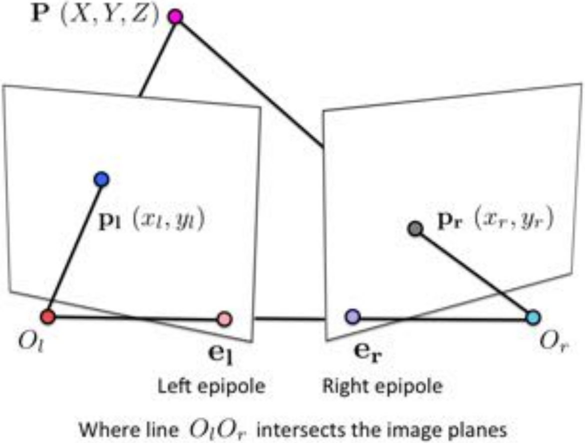
- Last World Cup was monitored with 14 high-speed cameras, capturing 500 frames per second, and could accurately detect ball motion to within 5mm.
- 2,000 tests performed, all successful. By German company Goal Control.

Stereo – General Case

Ready for the math?

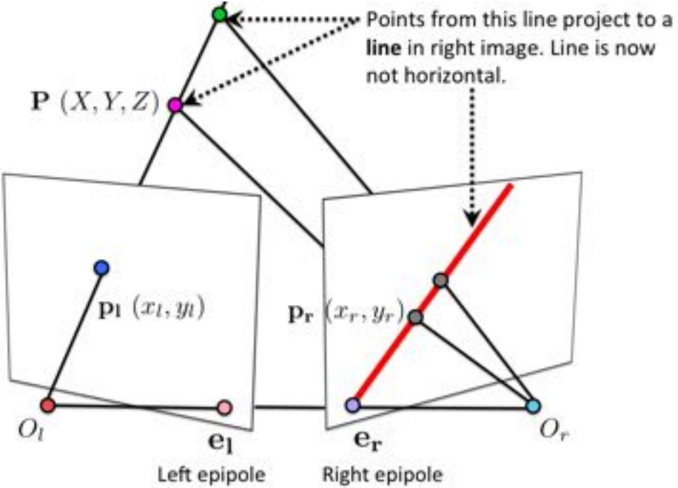
Stereo: Parallel Calibrated Cameras

- Some notation: the **left** and **right epipole**



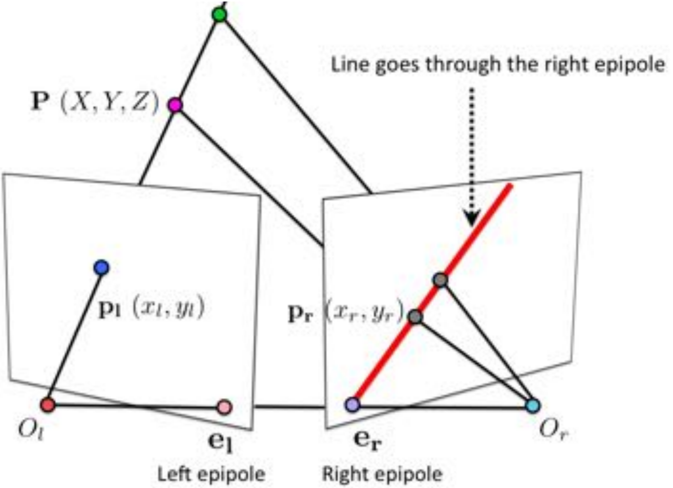
Stereo: Parallel Calibrated Cameras

- All points from the projective line $\mathbf{O_l p_l}$ project to a line on the right image plane. This time the line is not (necessarily) horizontal.



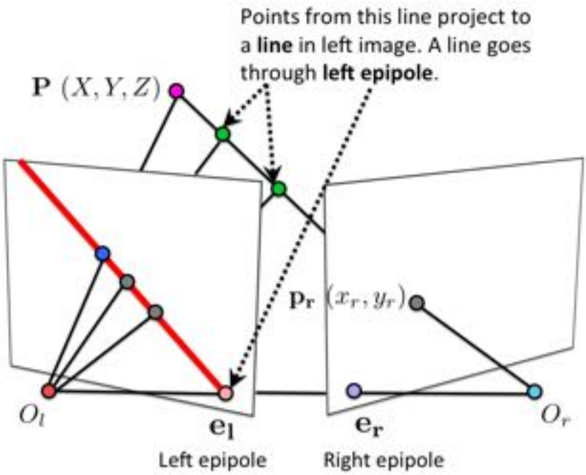
Stereo: Parallel Calibrated Cameras

- The line goes through the right epipole.



