

# Depth from Stereo

# Depth from Two Views: Stereo

- All points on the projective line to  $\mathbf{P}$  map to  $\mathbf{p}$

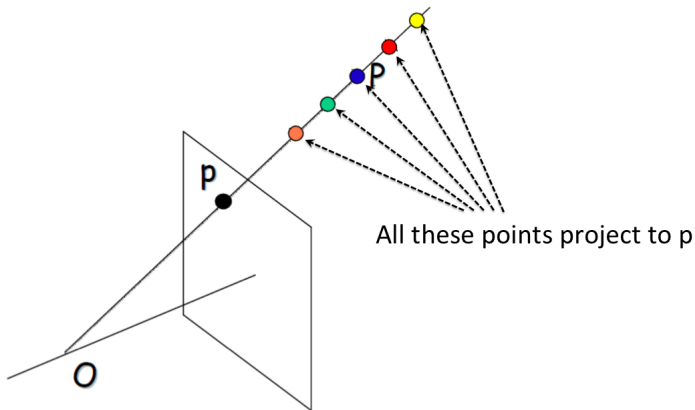
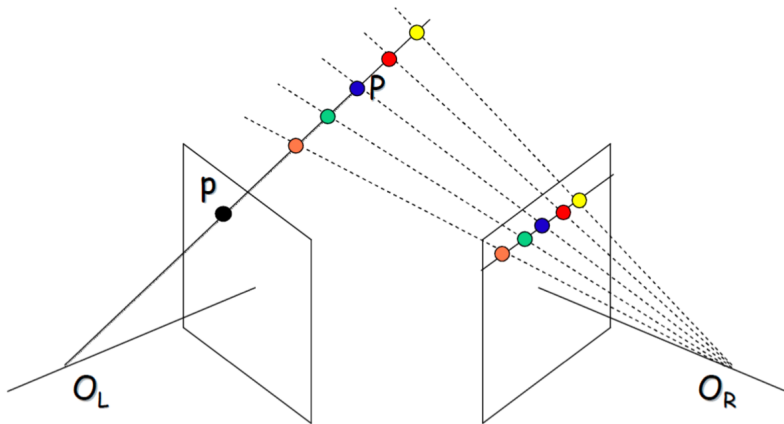


Figure: One camera

# Depth from Two Views: Stereo

- All points on projective line to  $\mathbf{P}$  in left camera map to a **line** in the image plane of the right camera



# Depth from Two Views: Stereo

- If I search this line to find correspondences...

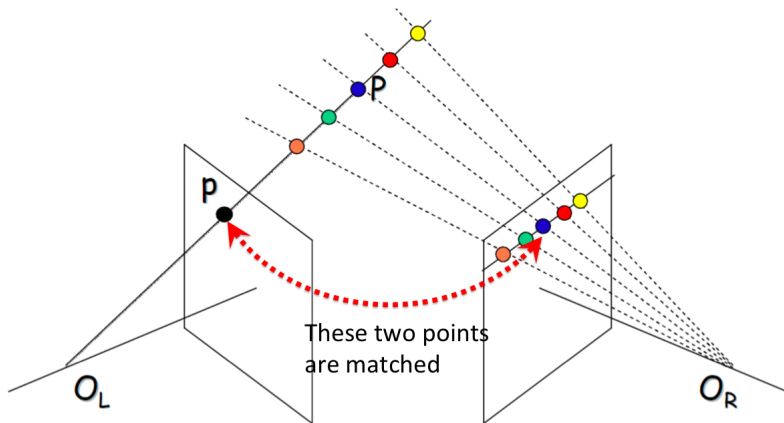


Figure: If I am able to find corresponding points in two images...



# Depth from Two Views: Stereo

- I can get 3D!

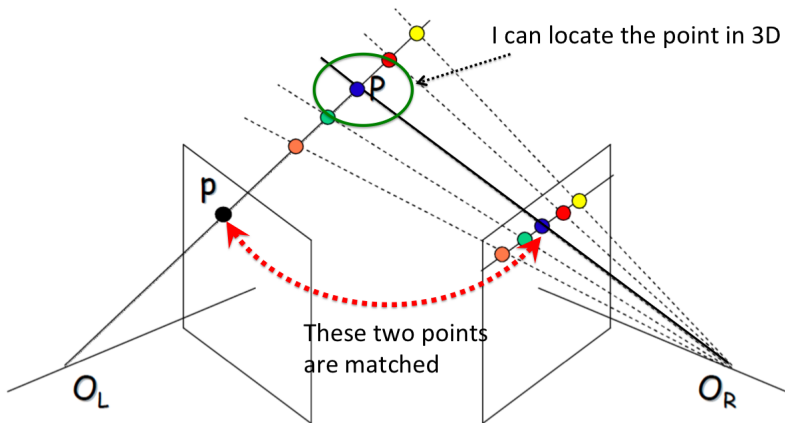
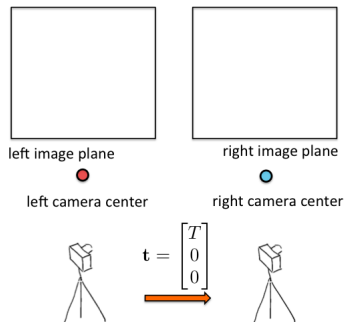


Figure: I can get a point in 3D by triangulation!

