Matching Planar Objects In New Viewpoints ... And Much More – via Homography

What Transformation Happened To My DVD?

• Rectangle goes to a parallelogram



Affine Transformations

Affine transformations are combinations of

- Linear transformations, and
- Translations

$$\begin{bmatrix} x'\\ y' \end{bmatrix} = \begin{bmatrix} a & b & e\\ c & d & f \end{bmatrix} \begin{bmatrix} x\\ y\\ 1 \end{bmatrix}$$

Properties of affine transformations:

- Origin does not necessarily map to origin
- Lines map to lines
- Parallel lines remain parallel
- Ratios are preserved
- Closed under composition
- Rectangles go to parallelograms
- [Source: N. Snavely, slide credit: R. Urtasun]

What Transformation Really Happened To My DVD?

• What about now?



What Transformation Really Happened To My DVD?

• Actually a rectangle goes to quadrilateral



2D Image Transformations



Transformation	Matrix	# DoF	Preserves	Icon
translation	$\left[egin{array}{c} I \mid t \end{array} ight]_{2 imes 3}$	2	orientation	
rigid (Euclidean)	$\left[egin{array}{c c} R & t \end{array} ight]_{2 imes 3}$	3	lengths	\Diamond
similarity	$\left[\begin{array}{c c} s m{R} & t \end{array} ight]_{2 imes 3}$	4	angles	\Diamond
affine	$\left[\begin{array}{c} A \end{array}\right]_{2 imes 3}$	6	parallelism	\square
projective	$\left[egin{array}{c} ilde{H} \end{array} ight]_{3 imes 3}$	8	straight lines	

- These transformations are a nested set of groups
- Closed under composition and inverse is a member

[source: R. Szeliski]

Projective Transformations

• Homography:

$$w \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Properties:

- Origin does not necessarily map to origin
- Lines map to lines
- Parallel lines do not necessarily remain parallel
- Ratios are **not** preserved
- Closed under composition
- Rectangle goes to quadrilateral
- Affine transformation is a special case, where g = h = 0 and i = 1
- [Source: N. Snavely, slide credit: R. Urtasun]

What Transformation Really Happened to My DVD?



For **planar** objects:

- Viewpoint change for planar objects is a homography
- Affine transformation approximates viewpoint change for planar objects that are far away from camera

What Transformation Happened to My DVD?

- Why should I care about homography?
- Now that I care, how should I estimate it?
- I want to understand the geometry behind homography. That is, why aren't parallel lines mapped to parallel lines in oblique viewpoints? How did we get that equation for computing the homography?

- Why should I care about homography? Let's answer this first
- Now that I care, how should I estimate it?
- I want to understand the geometry behind homography. That is, why aren't parallel lines mapped to parallel lines in oblique viewpoints? How did we get that equation for computing the homography?



- Why do we need homography? Can't we just assume that the transformation is affine? The approximation on the right looks pretty decent to me...
- That's right. If I want to detect (match) an object in a new viewpoint, an affine transformation is a relatively decent approximation



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- But for some applications I want to be more accurate. Which?



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- But for some applications I want to be more accurate. Which?



• Tom Cruise is taking an exam on Monday

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exam is here

• The professor keeps the exams in this office



• He enters (without permission) and takes a picture of the laptop screen



- His picture turns out to not be from a viewpoint he was shooting for (it's difficult to take pictures while hanging)
- Can he still read the exam?

Warping an Image with a Global Transformation



p = (x,y)

p' = (x',y')

• Transformation T is a coordinate-changing machine:

$$[x',y']=T(x,y)$$

- What does it mean that T is global?
 - Is the same for any point p
 - Can be described by just a few numbers (parameters)

[Source: N. Snavely, slide credit: R. Urtasun]

Warping an Image with a Global Transformation

• Example of warping for different transformations:



translation



rotation



aspect



affine



```
perspective
```

Forward and Inverse Warping

• Forward Warping: Send each pixel f(x) to its corresponding location (x', y') = T(x, y) in g(x', y')

procedure forwardWarp(f, h, out g):

For every pixel x in f(x)

- 1. Compute the destination location x' = h(x).
- 2. Copy the pixel f(x) to g(x').
- Inverse Warping: Each pixel at destination is sampled from original image

procedure inverseWarp(f, h, out g):

For every pixel x' in g(x')

- 1. Compute the source location $x = \hat{h}(x')$
- 2. Resample f(x) at location x and copy to g(x')

[source: R. Urtasun]



• We want to transform the picture (plane) inside these 4 points into a rectangle (laptop screen)



• We want it to look like this. How can we do this?

 A transformation that maps a projective plane (a quadrilateral) to another projective plane (another quadrilateral, in this case a rectangle) is a homography

homography H





• If we compute the homography and warp the image according to it, we get this



• If we used affine transformation instead, we'd get this. Would be even worse if our picture was taken closer to the laptop



The floor (enlarged)

Slide from Antonio Criminisi

Automatically rectified floor

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From Martin Kemp The Science of Art (manual reconstruction)



What is the (complicated) shape of the floor pattern?