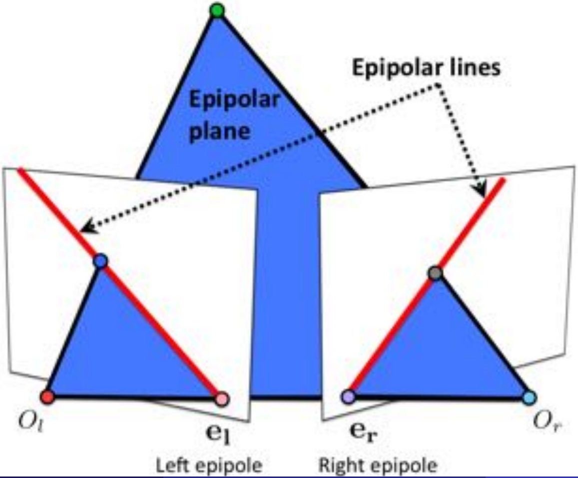
Stereo: Parallel Calibrated Cameras

- Similarly, All points from the projective line $\mathbf{O_r p_r}$ project to a line on the left image plane. This line goes through the left epipole.



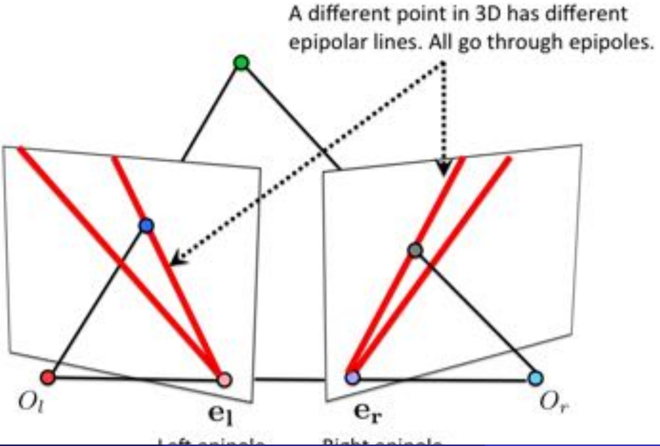
Stereo: Parallel Calibrated Cameras

- The reason for all this is simple: points $\mathbf{O_l}$, $\mathbf{O_r}$, and a point $\mathbf{P}$ in 3D lie on a plane. We call this the **epipolar plane**. This plane intersects each image plane in a line. We call these lines **epipolar lines**.



Stereo: Parallel Calibrated Cameras

- Obviously a different point in 3D will form a different epipolar plane and therefore different epipolar lines. But these epipolar lines go through epipoles as well.

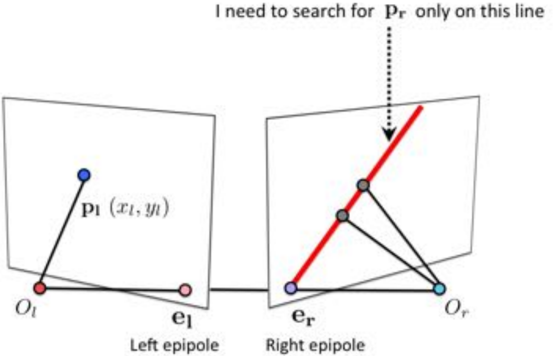


Stereo: Parallel Calibrated Cameras

- Why are we even dumping all this notation? Are epipolar lines, epipoles, etc somehow useful?