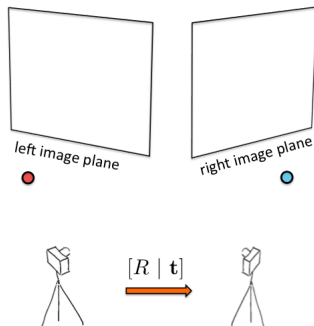
## Epipolar geometry

- Case with two cameras with parallel optical axes
- General case

### Parallel stereo cameras:



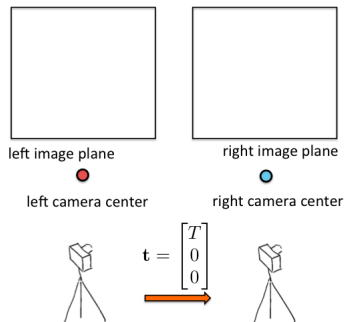
### General stereo cameras:



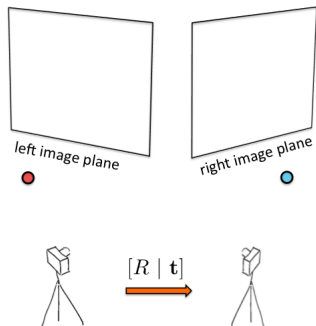
## Epipolar geometry

- Case with two cameras with parallel optical axes ← **First this**
- General case

### Parallel stereo cameras:

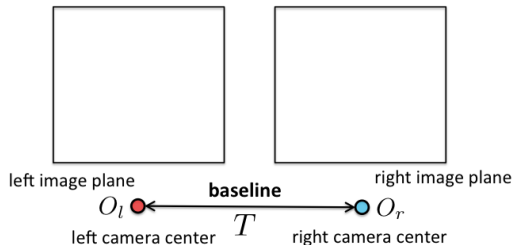


### General stereo cameras:



# Stereo: Parallel Calibrated Cameras

- We assume that the two calibrated cameras (we know intrinsics and extrinsics) are parallel, i.e. the right camera is just some distance to the right of left camera. We assume we know this distance. We call it the **baseline**.

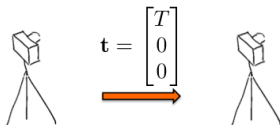
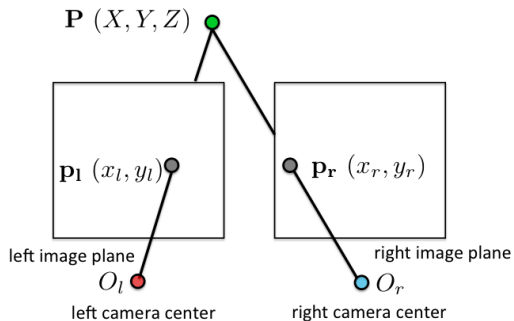


$$\mathbf{t} = \begin{bmatrix} T \\ 0 \\ 0 \end{bmatrix}$$

The right camera is shifted to the right in X direction

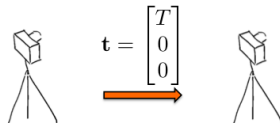
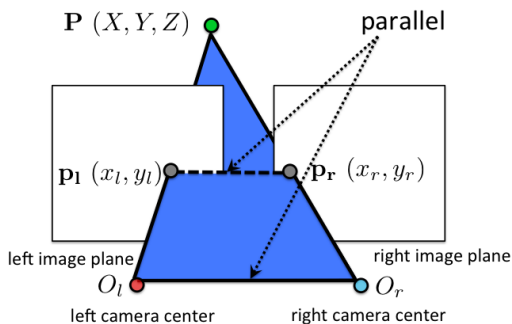
# Stereo: Parallel Calibrated Cameras

- Pick a point  $P$  in the world



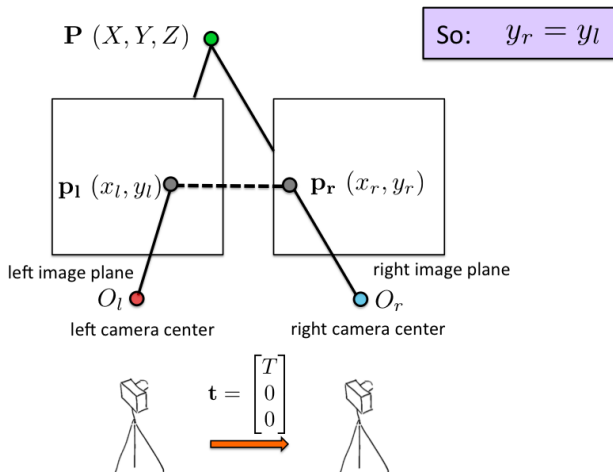
# Stereo: Parallel Calibrated Cameras

- Points  $O_l$ ,  $O_r$  and  $P$  (and  $p_l$  and  $p_r$ ) lie on a plane. Since two image planes lie on the same plane (distance  $f$  from each camera), the lines  $O_l O_r$  and  $p_l p_r$  are parallel.



