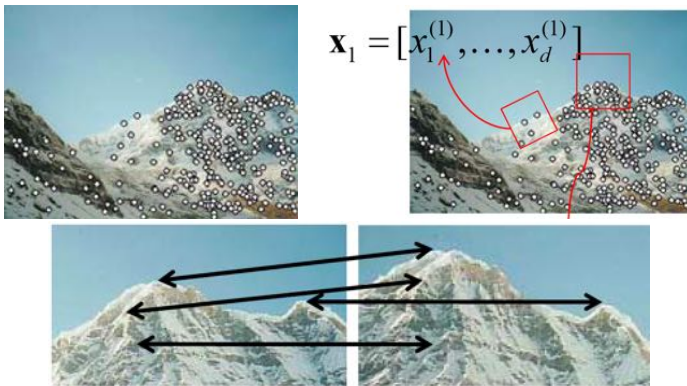


Image Features: Local Descriptors

Local Features

- **Detection:** Identify the interest points.
- **Description:** Extract a feature descriptor around each interest point.
- **Matching:** Determine correspondence between descriptors in two views.



[Source: K. Grauman]

The Ideal Feature Descriptor

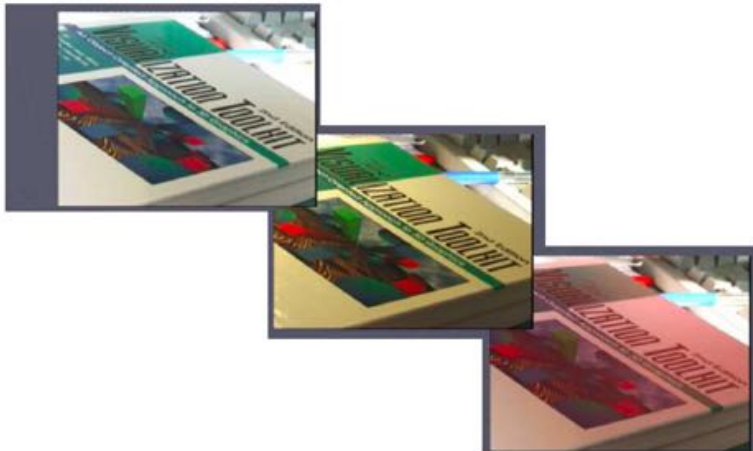
- **Repeatable:** Invariant to rotation, scale, photometric variations
- **Distinctive:** We will need to match it to lots of images/objects!
- **Compact:** Should capture rich information yet not be too high-dimensional (otherwise matching will be slow)
- **Efficient:** We would like to compute it (close-to) real-time

Invariances



[Source: T. Tuytelaars]

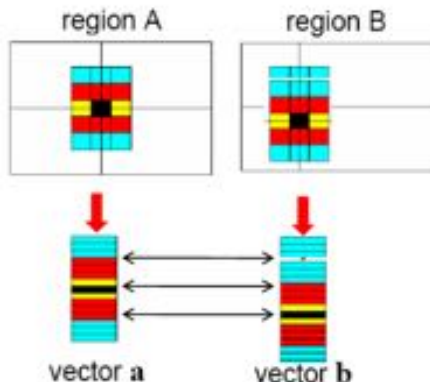
Invariances



[Source: T. Tuytelaars]

What If We Just Took Pixels?

- The simplest way is to write down the list of intensities to form a feature vector, and normalize them (i.e., mean 0, variance 1).
- Why normalization?
- But this is very sensitive to even small shifts, rotations and any affine transformation.



Tones Of Better Options

- SIFT
- PCA-SIFT
- GLOH
- HOG
- SURF
- DAISY
- LBP
- Shape Contexts
- Color Histograms

Tones Of Better Options

- **SIFT** **TODAY**
- PCA-SIFT
- GLOH
- HOG
- SURF
- DAISY
- LBP
- Shape Contexts
- Color Histograms

SIFT Descriptor [Lowe 2004]

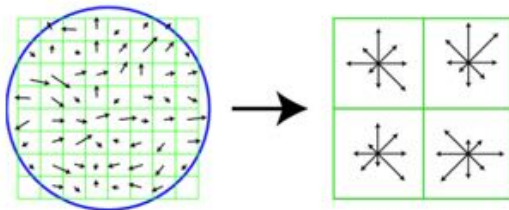
- SIFT stands for Scale Invariant Feature Transform
- Invented by David Lowe, who also did DoG scale invariant interest points
- Actually in the same paper, which you should read:

David G. Lowe

*Distinctive image features from scale-invariant
keypoints*

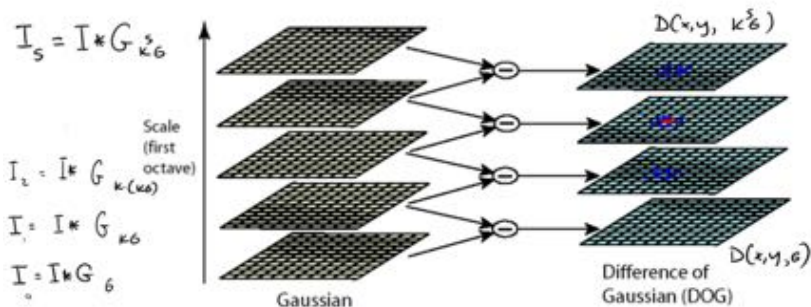
International Journal of Computer Vision, 2004

Paper: <http://www.cs.ubc.ca/~lowe/papers/ijcv04.pdf>



SIFT Descriptor

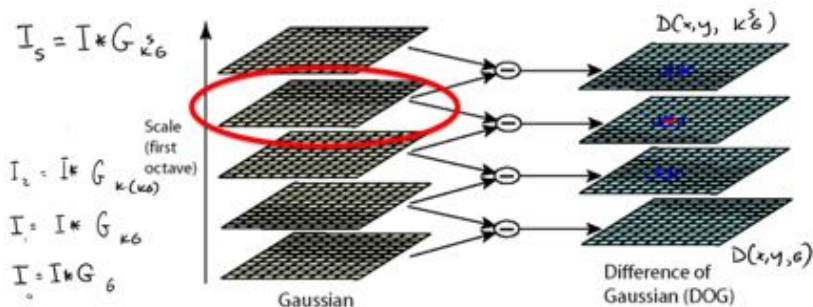
- 1 Our scale invariant interest point detector gives scale ρ for each keypoint



[Adopted from: F. Flores-Mangas]