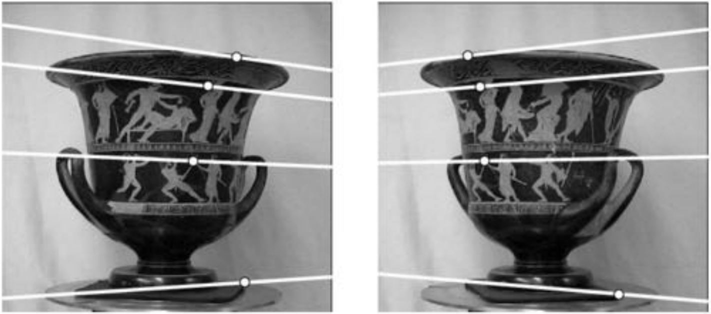
Stereo: Parallel Calibrated Cameras

- Remember what we did for parallel cameras? We were matching points in the left and right image, giving us a point in 3D. We want the same now.
- Epipolar geometry is useful because it constrains our search for the matches:
  - For each point $\mathbf{p_l}$ we need to search for $\mathbf{p_r}$ only on a epipolar line (much simpler than if I need to search in the full image)
  - All matches lie on lines that intersect in epipoles. This gives another constraint.



Epipolar geometry: Examples

- Example of epipolar lines for converging cameras



[Source: J. Hays, pic from Hartley & Zisserman]

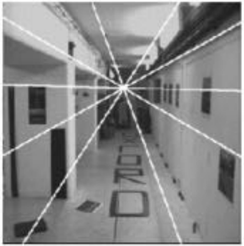
Epipolar geometry: Examples

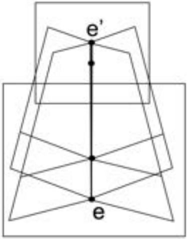
- How would epipolar lines look like if the camera moves directly forward?

[Source: J. Hays]

Epipolar geometry: Examples

- Example of epipolar lines for **forward motion**





Epipole has same coordinates in both images.
Points move along lines radiating from e: "Focus of expansion"

[Source: J. Hays, pic from Hartley & Zisserman]

Stereo for General Cameras

How we'll get 3D:

- We first need to figure out on which line we need to search for the matches for each $\mathbf{p_l}$
- Each point in left image maps to a line in right image. We will see that this mapping can be described by a single 3×3 matrix $\mathbf{F}$, called the **fundamental matrix**

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The Fundamental Matrix

- The fundamental matrix $\mathbf{F}$ is defined as $\mathbf{l_r} = \mathbf{F}\mathbf{p_l}$, where $\mathbf{l_r}$ is the right epipolar line corresponding to $\mathbf{p_l}$.
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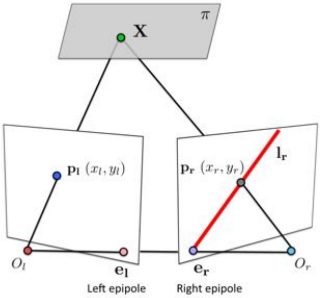

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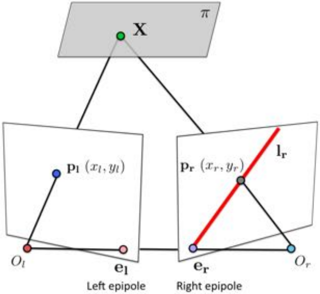

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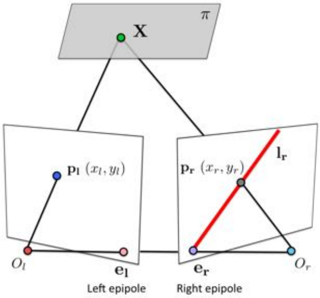

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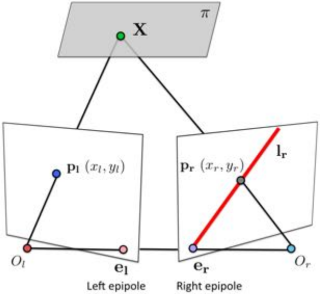

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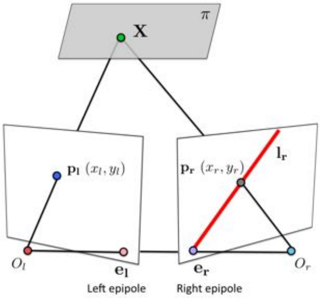

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[Adopted from: R. Urtasun]

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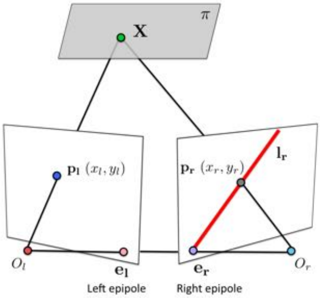

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- Do a trick:

$$\underbrace{\mathbf{p_r}^T \cdot \mathbf{l_r}}_{=0,\text{ because } \mathbf{p_r} \text{ lies on a line } \mathbf{l_r}} = \mathbf{p_r}^T \mathbf{F} \mathbf{p_l}$$

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for any match $(\mathbf{p_l}, \mathbf{p_r})$ (main thing to remember)!!