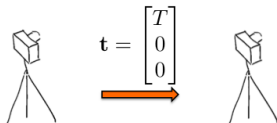
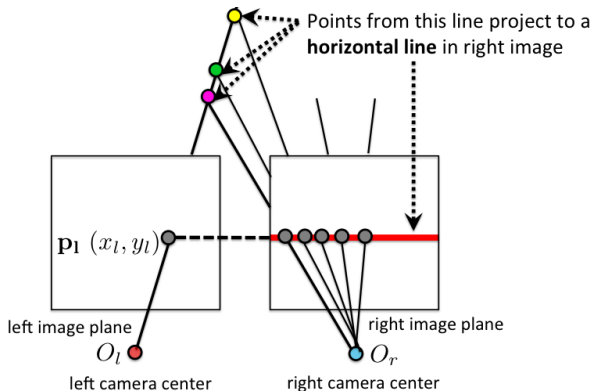
# Stereo: Parallel Calibrated Cameras

- Since lines  $\mathbf{O}_l\mathbf{O}_r$  and  $\mathbf{p}_l\mathbf{p}_r$  are parallel, and  $\mathbf{O}_l$  and  $\mathbf{O}_r$  have the same  $y$ , then also  $\mathbf{p}_l$  and  $\mathbf{p}_r$  have the same  $y$ :  $y_r = y_l$ !



# Stereo: Parallel Calibrated Cameras

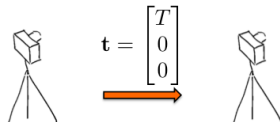
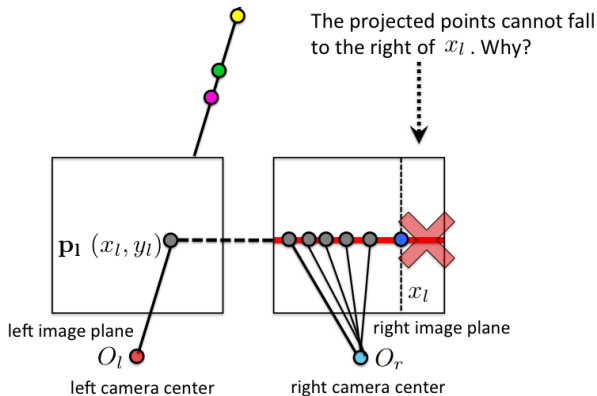
- So all points on the projective line  $\mathbf{O}_l \mathbf{p}_l$  project to a horizontal line with  $y = y_l$  on the right image. This is nice, let's remember this.





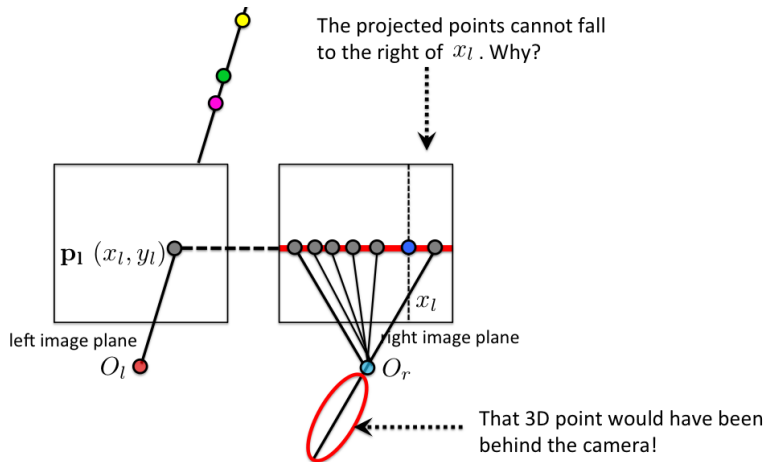
# Stereo: Parallel Calibrated Cameras

- Another observation: No point from  $\mathbf{O}_l \mathbf{p}_l$  can project to the right of  $x_l$  in the right image. **Why?**



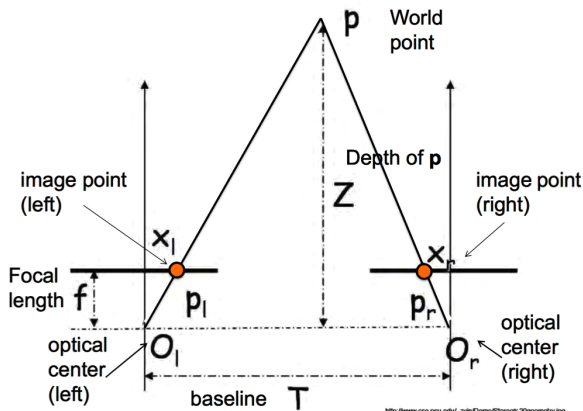
# Stereo: Parallel Calibrated Cameras

- Because that would mean our image can see behind the camera...



# Stereo: Parallel Calibrated Cameras

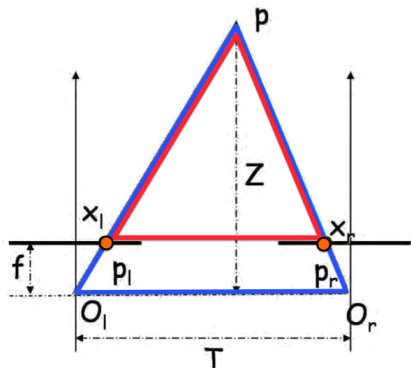
- Since our points  $\mathbf{p}_l$  and  $\mathbf{p}_r$  lie on a horizontal line, we can forget about  $y_l$  for a moment (it doesn't seem important). Let's look at the camera situation from the birdseye perspective instead. Let's see if we can find a connection between  $x_l$ ,  $x_r$  and  $Z$  (because  $Z$  is what we want).