SIFT Descriptor

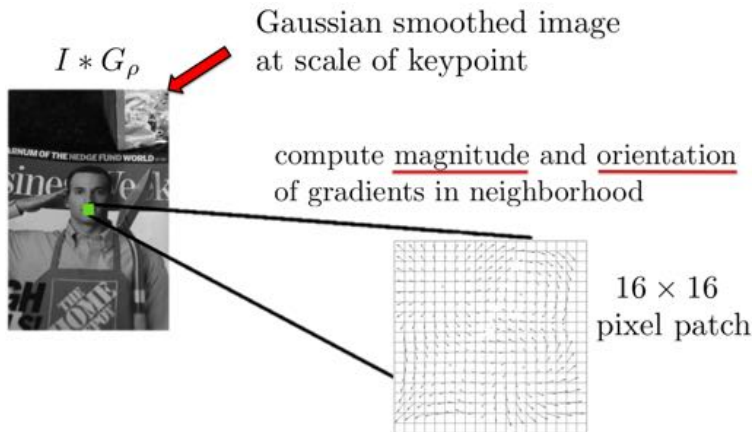
- ② For each keypoint, we take the Gaussian-blurred image at corresponding scale ρ



[Adopted from: F. Flores-Mangas]

SIFT Descriptor

- ③ Compute the gradient magnitude and orientation in neighborhood of each keypoint



[Adopted from: F. Flores-Mangas]

SIFT Descriptor

- ③ Compute the gradient magnitude and orientation in neighborhood of each keypoint

magnitude of gradient:

$$|\nabla I(x, y)| = \sqrt{\left(\frac{\partial(I(x, y) * G_\rho)}{\partial x}\right)^2 + \left(\frac{\partial(I(x, y) * G_\rho)}{\partial y}\right)^2}$$

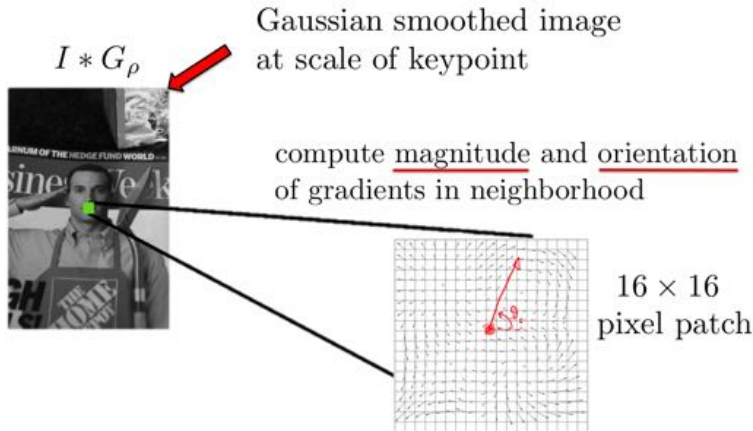
gradient orientation:

$$\theta(x, y) = \arctan\left(\frac{\partial I * G_\rho}{\partial y} / \frac{\partial I * G_\rho}{\partial x}\right)$$

(in case you forgot ;))

SIFT Descriptor

- 4 Compute dominant orientation of each keypoint. How?

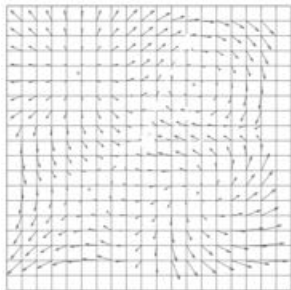


[Adopted from: F. Flores-Mangas]

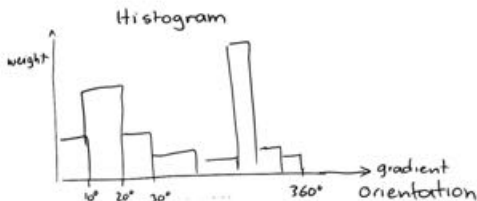
SIFT Descriptor: Computing Dominant Orientation

- Compute a histogram of gradient orientations, each bin covers 10°

16×16



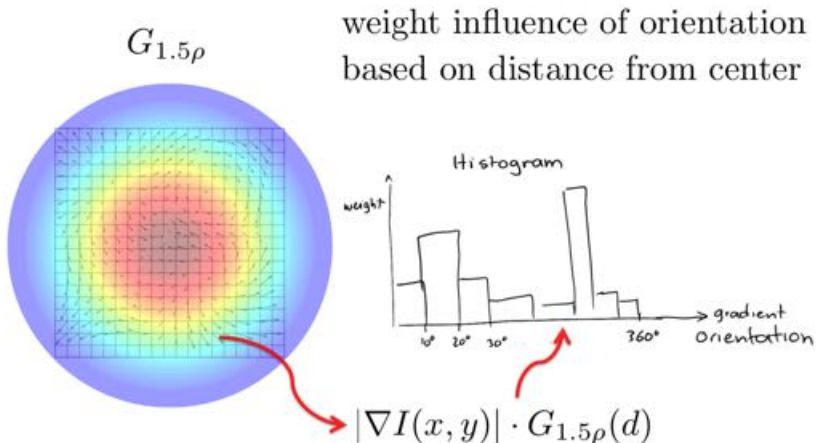
compute histograms of orientations
by orientation increments of 10°



[Adopted from: F. Flores-Mangas]

SIFT Descriptor: Computing Dominant Orientation

- Compute a histogram of gradient orientations, each bin covers 10°
- Orientations closer to the keypoint center should contribute more

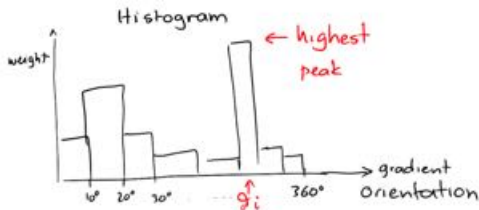
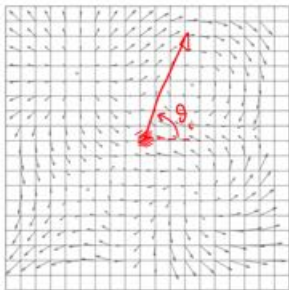


[Adopted from: F. Flores-Mangas]

SIFT Descriptor: Computing Dominant Orientation

- Compute a histogram of gradient orientations, each bin covers 10°
- Orientations closer to the keypoint center should contribute more
- Orientation giving the peak in the histogram is the keypoint's orientation

16×16

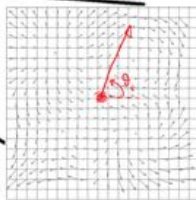


SIFT Descriptor

4 Compute dominant orientation



compute magnitude and orientation
of gradients in neighborhood

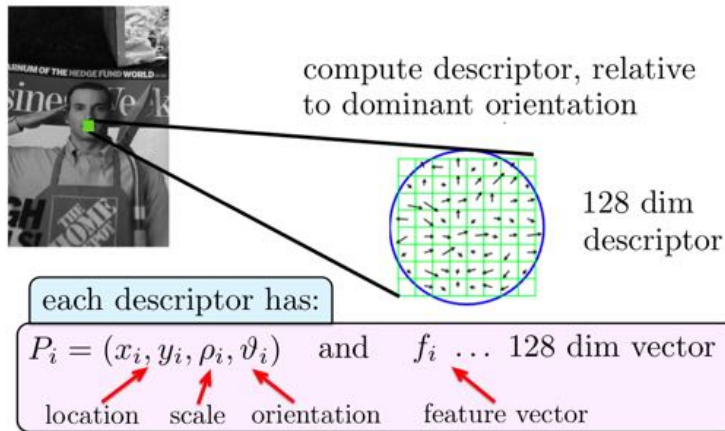


16×16
pixel patch

[Adopted from: F. Flores-Mangas]