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- Let's say that you found a few matching points in both images: $(x_{l,1}, y_{l,1}) \leftrightarrow (x_{r,1}, y_{r,1})$, $\ldots$, $(x_{l,n}, y_{l,n}) \leftrightarrow (x_{r,n}, y_{r,n})$, where $n \geq 7$
- Then you can get the parameters $\mathbf{f} := [F_{11}, F_{12}, \ldots, F_{33}]$ by solving:

$$\begin{bmatrix} x_{r,1}\, x_{l,1} & x_{r,1}\, y_{l,1} & x_{r,1} & y_{r,1}\, x_{l,1} & y_{r,1}\, y_{l,1} & y_{r,1} & x_{l,1} & y_{l,1} & 1 \\ & & & & \vdots & & & & \\ x_{r,n}\, x_{l,n} & x_{r,n}\, y_{l,n} & x_{r,n} & y_{r,n}\, x_{l,n} & y_{r,n}\, y_{l,n} & y_{r,n} & x_{l,n} & y_{l,n} & 1 \end{bmatrix} \mathbf{f} = \mathbf{0}$$

- How many correspondences do we need?
- **F** has 9 elements, but we don't care about scaling, so 8 elements.

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- See Zisserman & Hartley's book for details.

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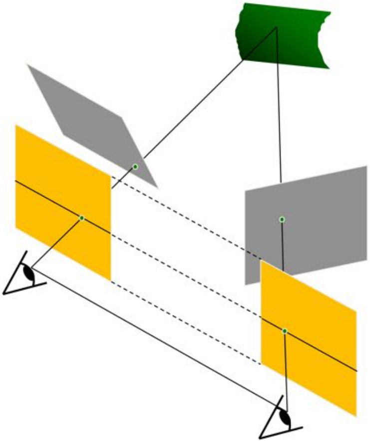
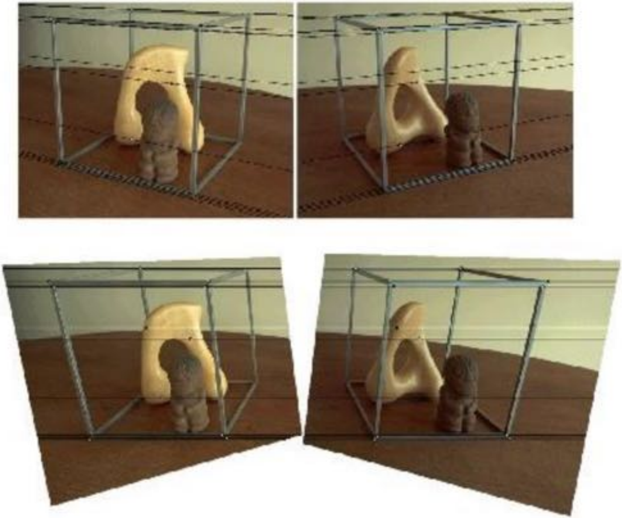
Rectification

- Once we have **F** we can compute homographies that transform each image plane such that they are parallel (see Zisserman & Hartley's book)
- Once they are parallel, we know how to proceed (matching, etc)



[Source: J. Hays]

Rectification Example



[Source: J. Hays]

The Fundamental Matrix: One Last Thing

- Once you have F you can even compute camera projection matrices $\mathbf{P_l}$ and $\mathbf{P_r}$ (under some ambiguity). You may choose the camera projection matrices like this:

$$P_{left} = [\, I_{3\times3} \,|\, \mathbf{0} \,] \qquad P_{right} = \left[\, [\mathbf{e_r}]_x F \,|\, \mathbf{e_r} \,\right]$$

where notation $[]_x$ stands for: $[\mathbf{a}]_x = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}$