[Adopted from: J. Hays]

# Stereo: Parallel Calibrated Cameras

- We can then use similar triangles to compute the depth of the point  $P$



Similar triangles:

$$\frac{T}{Z} = \frac{T + x_r - x_l}{Z - f}$$

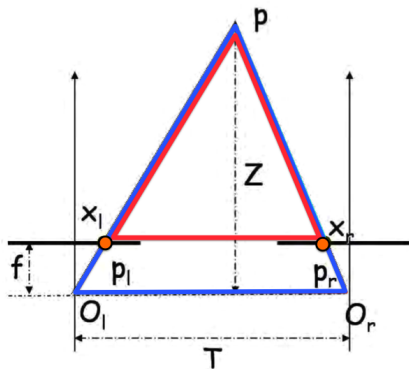
$$Z = \frac{f \cdot T}{x_l - x_r}$$

Labels for the equation above:  
-  $f$ : focal length  
-  $T$ : baseline  
-  $x_l - x_r$ : disparity

[Adopted from: J. Hays]

# Stereo: Parallel Calibrated Cameras

- We can then use similar triangles to compute the depth of the point  $P$



Similar triangles:

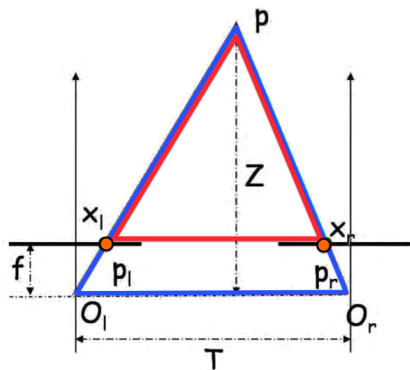
$$\frac{T}{Z} = \frac{T + x_l - x_r}{Z - f}$$

$$Z = \frac{f \cdot T}{x_r - x_l}$$

So if I know  $x_l$  and  $x_r$ , then I can compute  $Z$ !

# Stereo: Parallel Calibrated Cameras

- We can then use similar triangles to compute the depth of the point  $P$



Similar triangles:

$$\frac{T}{Z} = \frac{T + x_l - x_r}{Z - f}$$

$$Z = \frac{f \cdot T}{x_r - x_l}$$

$$x = \frac{f \cdot X}{Z} + p_x$$

And if I know  $Z$ , I can compute  $X$  and  $Y$ , which gives me the point in 3D

# Stereo: Parallel Calibrated Cameras

- For each point  $\mathbf{p}_l = (x_l, y_l)$ , how do I get  $\mathbf{p}_r = (x_r, y_r)$ ?



left image



right image

# Stereo: Parallel Calibrated Cameras

- For each point  $\mathbf{p}_l = (x_l, y_l)$ , how do I get  $\mathbf{p}_r = (x_r, y_r)$ ? By matching on line  $y_r = y_l$ .



left image

right image

the match will be on this line (same  $y$ )

(CAREFUL: this is only true for parallel cameras. Generally, line not horizontal)



# Stereo: Parallel Calibrated Cameras

- For each point  $\mathbf{p}_l = (x_l, y_l)$ , how do I get  $\mathbf{p}_r = (x_r, y_r)$ ? By matching on line  $y_r = y_l$ .

We are looking for this point



left image

$x_l$



right image

$x_l$

the match will be **on the left** of  $x_l$

**how do I find it?**

# Stereo: Parallel Calibrated Cameras

- For each point  $\mathbf{p}_l = (x_l, y_l)$ , how do I get  $\mathbf{p}_r = (x_r, y_r)$ ? By matching. Patch around  $(x_r, y_r)$  should look similar to the patch around  $(x_l, y_l)$ .

We call this line a **scanline**



left image

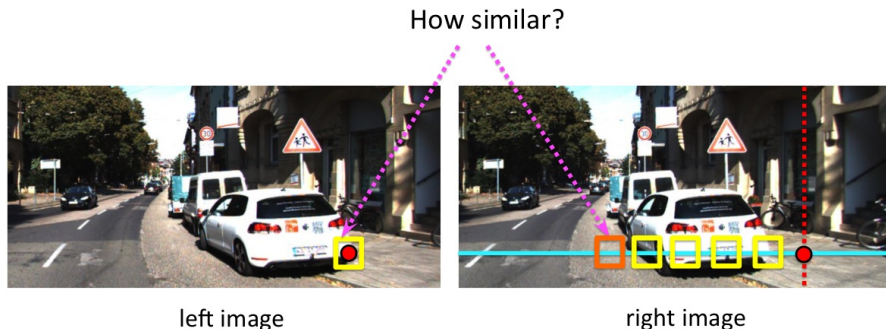


right image

we **scan** the line and **compare** patches to the one in the left image  
We are looking for a patch on scanline most similar to patch on the left