Automatically rectified floor

St. Lucy Altarpiece, D. Veneziano Slide from Criminisi



Automatic rectification



From Martin Kemp, The Science of Art (manual reconstruction)

Slide from Criminisi





• How many meters did this player run?

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• Field is planar. We know its dimensions (look on Wikipedia).

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• Let's take the 4 corner points of the field

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• We need to compute a homography that maps them to these 4 corners Sanja Fidler



• We need to compute a homography that maps the 4 corners. Any other point from this plane (the field) also maps to the right with the same homography



• Nice. What happened to the players?

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Application 2: How Much do Soccer Players Run?



 \bullet We can now also transform the player's trajectory \rightarrow and we have it in meters!

Application 2: How Much do Soccer Players Run?



• If we used affine transformation... Our estimations of running would not be accurate!

Application 3: Panorama Stitching





Take a tripod, rotate camera and take pictures

[Source: Fernando Flores-Mangas]

Application 3: Panorama Stitching



[Source: Fernando Flores-Mangas]

Application 3: Panorama Stitching





• Each pair of images is related by homography! If we also moved the camera, this wouldn't be true (next class) [Source: Fernando Flores-Mangas]

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- To do panorama stitching, we need to:
 - Match points between pairs of images I and J
 - $\bullet\,$ Compute a transformation between the between matches in I and J : a homography
 - Do it robustly (RANSAC)
 - Warp the first image to the second using the estimated homography
- Apart from the last point, this is exactly the same procedure as for the problem of matching planar objects across viewpoints
- So this should motivate the why do I care part of the homographies

Homography

- Why should I care about homography?
- Now that I care, how should I estimate it? Let's do this now
- I want to understand the geometry behind homography. That is, why aren't parallel lines mapped to parallel lines in oblique viewpoints? How did we get that equation for computing the homography?

- Let (x_i, y_i) be a point on the reference (model) image, and (x'_i, y'_i) its match in the test image
- A homography H maps (x_i, y_i) to (x'_i, y'_i) :

$$\begin{bmatrix} ax'_i \\ ay'_i \\ a \end{bmatrix} = \begin{bmatrix} h_{00} & h_{01} & h_{02} \\ h_{10} & h_{11} & h_{12} \\ h_{20} & h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}$$

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• We can get rid of that a on the left:

$$\begin{aligned} x'_i &= \frac{h_{00}x_i + h_{01}y_i + h_{02}}{h_{20}x_i + h_{21}y_i + h_{22}} \\ y'_i &= \frac{h_{10}x_i + h_{11}y_i + h_{12}}{h_{20}x_i + h_{21}y_i + h_{22}} \end{aligned}$$

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$$\begin{aligned} x_i' &= \frac{h_{00}x_i + h_{01}y_i + h_{02}}{h_{20}x_i + h_{21}y_i + h_{22}} \\ y_i' &= \frac{h_{10}x_i + h_{11}y_i + h_{12}}{h_{20}x_i + h_{21}y_i + h_{22}} \end{aligned}$$

• Hmmmm... Can I still rewrite this into a linear system in *h*? [Source: R. Urtasun]

• From:

$$\begin{aligned} x'_i &= \frac{h_{00}x_i + h_{01}y_i + h_{02}}{h_{20}x_i + h_{21}y_i + h_{22}} \\ y'_i &= \frac{h_{10}x_i + h_{11}y_i + h_{12}}{h_{20}x_i + h_{21}y_i + h_{22}} \end{aligned}$$

• We can easily get this:

$$\begin{aligned} x_i' \left(h_{20} x_i + h_{21} y_i + h_{22} \right) &= h_{00} x_i + h_{01} y_i + h_{02} \\ y_i' \left(h_{20} x_i + h_{21} y_i + h_{22} \right) &= h_{10} x_i + h_{11} y_i + h_{12} \end{aligned}$$

• Rewriting it a little:

$$\begin{aligned} h_{00}x_i + h_{01}y_i + h_{02} - x_i'(h_{20}x_i + h_{21}y_i + h_{22}) &= 0\\ h_{10}x_i + h_{11}y_i + h_{12} - y_i'(h_{20}x_i + h_{21}y_i + h_{22}) &= 0 \end{aligned}$$

• We can re-write these equations:

$$\begin{aligned} h_{00}x_i + h_{01}y_i + h_{02} - x_i' \left(h_{20}x_i - h_{21}y_i - h_{22} \right) &= 0 \\ h_{10}x_i + h_{11}y_i + h_{12} - y_i' \left(h_{20}x_i - h_{21}y_i - h_{22} \right) &= 0 \end{aligned}$$

• as a linear system!

$$\begin{bmatrix} x_i & y_i & 1 & 0 & 0 & 0 & -x'_i x_i & -x'_i y_i & -x'_i \\ 0 & 0 & 0 & x_i & y_i & 1 & -y'_i x_i & -y'_i y_i & -y'_i \end{bmatrix} \begin{bmatrix} h_{00} \\ h_{01} \\ h_{02} \\ h_{10} \\ h_{11} \\ h_{12} \\ h_{20} \\ h_{21} \\ h_{22} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