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- **This means that I don't need the relative poses of the two cameras, I can compute it!**
- This is very useful in scenarios where I just grab pictures from the web
- We need one last thing to compute P_{right}, and that's $\mathbf{e_r}$. But this is easy. We know that $\mathbf{e_r}$ lies on epipolar line $\mathbf{l_r}$, and so: $\mathbf{e_r}^T \mathbf{l_r} = 0$. We also know that $\mathbf{l_r} = F\mathbf{x_l}$. So: $\mathbf{e_r}^T F \mathbf{x_l} = \mathbf{0}$ for all $\mathbf{x_l}$, and therefore $\mathbf{e_r}^T F = \mathbf{0}$. So I can find $\mathbf{e_r}$ as the vector that maps F to $\mathbf{0}$.

The Fundamental Matrix: One Last Thing

- Once you have F you can even compute camera projection matrices $\mathbf{P_l}$ and $\mathbf{P_r}$ (under some ambiguity). You may choose the camera projection matrices like this:

$$P_{left} = [\, I_{3\times 3} \,|\, \mathbf{0} \,] \qquad P_{right} = \left[\, [\mathbf{e_r}]_x \, F \,|\, \mathbf{e_r} \,\right]$$

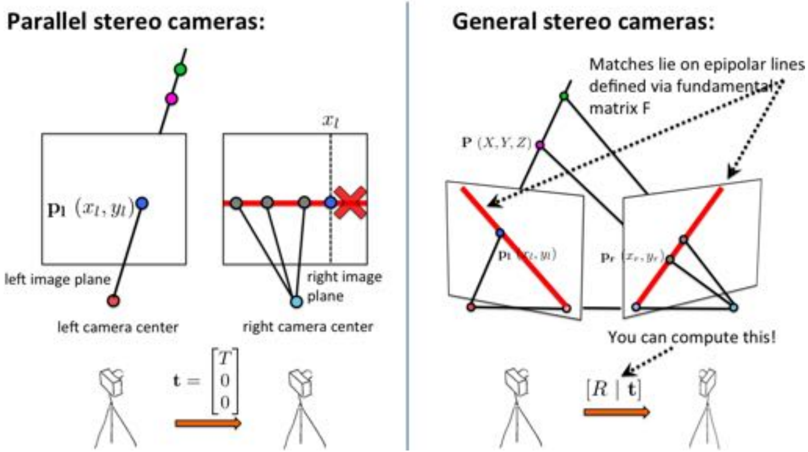
  where notation $[]_x$ stands for: $[\mathbf{a}]_x = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}$

- **This means that I don't need the relative poses of the two cameras, I can compute it!**
- This is very useful in scenarios where I just grab pictures from the web
- We need one last thing to compute P_{right}, and that's $\mathbf{e_r}$. But this is easy. We know that $\mathbf{e_r}$ lies on epipolar line $\mathbf{l_r}$, and so: $\mathbf{e_r}^T \mathbf{l_r} = 0$. We also know that $\mathbf{l_r} = F\mathbf{x_l}$. So: $\mathbf{e_r}^T F \mathbf{x_l} = \mathbf{0}$ for all $\mathbf{x_l}$, and therefore $\mathbf{e_r}^T F = \mathbf{0}$. So I can find $\mathbf{e_r}$ as the vector that maps F to $\mathbf{0}$.

Stereo: Summary

Epipolar geometry

- Case with two cameras with parallel optical axes
- General case



Summary – Stuff You Need To Know

Cameras with parallel optics and known intrinsics and extrinsics:

- You can search for correspondences along horizontal lines
- The difference in x direction between two correspondences is called disparity:

$$\mathrm{disparity} = x_l - x_r$$

- Assuming you know the camera intrinsics and the baseline (distance between the left and right camera canter in the world) you can compute the depth:

$$Z = \frac{f \cdot T}{\mathrm{disparity}}$$

- Once you have Z (depth), you can also compute X and Y, giving you full 3D
- Disparity and depth are inversely proportional

Matlab function:

- $\textsc{disparityMap} = \textsc{disparity}(I_{left}, I_{right})$;
- Function $\textsc{surf}$ is useful for plotting the point cloud