# Stereo: Parallel Calibrated Cameras

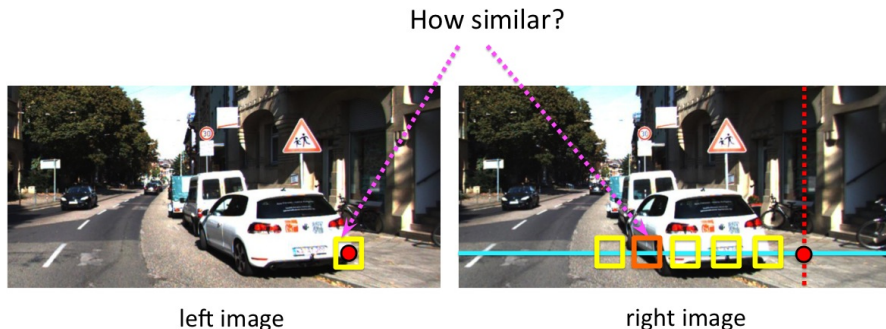
- For each point  $\mathbf{p}_l = (x_l, y_l)$ , how do I get  $\mathbf{p}_r = (x_r, y_r)$ ? By matching. Patch around  $(x_r, y_r)$  should look similar to the patch around  $(x_l, y_l)$ .



we **scan** the line and **compare** patches to the one in the left image  
We are looking for a patch on scanline most similar to patch on the left

# Stereo: Parallel Calibrated Cameras

- For each point  $\mathbf{p}_l = (x_l, y_l)$ , how do I get  $\mathbf{p}_r = (x_r, y_r)$ ? By matching. Patch around  $(x_r, y_r)$  should look similar to the patch around  $(x_l, y_l)$ .



we **scan** the line and **compare** patches to the one in the left image  
We are looking for a patch on scanline most similar to patch on the left

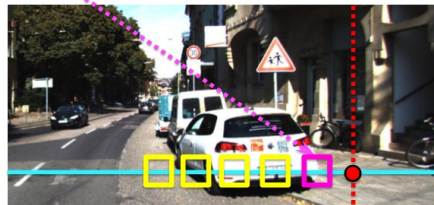
# Stereo: Parallel Calibrated Cameras

- For each point  $\mathbf{p}_l = (x_l, y_l)$ , how do I get  $\mathbf{p}_r = (x_r, y_r)$ ? By matching. Patch around  $(x_r, y_r)$  should look similar to the patch around  $(x_l, y_l)$ .

Most similar. A match!



left image



right image

we **scan** the line and **compare** patches to the one in the left image  
We are looking for a patch on scanline most similar to patch on the left

# Stereo: Parallel Calibrated Cameras

- For each point  $\mathbf{p}_l = (x_l, y_l)$ , how do I get  $\mathbf{p}_r = (x_r, y_r)$ ? By matching. Patch around  $(x_r, y_r)$  should look similar to the patch around  $(x_l, y_l)$ .



left image



At each point on the scanline: Compute a **matching cost**

Matching cost: **SSD** or **normalized correlation**

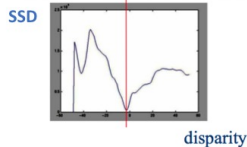
# Stereo: Parallel Calibrated Cameras

- For each point  $\mathbf{p}_l = (x_l, y_l)$ , how do I get  $\mathbf{p}_r = (x_r, y_r)$ ? By matching. Patch around  $(x_r, y_r)$  should look similar to the patch around  $(x_l, y_l)$ .

$$SSD(\text{patch}_l, \text{patch}_r) = \sum_x \sum_y (I_{\text{patch}_l}(x, y) - I_{\text{patch}_r}(x, y))^2$$



left image



Compute a matching cost

Matching cost: **SSD (look for minima)**

# Stereo: Parallel Calibrated Cameras

- For each point  $\mathbf{p}_l = (x_l, y_l)$ , how do I get  $\mathbf{p}_r = (x_r, y_r)$ ? By matching. Patch around  $(x_r, y_r)$  should look similar to the patch around  $(x_l, y_l)$ .

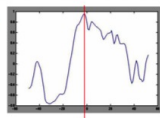
$$NC(\text{patch}_l, \text{patch}_r) = \frac{\sum_x \sum_y (I_{\text{patch}_l}(x, y) \cdot I_{\text{patch}_r}(x, y))}{\|I_{\text{patch}_l}\| \cdot \|I_{\text{patch}_r}\|}$$



left image



Norm.  
Corr.



disparity

Compute a matching cost

Matching cost: **Normalized Corr. (look for maxima)**

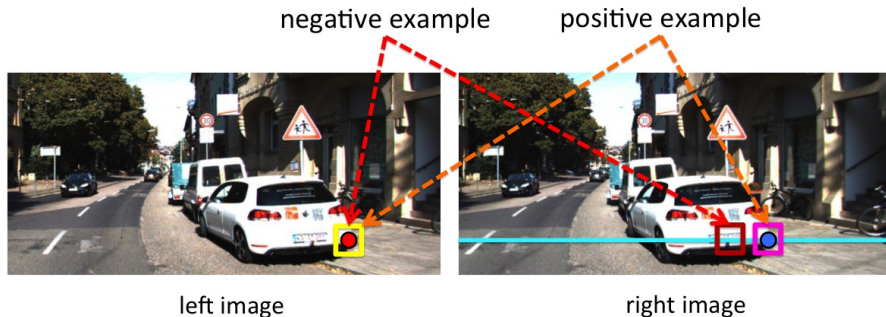


# Stereo: Parallel Calibrated Cameras

- Version'2015: Can I do this task even better?