[Source: R. Urtasun]

• Taking all our matches into account:

$$\begin{bmatrix} x_{1} & y_{1} & 1 & 0 & 0 & 0 & -x_{1}'x_{1} & -x_{1}'y_{1} & -x_{1}'\\ 0 & 0 & 0 & x_{1} & y_{1} & 1 & -y_{1}'x_{1} & -y_{1}'y_{1} & -y_{1}'\\ x_{n} & y_{n} & 1 & 0 & 0 & 0 & -x_{n}'x_{n} & -x_{n}'y_{n} & -x_{n}'\\ 0 & 0 & 0 & x_{n} & y_{n} & 1 & -y_{n}'x_{n} & -y_{n}'y_{n} & -y_{n}' \end{bmatrix} \begin{bmatrix} h_{00} \\ h_{02} \\ h_{11} \\ h_{12} \\ h_{20} \\ h_{21} \\ h_{22} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ h_{21} \\ h_{22} \end{bmatrix}$$

$$A \begin{bmatrix} A \\ B \\ 2n \\ B \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

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- This defines a least squares problem:

 $\min_{\mathbf{h}} ||\mathbf{A}\mathbf{h}||_2^2$

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$$\min_{\mathbf{h}} ||\mathbf{A}\mathbf{h}||_2^2$$

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- $\bullet~$ Solution: $\hat{\boldsymbol{h}}=$ eigenvector of $\boldsymbol{A}^{\mathcal{T}}\boldsymbol{A}$ with smallest eigenvalue

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- How many matches do I need to estimate H?
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- Since **h** is only defined up to scale, solve for unit vector
- Solution: $\hat{\boldsymbol{h}} = \text{eigenvector}$ of $\boldsymbol{A}^{\mathcal{T}}\boldsymbol{A}$ with smallest eigenvalue
- Works with 4 or more points

[Source: R. Urtasun]

Image Alignment Algorithm: Homography

Given images I and J

- Compute image features for I and J
- 2 Match features between I and J
- Sompute homography transformation A between I and J (with RANSAC)

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[Source: N. Snavely]



• Compute the matches

[Source: R. Queiroz Feitosa]



• Estimate the homography and warp

[Source: R. Queiroz Feitosa]



Stitch

[Source: R. Queiroz Feitosa]



[Source: Fernando Flores-Mangas]



[Source: Fernando Flores-Mangas]



[Source: Fernando Flores-Mangas]

Summary – Stuff You Need To Know

- A homography is a mapping between projective planes
- You need at least 4 correspondences (matches) to compute it

Matlab functions:

- TFORM = MAKETFORM('AFFINE', [X1,Y1], [X2,Y2]); % Computes affine transformation between points [x₁, y₁] and [x₂, y₂]. Needs 3 pairs of matches (x₁, y₁, x₂, y₂ have three rows)
- TFORM = MAKETFORM('PROJECTIVE',[X1,Y1],[X2,Y2]); % Computes homography between points [*x*₁, *y*₁] and [*x*₂, *y*₂]. Needs 4 pairs of matches
- IMW = IMTRANSFORM(IM, TFORM, 'BICUBIC', 'FILL', 0); % Warps the image according to transformation

Birdseye View on What We Learned So Far

Problem	Detection	Description	Matching	
Find Planar	Scale Invariant Local feature:		All features to all features	
Distinctive Objects	Interest Points	SIFT	+ Affine / Homography	
Panorama Stitching		Local feature:	All features to all features	
	Interest Points	SIFT	+ Homography	

• Can I walk here during the night? Can we tell this from an image?



• Can I walk here during the night? Can we tell this from an image?



It's Chicago…



Chicago violent crimes			Population 2,708,382	
	MURDER	RAPE	ROBBERY	ASSAULT
REPORT TOTAL	500	UNREPORTED	13,506	12,277
RATE PER 1,000	0.18	UNREPORTED	4.99	4.53

http://www.neighborhoodscout.com/il/chicago/crime/

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• It's Chicago... Can I walk here during the day?





• Idea: Match image to Google's StreetView images of Chicago!

• Our match to StreetView



• Lookup the GPS location...



• Lookup the crime map for that GPS location



http://www.neighborhoodscout.com/il/chicago/crime/

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• We're in 2017...

Think not (only) what you can do with one image, but what **lots and lots** of images can do for you

• We're in 2017...

Think not (only) what you can do with one image, but what **lots and lots** of images can do for you

• Would our current matching method work with lots of data?

- So far we matched a known object in a new viewpoint
- What if we have to match an object to LOTS of images? Or LOTS of objects to one image?
- Please read this and we will discuss:

Josef Sivic, Andrew Zisserman

Video Google: A Text Retrieval Approach to Object Matching in Videos ICCV 2003

Paper link: http://www.robots.ox.ac.uk/~vgg/publications/papers/sivic03.pdf

Next Time: Camera Models