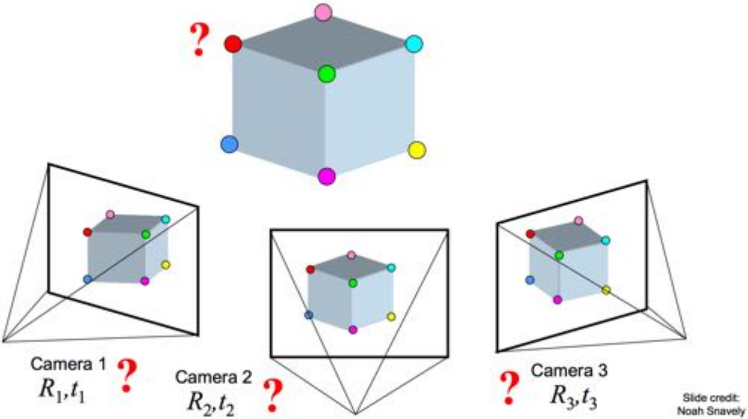
Summary – Stuff You Need To Know

General cameras:

- You first find matches in both images without any restriction. You need at least 7 matches, but the more (reliable) matches the better
- Solve a homogeneous linear system to get the fundamental matrix F
- Given F, you can compute homographies that can rectify both images to be parallel.
- Given F, you can also compute the relative pose between cameras.

Structure From Motion

- What if you have more than two views of the same scene?
- This problem is called **structure-from-motion**



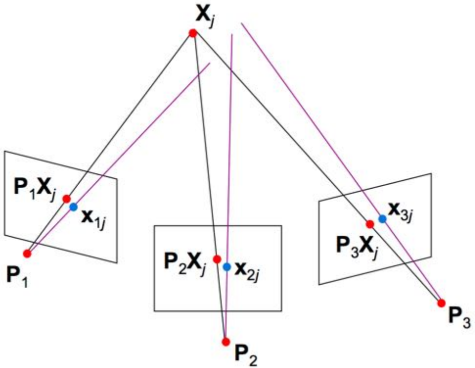
[Source: J. Hays]

Structure From Motion

- Solve a non-linear optimization problem minimizing re-projection error:

$$E(\mathbf{P}, \mathbf{X}) = \sum_{i=1}^{\#cameras} \sum_{j=1}^{\#points} \mathrm{dist}(\mathbf{x_{ij}}, P_i X_j)$$

- This can be done via technique called **bundle adjustment**



[Source: J. Hays]

Lost in Translation

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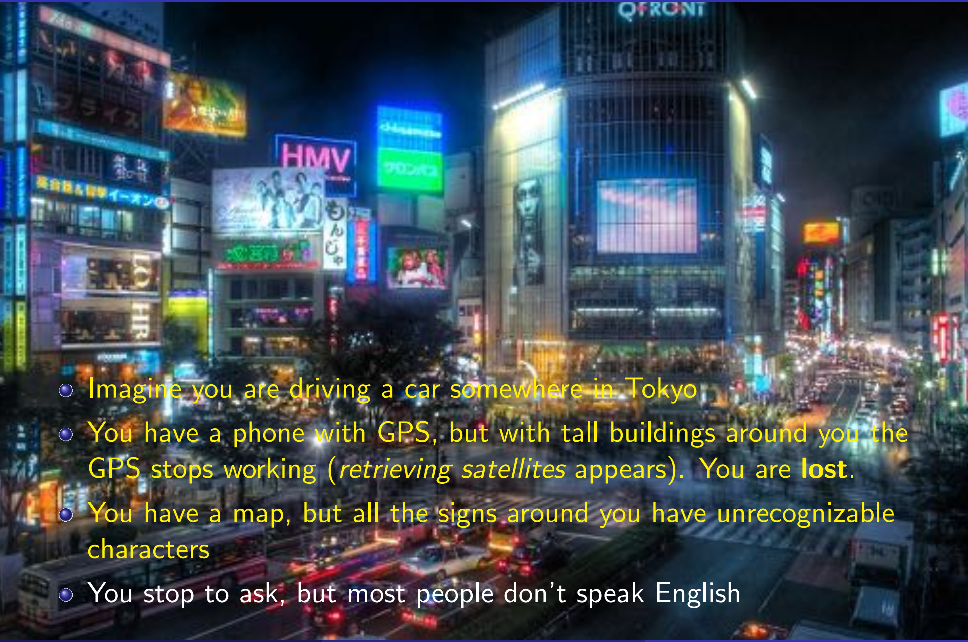

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- You stop to ask, but most people don't speak English

Lost in Translation

What can you do?