# Stereo: Parallel Calibrated Cameras

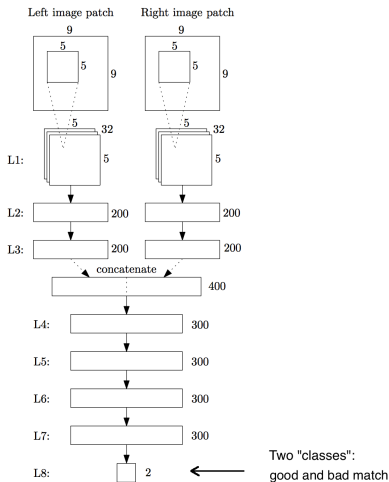
- Version'2015: Train a classifier! How can I get ground-truth?



**Training examples:** get positive and negative matches

# Stereo: Parallel Calibrated Cameras

- Version'2015: Train a Neural Network classifier!



[J. Zbontar and Y. LeCun: Computing the Stereo Matching Cost with a Convolutional Neural Network, CVPR'15]

# Stereo-CNN: Parallel Calibrated Cameras

- Version'2015: Train a Neural Network classifier!
- To get the most amazing performance





	Method	Setting	Code	Out-Noc	Out-All	Avg-Noc	Avg-All	Density	Runtime	Environment	Compare
1	<a href="#">MC-CNN-acrt</a>		<a href="#">code</a>	2.43 %	3.63 %	0.7 px	0.9 px	100.00 %	67 s	Nvidia GTX Titan X (CUDA, Lua/Torch7)	<input type="checkbox"/>
J. Zbontar and Y. LeCun: <a href="#">Stereo Matching by Training a Convolutional Neural Network to Compare Image Patches</a> . Submitted to JMLR.											
2	<a href="#">Displets</a>		<a href="#">code</a>	2.47 %	3.27 %	0.7 px	0.9 px	100.00 %	265 s	>8 cores @ 3.0 Ghz (Matlab + C/C++)	<input type="checkbox"/>
F. Guey and A. Geiger: <a href="#">Displets: Resolving Stereo Ambiguities using Object Knowledge</a> . Conference on Computer Vision and Pattern Recognition (CVPR) 2015.											
3	<a href="#">MC-CNN</a>			2.61 %	3.84 %	0.8 px	1.0 px	100.00 %	100 s	Nvidia GTX Titan (CUDA, Lua/Torch7)	<input type="checkbox"/>
J. Zbontar and Y. LeCun: <a href="#">Computing the Stereo Matching Cost with a Convolutional Neural Network</a> . Conference on Computer Vision and Pattern Recognition (CVPR) 2015.											
4	<a href="#">PRSM</a>		<a href="#">code</a>	2.78 %	3.00 %	0.7 px	0.7 px	100.00 %	300 s	1 core @ 2.5 Ghz (C/C++)	<input type="checkbox"/>
C. Vogel, K. Schindler and S. Roth: <a href="#">3D Scene Flow Estimation with a Piecewise Rigid Scene Model</a> . ijcv 2015.											
5	<a href="#">SPS-StFl</a>			2.83 %	3.64 %	0.8 px	0.9 px	100.00 %	35 s	1 core @ 3.5 Ghz (C/C++)	<input type="checkbox"/>
K. Yamaguchi, D. McAllester and R. Urtasun: <a href="#">Efficient Joint Segmentation, Occlusion Labeling, Stereo and Flow Estimation</a> . ECCV 2014.											
6	<a href="#">YC-SF</a>			3.05 %	3.31 %	0.8 px	0.8 px	100.00 %	300 s	1 core @ 2.5 Ghz (C/C++)	<input type="checkbox"/>
C. Vogel, S. Roth and K. Schindler: <a href="#">View-Consistent 3D Scene Flow Estimation over Multiple Frames</a> . Proceedings of European Conference on Computer Vision. Lecture Notes in, Computer Science 2014.											
7	<a href="#">Deep Embed</a>			3.10 %	4.24 %	0.9 px	1.1 px	100.00 %	3 s	1 core @ 2.5 Ghz (C/C++)	<input type="checkbox"/>
Z. Chen, X. Sun, Y. Yu, L. Wang and C. Huang: <a href="#">A Deep Visual Correspondence Embedding Model for Stereo Matching Costs</a> . ICCV 2015.											
8	<a href="#">JSOSM</a>			3.15 %	3.94 %	0.8 px	0.9 px	100.00 %	105 s	8 cores @ 2.5 Ghz (C/C++)	<input type="checkbox"/>
Anonymous submission											
9	<a href="#">OSF</a>		<a href="#">code</a>	3.28 %	4.07 %	0.8 px	0.9 px	99.98 %	50 min	1 core @ 3.0 Ghz (Matlab + C/C++)	<input type="checkbox"/>
M. Menze and A. Geiger: <a href="#">Object Scene Flow for Autonomous Vehicles</a> . Conference on Computer Vision and Pattern Recognition (CVPR) 2015.											
10	<a href="#">CoR</a>		<a href="#">code</a>	3.30 %	4.10 %	0.8 px	0.9 px	100.00 %	6 s	6 cores @ 3.3 Ghz (Matlab + C/C++)	<input type="checkbox"/>
A. Chakrabarti, Y. Xiong, S. Gortler and T. Zickler: <a href="#">Low-level Vision by Consensus in a Spatial Hierarchy of Regions</a> . CVPR 2015.											