SIFT Descriptor

- 5 Compute a 128 dimensional descriptor: 4×4 grid, each cell is a histogram of 8 orientation bins relative to dominant orientation

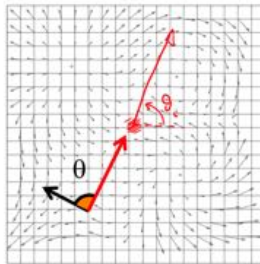
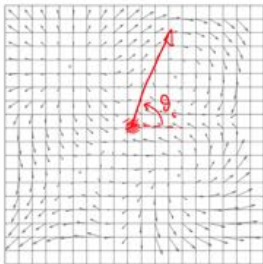


[Adopted from: F. Flores-Mangas]

SIFT Descriptor: Computing the Feature Vector

- Compute the orientations **relative** to the **dominant orientation**

16×16 patch
centered in (x_i, y_i)

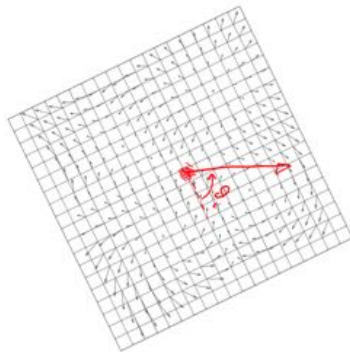
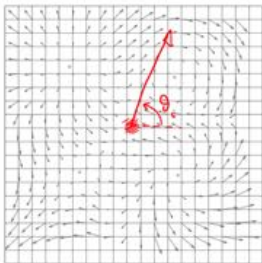


[Adopted from: F. Flores-Mangas]

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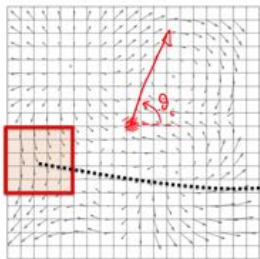


[Adopted from: F. Flores-Mangas]

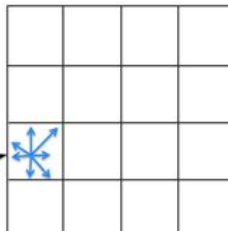
SIFT Descriptor: Computing the Feature Vector

- Compute the orientations **relative** to the **dominant orientation**
- Form a 4×4 grid. For each grid cell compute a histogram of orientations for 8 orientation bins spaced apart by 45°

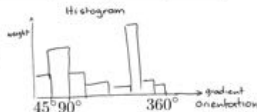
16×16 patch
centered in (x_i, y_i)



SIFT descriptor



compute histogram of orientations
this time 8 bins spaced by 45°

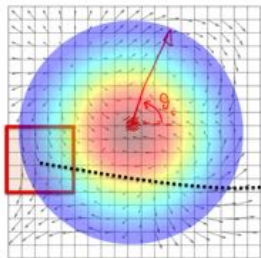


[Adopted from: F. Flores-Mangas]

SIFT Descriptor: Computing the Feature Vector

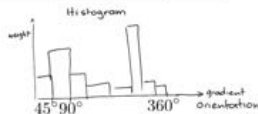
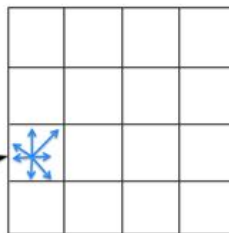
- Compute the orientations **relative** to the **dominant orientation**
- Form a 4×4 grid. For each grid cell compute a histogram of orientations for 8 orientation bins spaced apart by 45°

16×16 patch
centered in (x_i, y_i)



again weigh contributions
this time: $|\nabla I(x, y)| \cdot G_{0.5\rho}$

SIFT descriptor

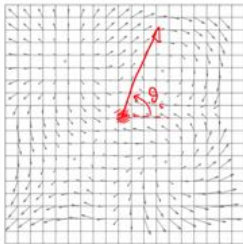


[Adopted from: F. Flores-Mangas]

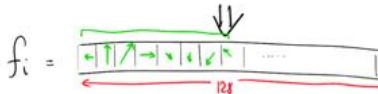
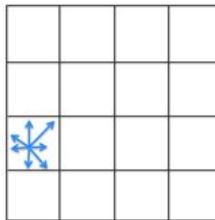
SIFT Descriptor: Computing the Feature Vector

- Compute the orientations **relative** to the **dominant orientation**
- Form a 4×4 grid. For each grid cell compute a histogram of orientations for 8 orientation bins spaced apart by 45°
- Form the 128 dimensional feature vector

16×16 patch
centered in (x_i, y_i)



SIFT descriptor



[Adopted from: F. Flores-Mangas]

SIFT Descriptor: Post-processing

- The resulting 128 non-negative values form a **raw version** of the SIFT descriptor vector.
- To reduce the **effects of contrast or gain** (additive variations are already removed by the gradient), the 128-D vector is normalized to unit length: $f_i = f_i / ||f_i||$

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- What is SIFT invariant to?

SIFT Descriptor: Post-processing

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- Great engineering effort!
- What is SIFT invariant to?

Properties of SIFT

Invariant to:

- Scale
- Rotation

Partially invariant to:

- Illumination changes (sometimes even day vs. night)
- Camera viewpoint (up to about 60 degrees of out-of-plane rotation)
- Occlusion, clutter (why?)

Also important:

- Fast and efficient – can run in real time
- Lots of code available

Examples



Figure: Matching in day / night under viewpoint change

[Source: S. Seitz]

Example

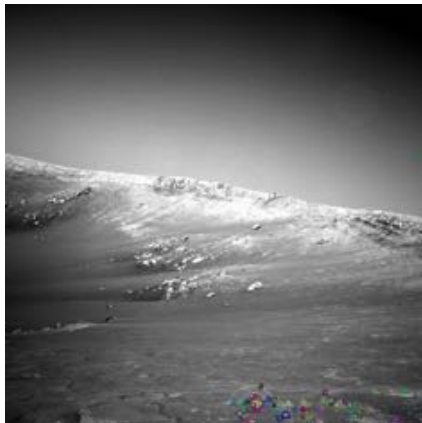
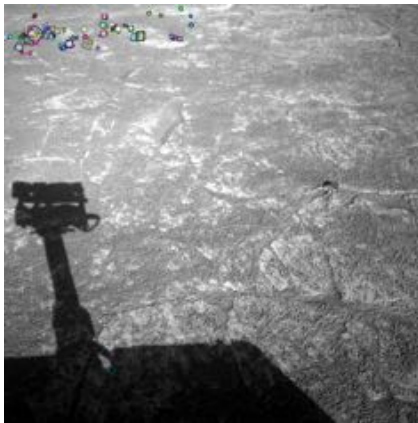