- Imagine you are driving a car somewhere in Tokyo
- You have a phone with GPS, but with tall buildings around you the GPS stops working (*retrieving satellites* appears). You are **lost**.
- You have a map, but all the signs around you have unrecognizable characters
- You stop to ask, but most people don't speak English

Take out your phone, start recording the road and

Drive!

M. Brubaker, A. Geiger and R. Urtasun

Lost! Leveraging the Crowd for Probabilistic Visual Self-Localization

CVPR 2013

Paper & Code: `http://www.cs.toronto.edu/~mbrubake/projects/map/`

Lost

[M. Brubaker, A. Geiger and R. Urtasun, CVPR13]

- From consecutive frames you can compute relative camera poses
- The recorded video stream therefore gives you a trajectory you are driving

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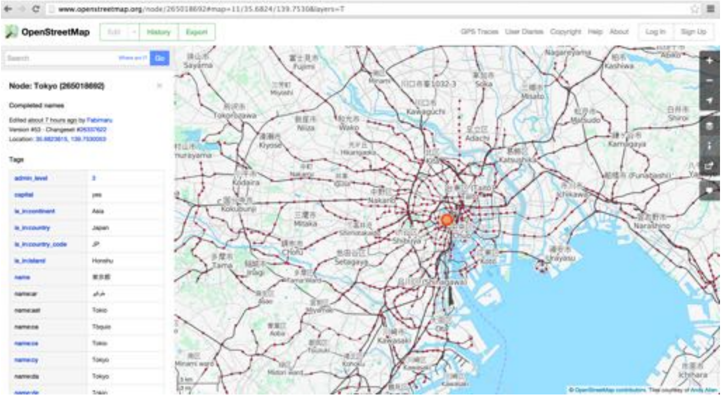

Figure: OpenStreetMap are free downloadable maps (with GPS) of the world

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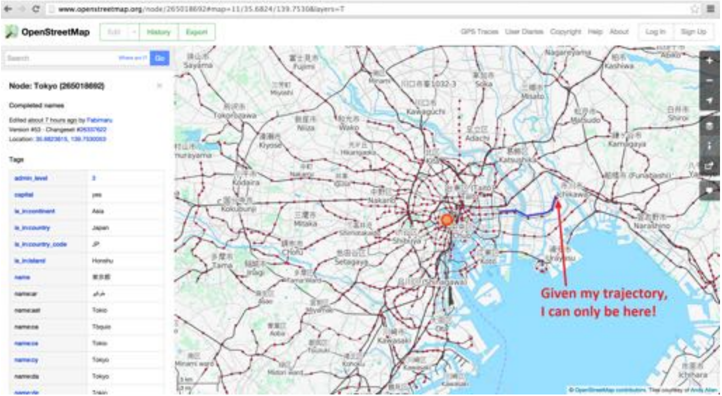

Figure: The shape of my trajectory reveals where I am

Lost

[M. Brubaker, A. Geiger and R. Urtasun, CVPR13]

- From consecutive frames you can compute relative camera poses
- The recorded video stream therefore gives you a trajectory you are driving
- Probabilistic model reasons where you can be on a map given your trajectory
- This gives you the GPS location!
- With 1 camera up to 18m accuracy, 2 cameras up to 3m accuracy

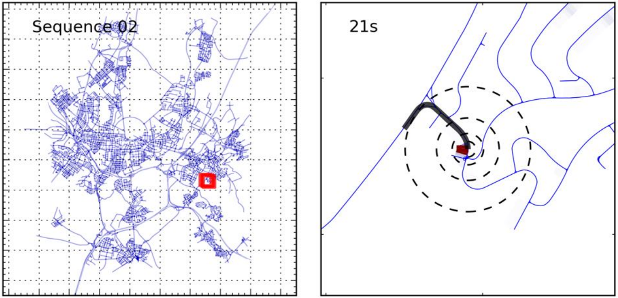

Figure: `https://www.youtube.com/watch?v=4Z3shNPOdQA&feature=youtu.be`

Vision for Visually Impaired

- You can imagine a more complex version of the system for visually impaired

Pic from: http://www.blogcdn.com/www.engadget.com/media/2012/05/wxzfdgrs.jpg

Vision for Visually Impaired

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- How else could depth / 3D help me?

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Vision for Visually Impaired

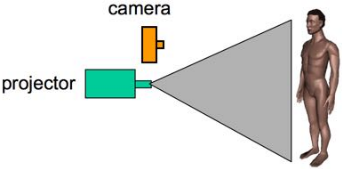
- You can imagine a more complex version of the system for visually impaired
- How else could depth / 3D help me?
- What else do we need to solve to make a vision system for visually impaired functional?

Pic from: http://www.blogcdn.com/www.engadget.com/media/2012/05/wxzfdgrs.jpg

Another Way to get Stereo: Stereo with Structured Light

Project “structured” light patterns onto the object

- Simplifies the correspondence problem
- Allows us to use only one camera



L. Zhang, B. Curless, and S. M. Seitz. Rapid Shape Acquisition Using Color Structured Light and Multi-pass Dynamic Programming. 3DPVT 2002
[Source: J. Hays]

Kinect: Structured infrared light

Figure: https://www.youtube.com/watch?v=uq9SEJxZiUg

[Source: J. Hays]

Stereo Vision in the Wild

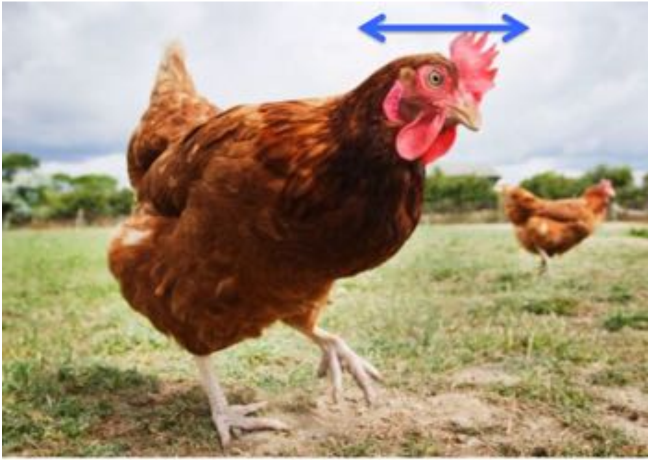
- Humans and a lot of animals (particularly cute ones) have stereoscopic vision

Stereo Vision in the Wild

- Most birds don't see in stereo (each eye gets its own picture, no overlap)
- How do these animals get depth? E.g., how can a chicken beak the corn without smashing the head against the floor?

Stereo Vision in the Wild

- Most birds don't see in stereo (each eye gets its own picture, no overlap)
- How do these animals get depth? E.g., how can a chicken beak the corn without smashing the head against the floor? **Structure-from-motion**



Stereo Vision in the Wild

- Owls are one of the exceptions (they see stereo)

Birdseye View on What We Learned So Far

Problem	**Detection**	**Description**	**Matching**
Find Planar Distinctive Objects	Scale Invariant Interest Points	Local feature: SIFT	All features to all features + Affine / Homography
Panorama Stitching	Scale Invariant Interest Points	Local feature: SIFT	All features to all features + Homography
Stereo	Compute in every point	Intensity or Gradient patch	For each point search on epipolar line

Towards Semantics

- 3D and Projective Geometry can explain a lot of things in the image.
- However, some of the most valuable images cannot be explained by 3D at all.

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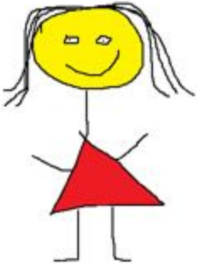
100 million $

"Dora Maar au Chat"
Pablo Picasso, 1941



1 cent

"La Picture"
Sanja Fidler, yesterday

[Adopted from: A. Torralba]

Towards Semantics

- We shouldn't only look at the 3D behind the image but also at the **story** behind it.
- We need to also understand the image **semantics**.

It's Fine Without Depth Too

https://www.youtube.com/watch?v=_dPlkFPowCc

It's Fine Without Depth Too

- Chickens don't want depth, they want story ;)

https://www.youtube.com/watch?v=_dPlkFPowCc

Next Time:

Recognition