Figure: Performance on KITTI (metrics is error, so lower is better)

# Stereo: Parallel Calibrated Cameras

- For each point  $\mathbf{p}_l = (x_l, y_l)$ , how do I get  $\mathbf{p}_r = (x_r, y_r)$ ? By matching. Patch around  $(x_r, y_r)$  should look similar to the patch around  $(x_l, y_l)$ .



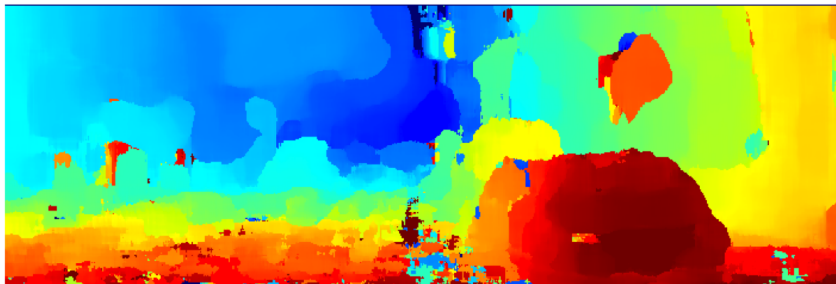
left image



Do this for all the points in the left image!

# Stereo: Parallel Calibrated Cameras

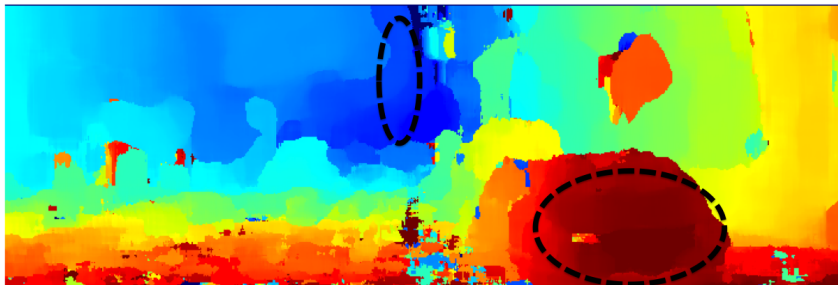
- We get a disparity map as a result



Result: **Disparity map**  
(red values large disp., blue small disp.)

# Stereo: Parallel Calibrated Cameras

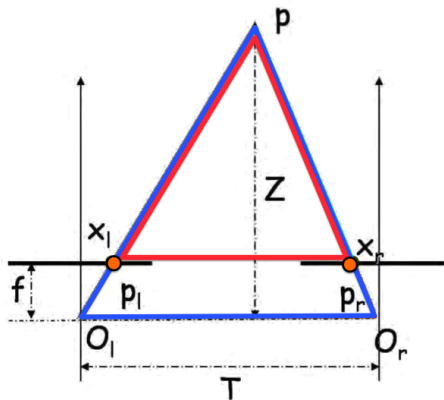
- We get a disparity map as a result



Things that are closer have **larger disparity** than those that are far away from camera. Why?

# Stereo: Parallel Calibrated Cameras

- Depth and disparity are inversely proportional



Similar triangles:

$$\frac{T}{Z} = \frac{T + x_l - x_r}{Z - f}$$

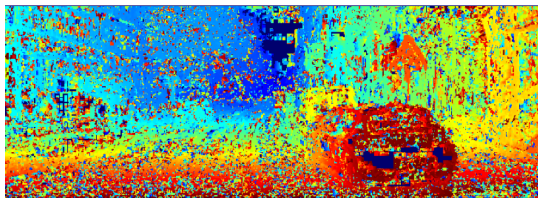
$$Z = \frac{f \cdot T}{x_r - x_l}$$

Depth ( $Z$ ) and disparity are **inversely proportional**

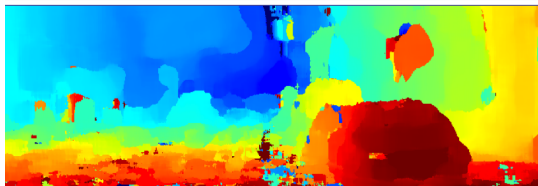


# Stereo: Parallel Calibrated Cameras

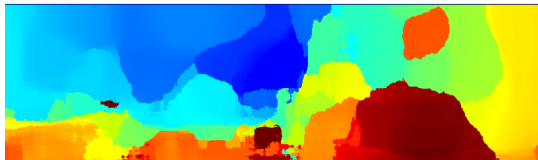
- Smaller patches: more detail, but noisy. Bigger: less detail, but smooth