Figure: NASA Mars Rover images with SIFT feature matches

[Source: N. Snavely]

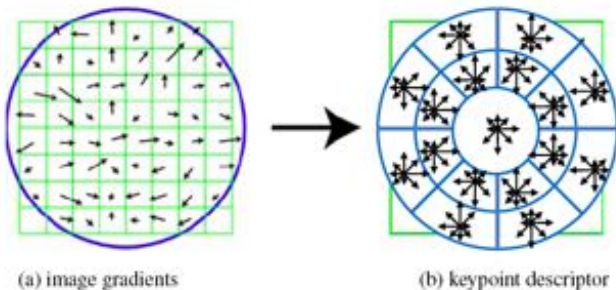
PCA-SIFT

- The dimensionality of SIFT is pretty high, i.e., 128D for each keypoint
- Reduce the dimensionality using linear dimensionality reduction
- In this case, principal component analysis (PCA)
- Use 10D or so descriptor

[Source: R. Urtasun]

Gradient location-orientation histogram (GLOH)

- Developed by Mikolajczyk and Schmid (2005): variant of SIFT that uses a log-polar binning structure instead of the four quadrants.
- The spatial bins are 11, and 15, with eight angular bins (except for the central region), for a total of 17 spatial bins and 16 orientation bins.
- The 272D histogram is then projected onto a 128D descriptor using PCA trained on a large database.



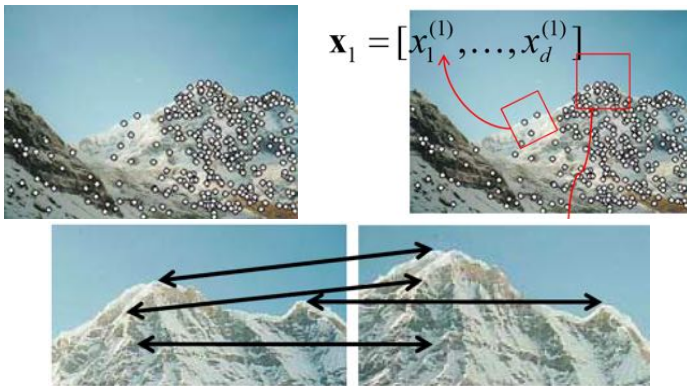
[Source: R. Szeliski]

Other Descriptors

- SURF
- DAISY
- LBP
- HOG
- Shape Contexts
- Color Histograms

Local Features

- **Detection:** Identify the interest points.
- **Description:** Extract feature descriptor around each interest point.
- **Matching:** Determine correspondence between descriptors in two views.



[Source: K. Grauman]

Image Features: Matching the Local Descriptors

Matching the Local Descriptors

Once we have extracted keypoints and their descriptors, we want to match the features between pairs of images.

- Ideally a match is a correspondence between a local part of the object on one image to the same local part of the object in another image
- How should we compute a match?



Figure: Images from K. Grauman

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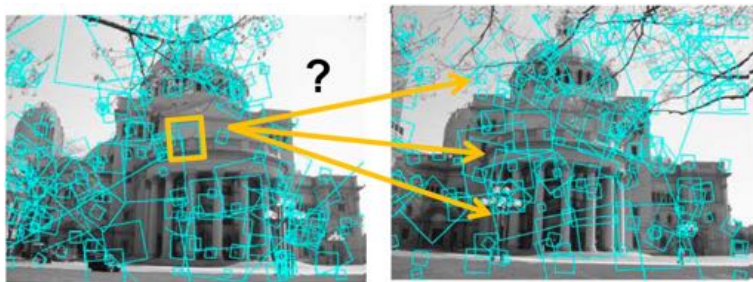
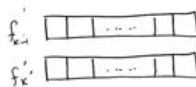
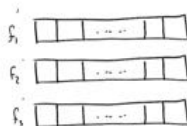
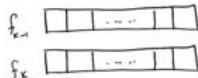
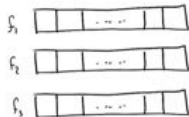
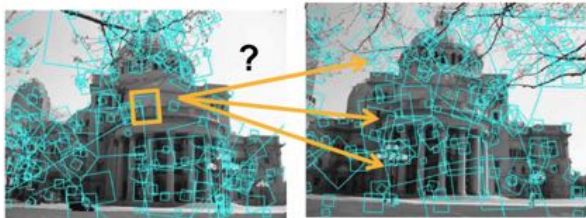


Figure: Images from K. Grauman

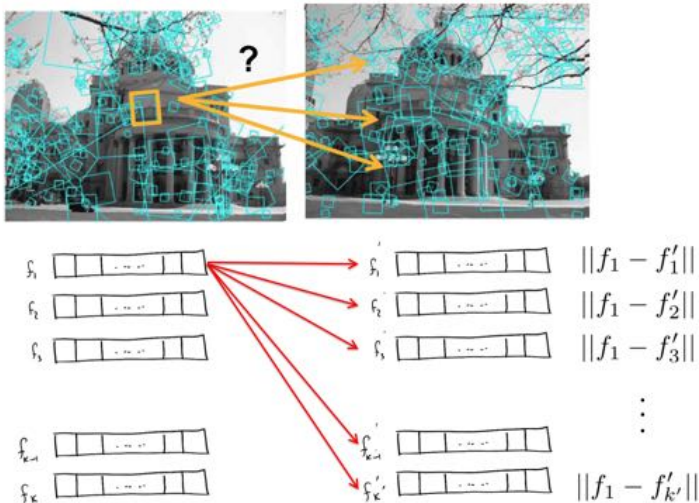
Matching the Local Descriptors

- Simple: **Compare them all**, compute Euclidean distance



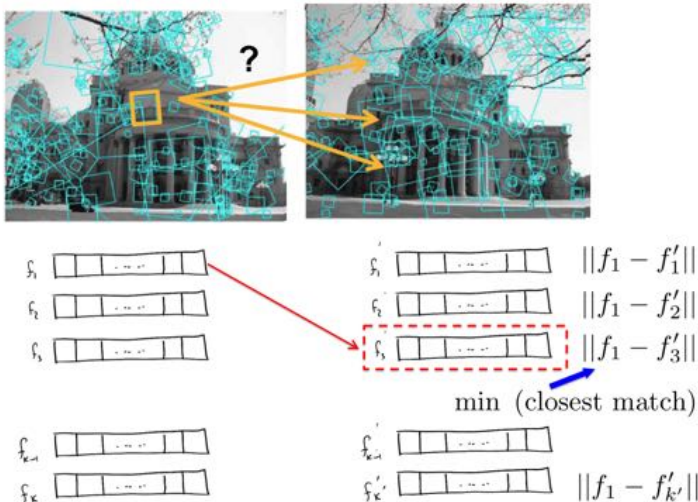
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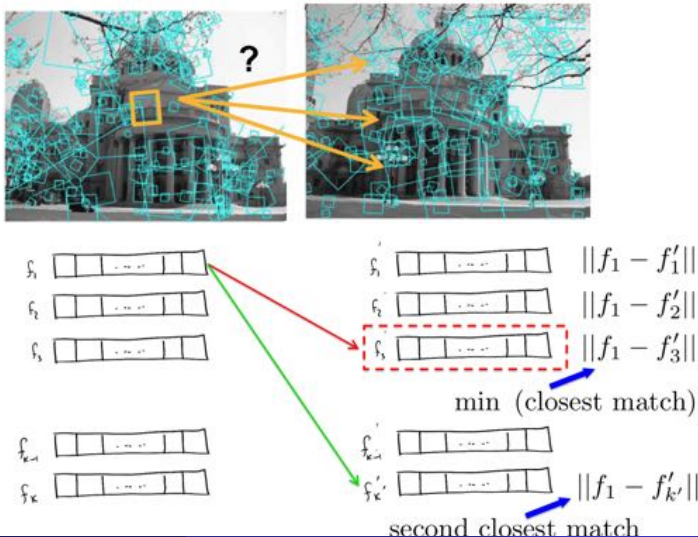
Matching the Local Descriptors

- Find closest match (min distance). How do we know if match is **reliable**?



Matching the Local Descriptors

- Find also the second closest match. Match reliable if first distance “much” smaller than second distance

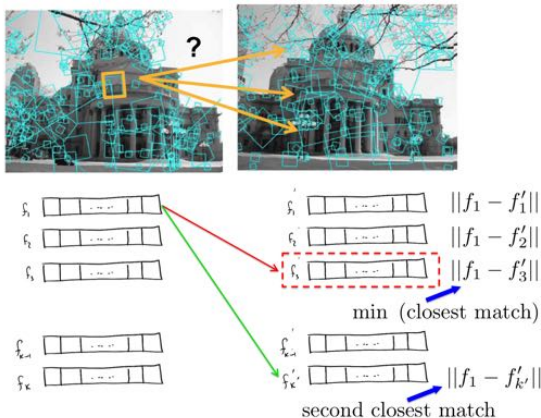


Matching the Local Descriptors

- Compute the ratio:

$$\phi_i = \frac{\|f_i - f_i^*\|}{\|f_i - f_i^{**}\|}$$

where f_i^* is the closest and f_i^{**} second closest match to f_i .



Which Threshold to Use?

- Setting the threshold too high results in too many false positives, i.e., incorrect matches being returned.
- Setting the threshold too low results in too many false negatives, i.e., too many correct matches being missed

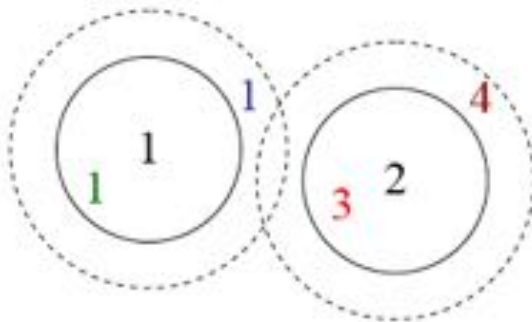


Figure: Images from R. Szeliski

Which Threshold to Use?

- Threshold ratio of nearest to 2nd nearest descriptor
- Typically: $\phi_i < 0.8$

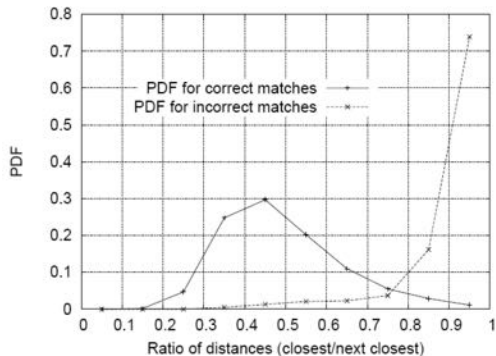


Figure: Images from D. Lowe

[Source: K. Grauman]

Applications of Local Invariant Features

- Wide baseline stereo
- Motion tracking
- Panorama stitching
- Mobile robot navigation
- 3D reconstruction
- Recognition
- Retrieval

[Source: K. Grauman]

Wide Baseline Stereo



[Source: T. Tuytelaars]

Recognizing the Same Object



Schmid and Mohr 1997



Sivic and Zisserman, 2003



Rothganger et al. 2003



Lowe 2002

[Source: K. Grauman]

Motion Tracking



Figure: Images from J. Pilet

Now What

- Now we know how to extract scale and rotation invariant features
- We even know how to match features across images
- Can we use this to find Waldo in an even more sneaky scenario?

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template

Waldo on the road

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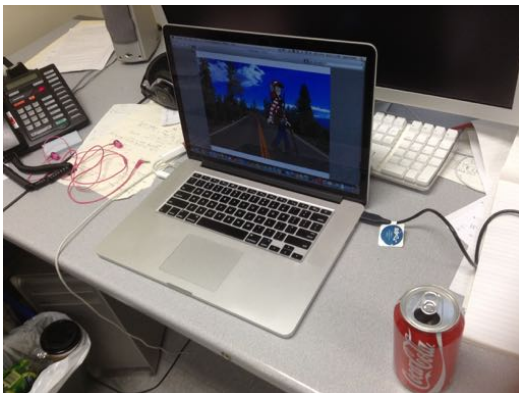


template

He comes closer... We know how to solve this

Now What

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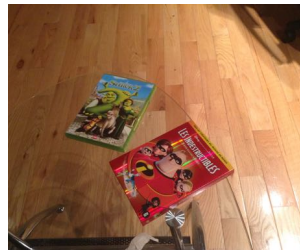
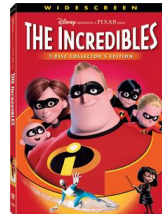


template

Someone takes a (weird) picture of him!

Find My DVD!

- More interesting: If we have DVD covers (e.g., from Amazon), can we match them to DVDs in real scenes?



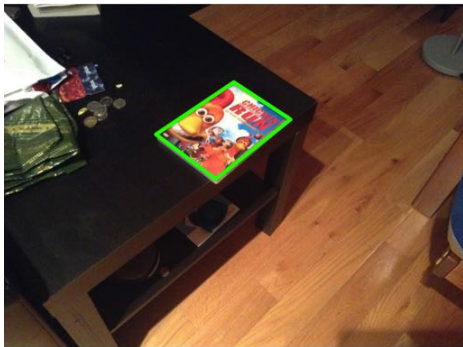
Matching Planar Objects In New Viewpoints

What Kind of Transformation Happened To My DVD?



What Kind of Transformation Happened To My DVD?

- Rectangle goes to a parallelogram (almost but not really, but let's believe that for now)



All 2D Linear Transformations

Linear transformations are combinations of

- Scale,
- Rotation
- Shear
- Mirror

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

[Source: N. Snavely]

All 2D Linear Transformations

Properties of linear transformations:

- Origin maps to origin
- Lines map to lines

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What about the translation?

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What about the translation?

[Source: N. Snavely]

Affine Transformations

Affine transformations are combinations of

- Linear transformations, and
- Translations

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same as:

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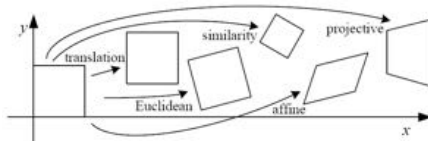
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Properties of affine transformations:

- Origin does not necessarily map to origin
- Lines map to lines
- Parallel lines remain parallel
- Ratios are preserved
- Closed under composition
- Rectangles go to parallelograms

[Source: N. Snavely]

2D Image Transformations

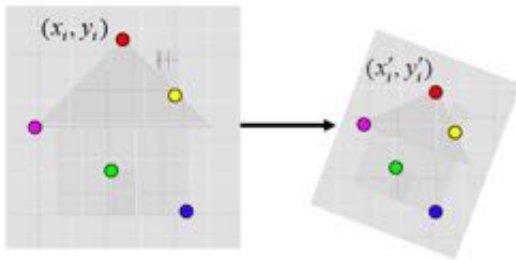


Transformation	Matrix	# DoF	Preserves	Icon
translation	$\begin{bmatrix} I & t \end{bmatrix}_{2 \times 3}$	2	orientation	
rigid (Euclidean)	$\begin{bmatrix} R & t \end{bmatrix}_{2 \times 3}$	3	lengths	
similarity	$\begin{bmatrix} sR & t \end{bmatrix}_{2 \times 3}$	4	angles	
affine	$\begin{bmatrix} A \end{bmatrix}_{2 \times 3}$	6	parallelism	
projective	$\begin{bmatrix} \tilde{H} \end{bmatrix}_{3 \times 3}$	8	straight lines	

- These transformations are a nested set of groups
- Closed under composition and inverse is a member

What Transformation Happened to My DVD?

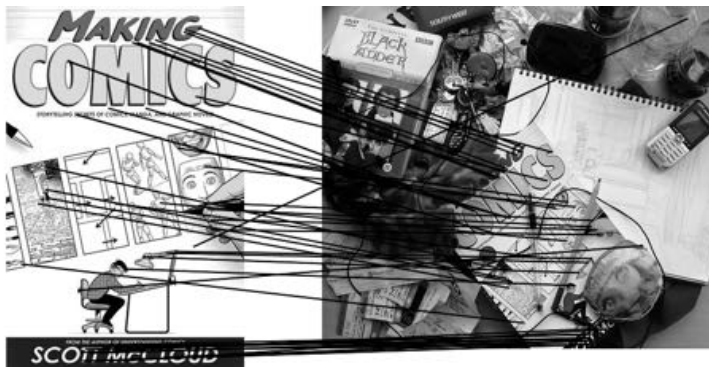
- Affine transformation approximates viewpoint changes for roughly **planar objects** and roughly **orthographic cameras** (more about these later in class)
- DVD went affine!



Computing the (Affine) Transformation

Given a set of matches between images I and J

- How can we compute the affine transformation A from I to J ?
- Find transform A that best agrees with the matches

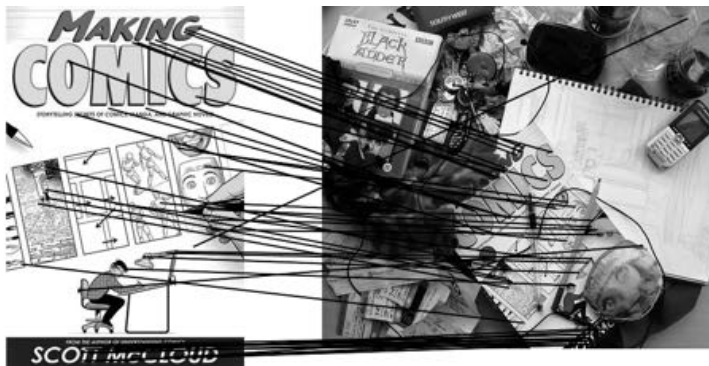


[Source: N. Snavely]

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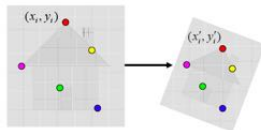
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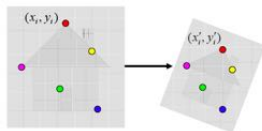
Computing the Affine Transformation



- Let (x_i, y_i) be a point on the reference (model) image, and (x'_i, y'_i) its match in the test image
- An affine transformation A maps (x_i, y_i) to (x'_i, y'_i) :

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- We can rewrite this into a simple linear system:

$$\begin{bmatrix} x_i & y_i & 0 & 0 & 1 & 0 \\ 0 & 0 & x_i & y_i & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix} = \begin{bmatrix} x'_i \\ y'_i \end{bmatrix}$$

Computing the Affine Transformation

- But we have many matches:

$$\underbrace{\begin{bmatrix} \vdots \\ x_i & y_i & 0 & 0 & 1 & 0 \\ 0 & 0 & x_i & y_i & 0 & 1 \\ \vdots \end{bmatrix}}_{\mathbf{P}} \underbrace{\begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix}}_{\mathbf{a}} = \underbrace{\begin{bmatrix} \vdots \\ x'_i \\ y'_i \\ \vdots \end{bmatrix}}_{\mathbf{P}'}$$

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- For each match we have two more equations
- How many matches do we need to compute \mathbf{A} ?
- 6 parameters \rightarrow 3 matches
- But the more, the better (more reliable)
- How do we compute \mathbf{A} ?

Computing the Affine Transformation

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- If we have more than 3, then we do **least-squares estimation**:

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- Which has a closed form solution:

$$\mathbf{a} = (\mathbf{P}^T \mathbf{P})^{-1} \mathbf{P}^T \mathbf{P}'$$

Image Alignment Algorithm: Affine Case

Given images I and J

- 1 Compute image features for I and J
- 2 Match features between I and J
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Image Alignment Algorithm: Affine Case

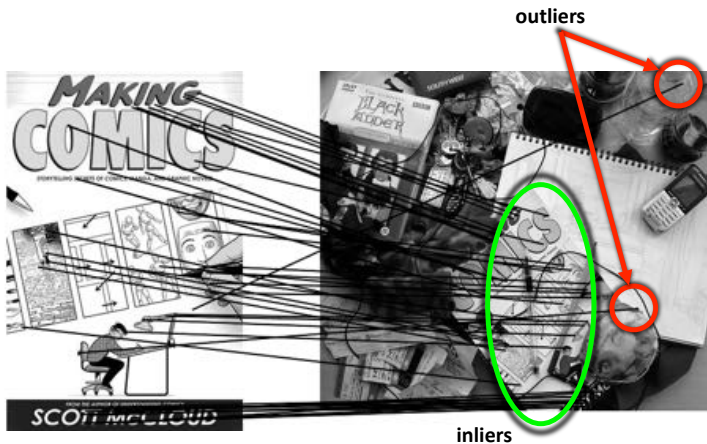
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Is there a problem with this?

[Source: N. Snavely]

Robustness

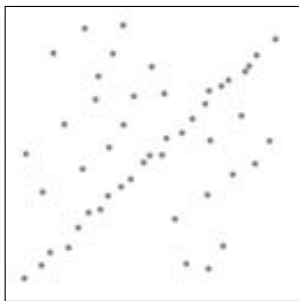


[Source: N. Snavely]

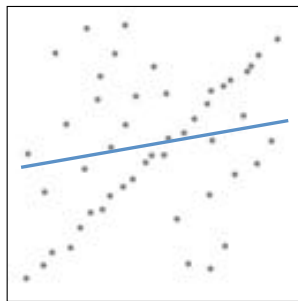
Simple Case

(This example is unrelated to the object matching example, but it nicely shows how to robustify estimation)

- Lets consider a simpler example ... Fit a line to the points below!



Problem: Fit a line to these datapoints

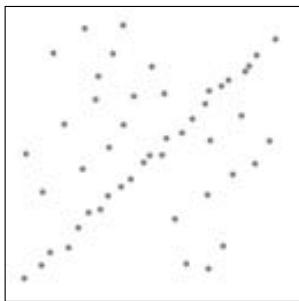


Least squares fit

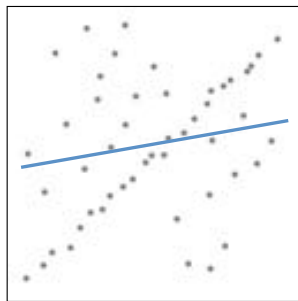
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Problem: Fit a line to these datapoints



Least squares fit

- How can we fix this?

[Source: N. Snavely]

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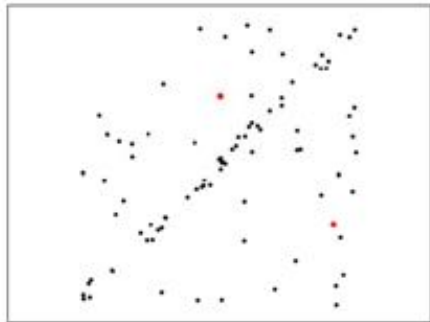
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- Repeat this many times, remember the number of inliers for each trial
- Among several trials, select the one with the largest number of inliers

This procedure is called **RA**ndom **SA**mple **C**onsensus

RANSAC for Line Fitting Example

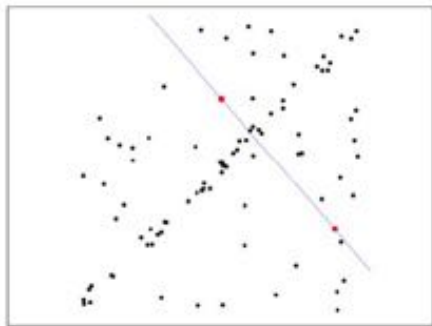
- 1 Randomly select minimal subset of points



[Source: R. Raguram]

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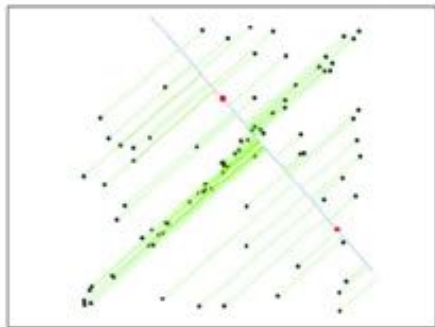
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[Source: R. Raguram]

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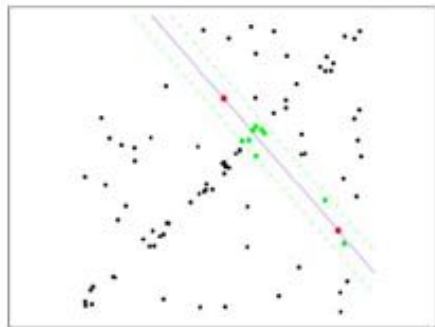
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- 3 Compute error function



[Source: R. Raguram]

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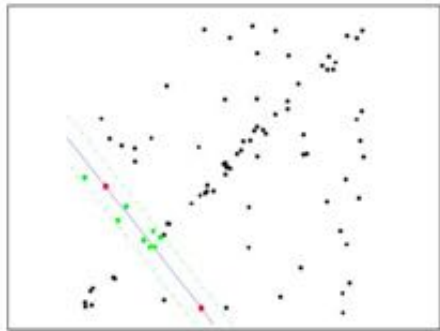
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- 4 Select points consistent with model



[Source: R. Raguram]

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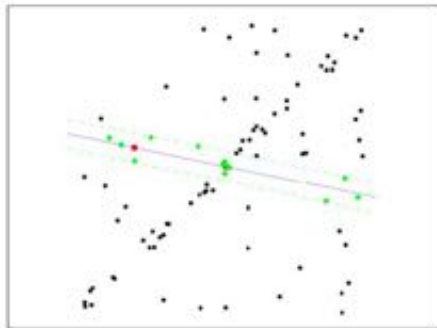
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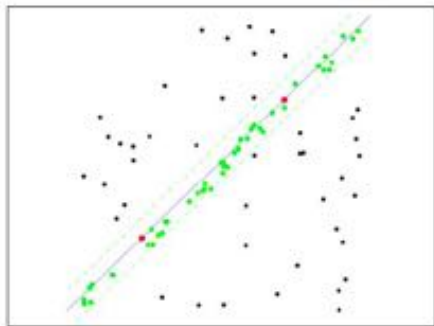
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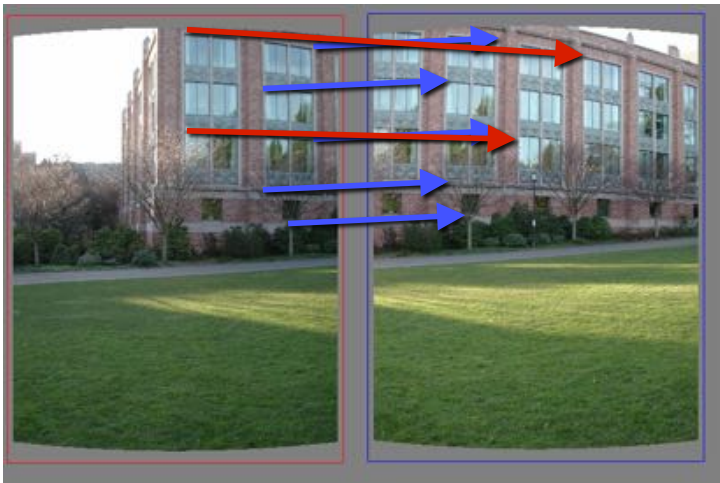
RANSAC for Line Fitting Example

- 1 Randomly select minimal subset of points
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- 6 Choose model with largest set of inliers



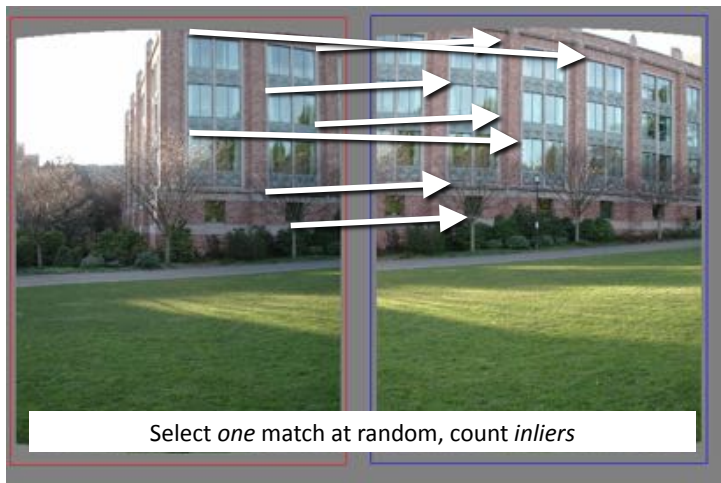
[Source: R. Raguram]

Translations



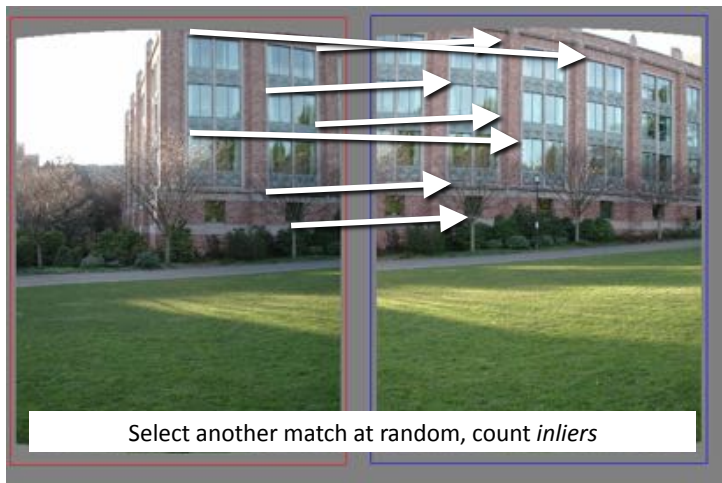
[Source: N. Snavely]

RANdom SAmple Consensus



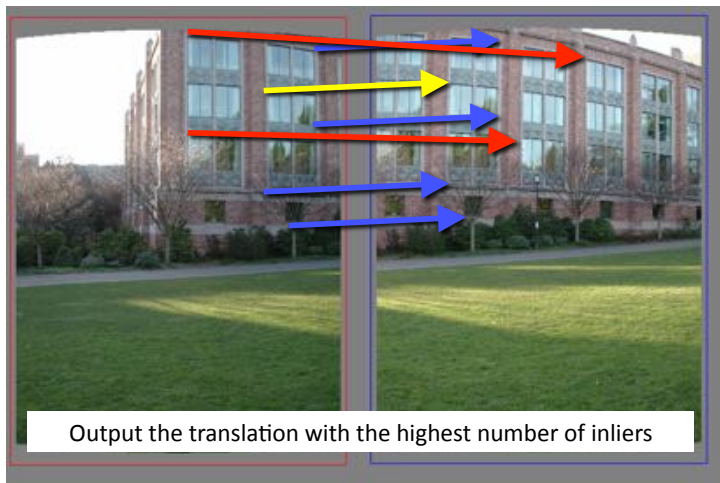
[Source: N. Snavely]

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- RANSAC only has guarantees if there are $< 50\%$ outliers
- "All good matches are alike; every bad match is bad in its own way." – [Tolstoy via Alyosha Efros]

[Source: N. Snavely]

Affine Transformation

How?

Affine Transformation

How?

- Find matches across images I and J . This gives us points X_I in image I and X_J in J , where we know that the point X_I^k is a match with X_J^k
- Iterate:
 - Choose 3 pairs of matches randomly
 - Compute the affine transformation
 - Project all matched points X_I from I to J via the computed transformation. This gives us \hat{X}_I
 - Find how many matches are inliers, i.e., $\|\hat{X}_I^k - X_J^k\| < thresh.$
- Choose the transformation with the most inliers

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- **Number of rounds** related to the percentage of outliers we expect, and the probability of success we'd like to guarantee
- Suppose there are 20% outliers, and we want to find the correct answer with 99% probability
- How many rounds do we need?

[Source: R. Urtasun]

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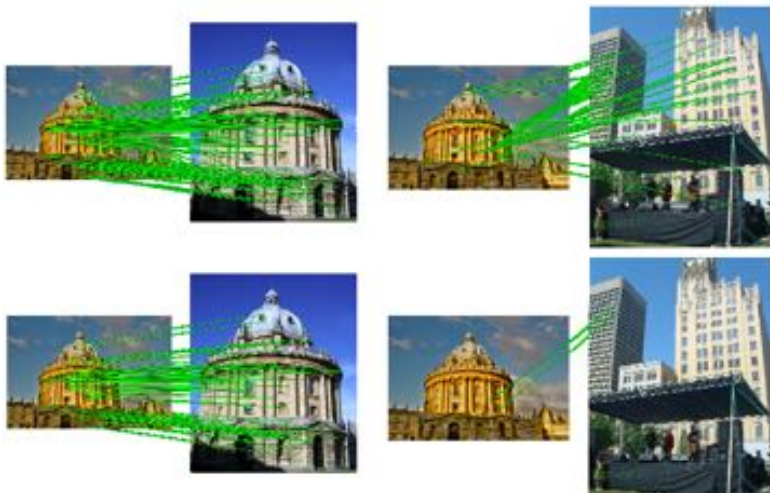
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Cons

- Parameters to tune
- Sometimes too many iterations are required
- Can fail for extremely low inlier ratios
- We can often do better than brute-force sampling

[Source: N. Snavely, slide credit: R. Urtasun]

Ransac Verification



[Source: K. Grauman, slide credit: R. Urtasun]

Summary – Stuff You Need To Know

To match image I and J under affine transformation:

- Compute scale and rotation invariant keypoints in both images
- Compute a (rotation invariant) feature vector in each keypoint (e.g., SIFT)
- Match all features in I to all features in J
- For each feature in reference image I find closest match in J
- If ratio between closest and second closest match is < 0.8 , keep match
- Do RANSAC to compute affine transformation A :
 - Select 3 matches at random
 - Compute A
 - Compute the number of inliers
 - Repeat
 - Find A that gave the most inliers