patch size = 5



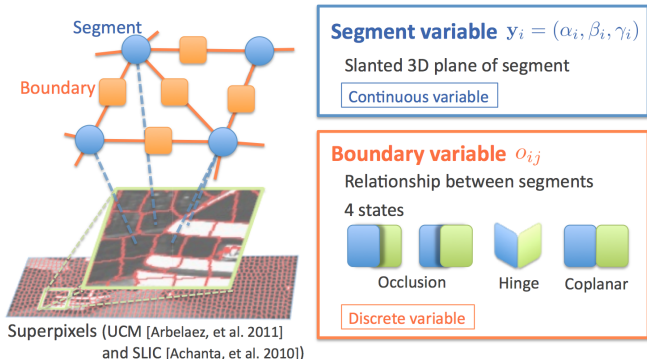
patch size = 35



patch size = 85

# You Can Do It Much Better...

- With Energy Minimization on top, e.g., a Markov Random Field (MRF)



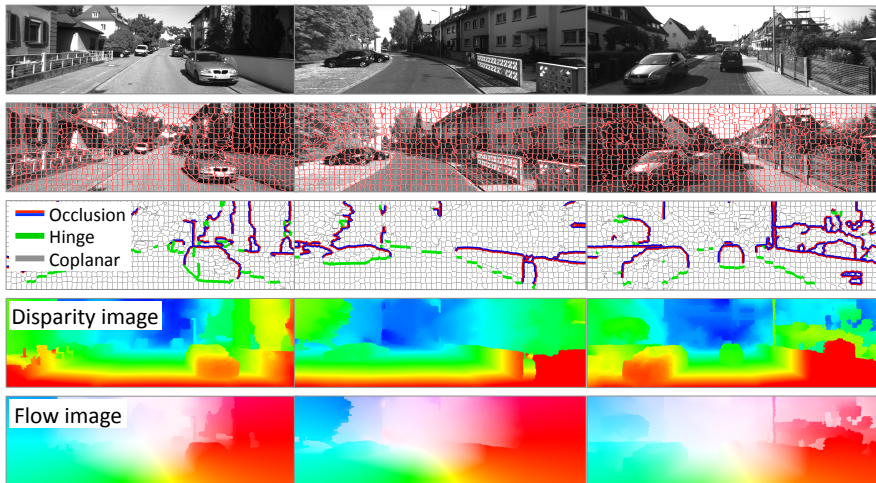
K. Yamaguchi, D. McAllester, R. Urtasun, *Efficient Joint Segmentation, Occlusion Labeling, Stereo and Flow Estimation*, ECCV 2014

Paper: [http://www.cs.toronto.edu/~urtasun/publications/yamaguchi\\_et\\_al\\_eccv14.pdf](http://www.cs.toronto.edu/~urtasun/publications/yamaguchi_et_al_eccv14.pdf)

Code: <http://ttic.uchicago.edu/~dmcallester/SPS/index.html>

# You Can Do It Much Better...

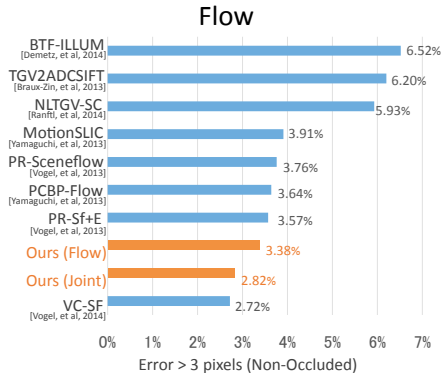
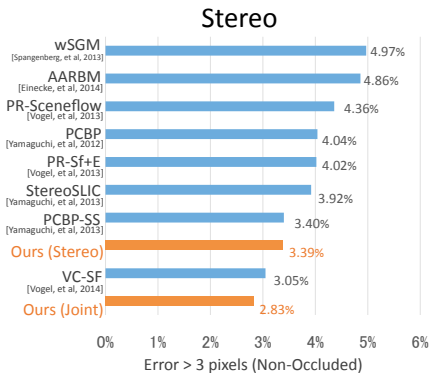
[K. Yamaguchi, D. McAllester and R. Urtasun, ECCV 2014]



# Look at State-of-the-art on KITTI

Where "Ours" means: [K. Yamaguchi, D. McAllester and R. Urtasun, ECCV 2014]

- How can we evaluate the performance of a stereo algorithm?



- Autonomous driving dataset KITTI: <http://www.cvlibs.net/datasets/kitti/>

# From Disparity We Get...

- Depth: Once you have disparity, you have 3D



Figure: K. Yamaguchi, D. McAllester and R. Urtasun, ECCV 2014

# From Disparity We Get...

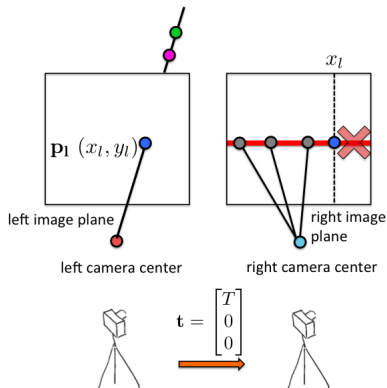
- Money ;) )



## Epipolar geometry

- Case with two cameras with parallel optical axes
- General case ← **Next time**

### Parallel stereo cameras:



### General stereo cameras:

