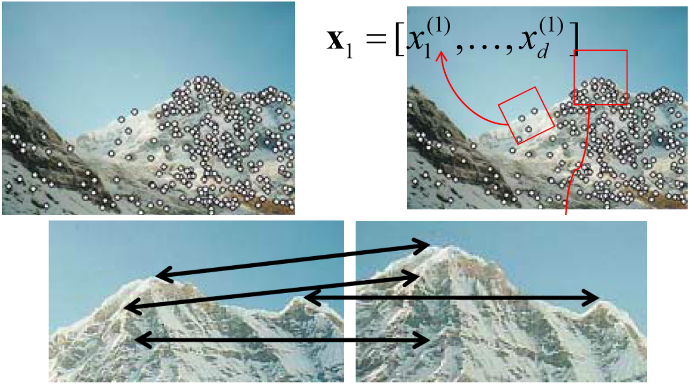
Image Features:

Local Descriptors

Local Features

- **Detection**: Identify the interest points.
- **Description**: Extract a feature descriptor around each interest point.
- **Matching**: Determine correspondence between descriptors in two views.



[Source: K. Grauman]

The Ideal Feature Descriptor

- **Repeatable**: Invariant to rotation, scale, photometric variations
- **Distinctive**: We will need to match it to lots of images/objects!
- **Compact**: Should capture rich information yet not be too high-dimensional (otherwise matching will be slow)
- **Efficient**: We would like to compute it (close-to) real-time

Invariances

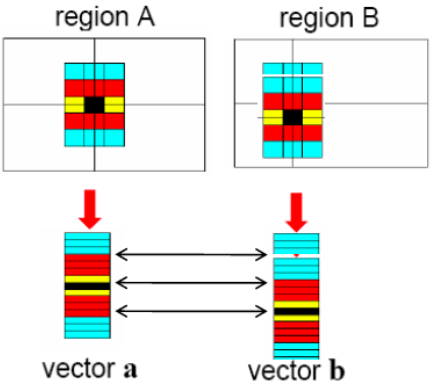
[Source: T. Tuytelaars]

Invariances

[Source: T. Tuytelaars]

What If We Just Took Pixels?

- The simplest way is to write down the list of intensities to form a feature vector, and normalize them (i.e., mean 0, variance 1).
- Why normalization?
- But this is very sensitive to even small shifts, rotations and any affine transformation.



Tones Of Better Options

- SIFT
- PCA-SIFT
- GLOH
- HOG
- SURF
- DAISY
- LBP
- Shape Contexts
- Color Histograms

Tones Of Better Options

- **SIFT** **TODAY**
- PCA-SIFT
- GLOH
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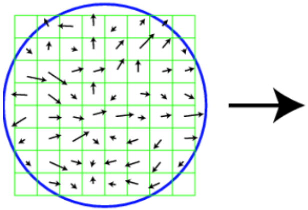
- Color Histograms

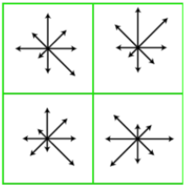
SIFT Descriptor [Lowe 2004]

- SIFT stands for Scale Invariant Feature Transform
- Invented by David Lowe, who also did DoG scale invariant interest points
- Actually in the same paper, which you should read:

David G. Lowe
Distinctive image features from scale-invariant keypoints
International Journal of Computer Vision, 2004
Paper: http://www.cs.ubc.ca/~lowe/papers/ijcv04.pdf



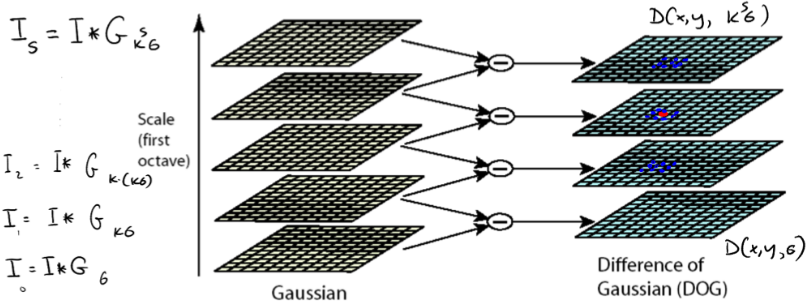
(a) image gradients



(b) keypoint descriptor

SIFT Descriptor

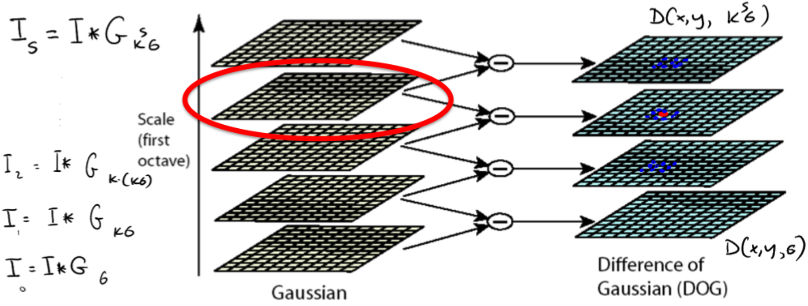
1. Our scale invariant interest point detector gives scale ρ for each keypoint



[Adopted from: F. Flores-Mangas]

SIFT Descriptor

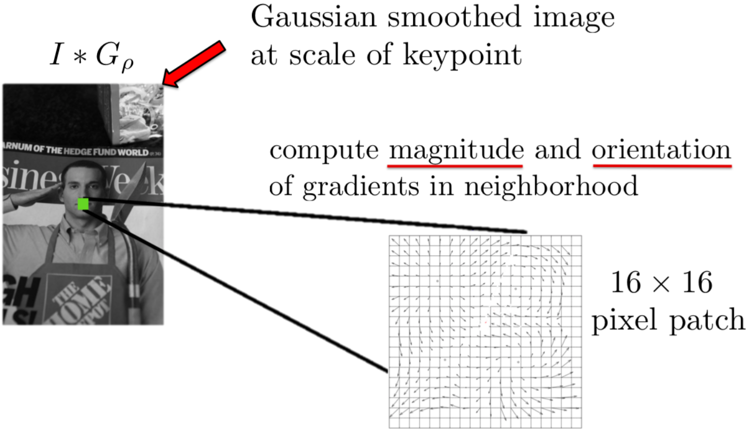
2. For each keypoint, we take the Gaussian-blurred image at corresponding scale ρ



[Adopted from: F. Flores-Mangas]

SIFT Descriptor

3. Compute the gradient magnitude and orientation in neighborhood of each keypoint



[Adopted from: F. Flores-Mangas]

SIFT Descriptor

3. Compute the gradient magnitude and orientation in neighborhood of each keypoint

magnitude of gradient:

$$|\nabla I(x,y)| = \sqrt{\left(\frac{\partial(I(x,y) * G_\rho)}{\partial x}\right)^2 + \left(\frac{\partial(I(x,y) * G_\rho)}{\partial y}\right)^2}$$

gradient orientation:

$$\theta(x,y) = \arctan\left(\frac{\partial I * G_\rho}{\partial y} / \frac{\partial I * G_\rho}{\partial x}\right)$$

(in case you forgot ;))

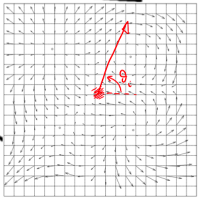
SIFT Descriptor

4. Compute dominant orientation of each keypoint. How?

$I * G_\rho$

Gaussian smoothed image
at scale of keypoint

compute magnitude and orientation
of gradients in neighborhood



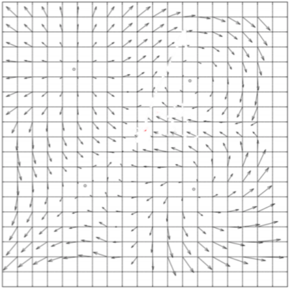
16×16
pixel patch

[Adopted from: F. Flores-Mangas]

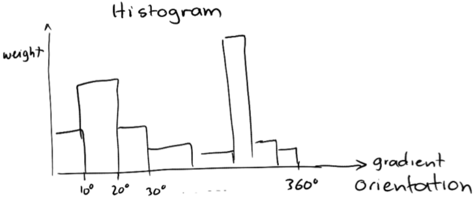
SIFT Descriptor: Computing Dominant Orientation

- Compute a histogram of gradient orientations, each bin covers 10°

16×16



compute histograms of orientations
by orientation increments of 10°



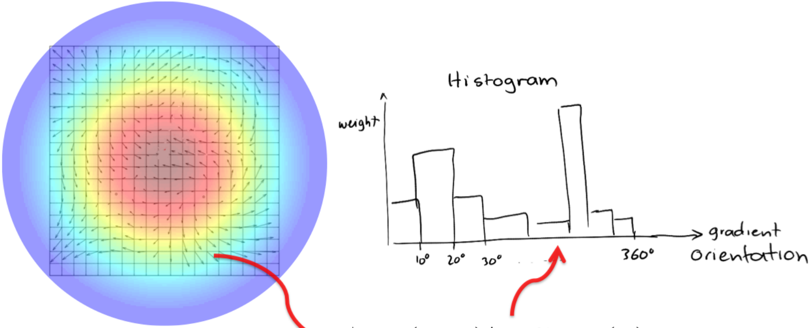
[Adopted from: F. Flores-Mangas]

SIFT Descriptor: Computing Dominant Orientation

- Compute a histogram of gradient orientations, each bin covers 10°
- Orientations closer to the keypoint center should contribute more

$G_{1.5\rho}$

weight influence of orientation based on distance from center



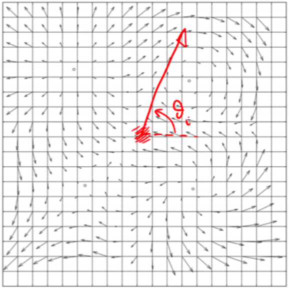
$$|\nabla I(x,y)| \cdot G_{1.5\rho}(d)$$

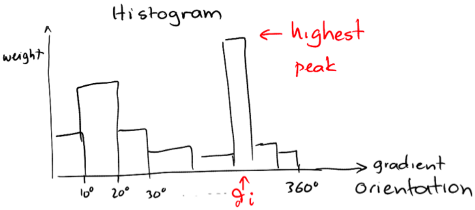
[Adopted from: F. Flores-Mangas]

SIFT Descriptor: Computing Dominant Orientation

- Compute a histogram of gradient orientations, each bin covers 10°
- Orientations closer to the keypoint center should contribute more
- Orientation giving the peak in the histogram is the keypoint's orientation

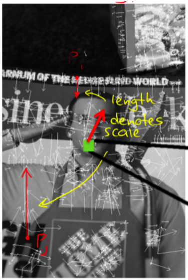
16×16



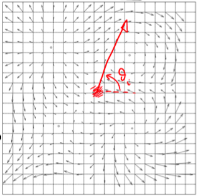


[Adapted from: F. Flores-Mangas]

4. Compute dominant orientation



compute magnitude and orientation of gradients in neighborhood

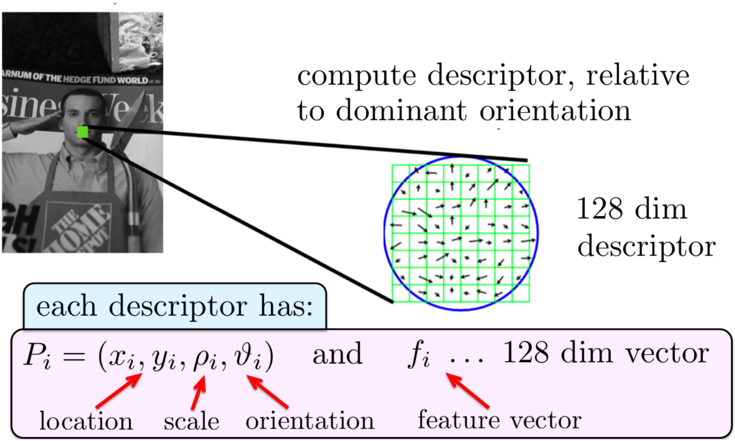


16×16 pixel patch

[Adopted from: F. Flores-Mangas]

SIFT Descriptor

5. Compute a 128 dimensional descriptor: 4×4 grid, each cell is a histogram of 8 orientation bins relative to dominant orientation

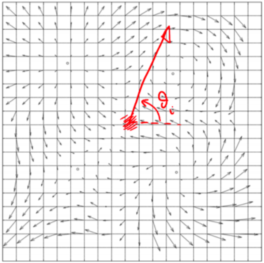


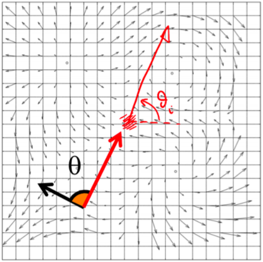
[Adopted from: F. Flores-Mangas]

SIFT Descriptor: Computing the Feature Vector

- Compute the orientations **relative** to the **dominant orientation**

16 $\times$ 16 patch
centered in (x_i, y_i)





[Adopted from: F. Flores-Mangas]

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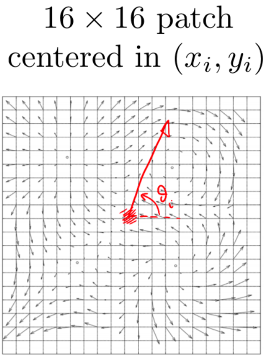
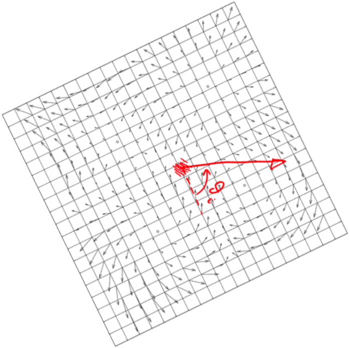
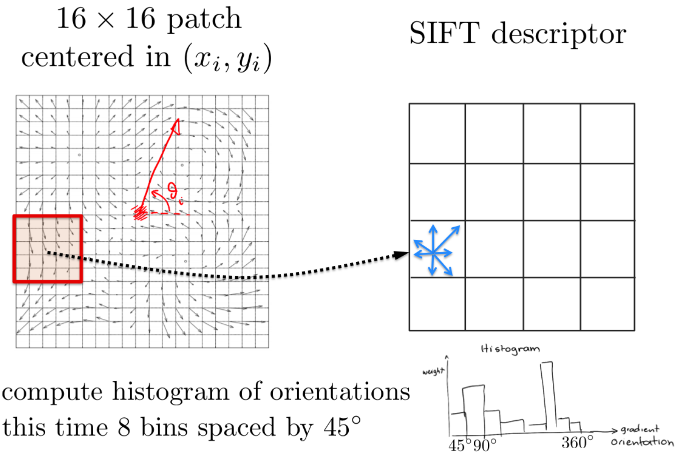




[Adopted from: F. Flores-Mangas]

SIFT Descriptor: Computing the Feature Vector

- Compute the orientations **relative** to the **dominant orientation**
- Form a 4×4 grid. For each grid cell compute a histogram of orientations for 8 orientation bins spaced apart by 45°



[Adopted from: F. Flores-Mangas]

SIFT Descriptor: Computing the Feature Vector

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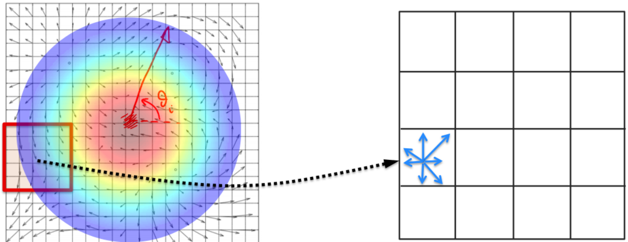
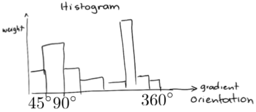
16×16 patch centered in (x_i, y_i)

SIFT descriptor



again weigh contributions this time: $|\nabla I(x, y)| \cdot G_{0.5\rho}$



[Adopted from: F. Flores-Mangas]

SIFT Descriptor: Computing the Feature Vector

- Compute the orientations **relative** to the **dominant orientation**
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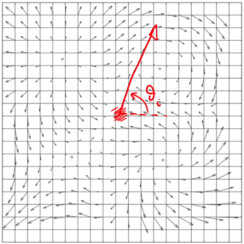
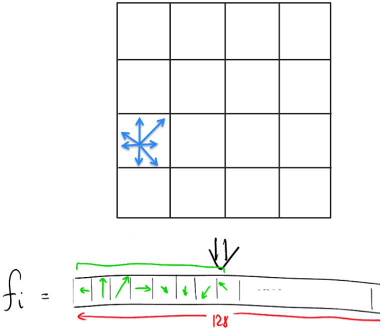
- Form the 128 dimensional feature vector

16×16 patch centered in (x_i, y_i)



SIFT descriptor



[Adopted from: F. Flores-Mangas]

SIFT Descriptor: Post-processing

- The resulting 128 non-negative values form a **raw version** of the SIFT descriptor vector.
- To reduce the **effects of contrast or gain** (additive variations are already removed by the gradient), the 128-D vector is normalized to unit length: $f_i = f_i/||f_i||$

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- What is SIFT invariant to?

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Properties of SIFT

Invariant to:

- Scale
- Rotation

Partially invariant to:

- Illumination changes (sometimes even day vs. night)
- Camera viewpoint (up to about 60 degrees of out-of-plane rotation)
- Occlusion, clutter (why?)

Also important:

- Fast and efficient – can run in real time
- Lots of code available

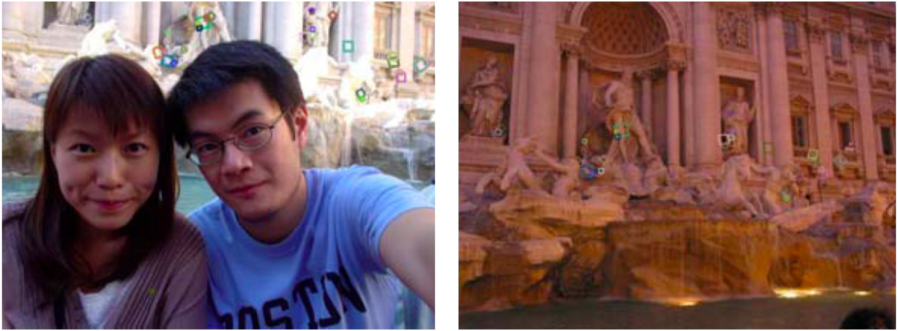


Figure: Matching in day / night under viewpoint change

[Source: S. Seitz]

Example

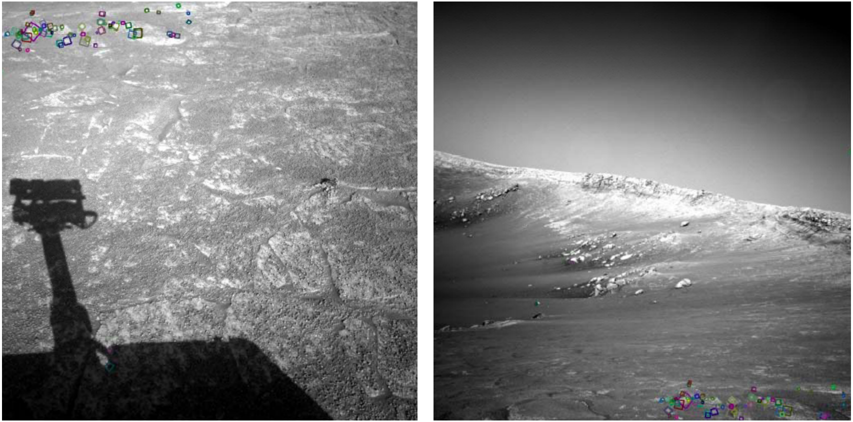


Figure: NASA Mars Rover images with SIFT feature matches

[Source: N. Snavely]

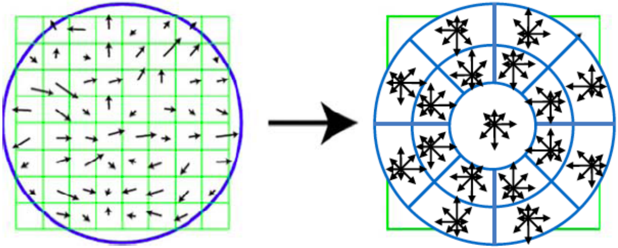
PCA-SIFT

- The dimensionality of SIFT is pretty high, i.e., 128D for each keypoint
- Reduce the dimensionality using linear dimensionality reduction
- In this case, principal component analysis (PCA)
- Use 10D or so descriptor

[Source: R. Urtasun]

Gradient location-orientation histogram (GLOH)

- Developed by Mikolajczyk and Schmid (2005): variant of SIFT that uses a log-polar binning structure instead of the four quadrants.
- The spatial bins are 11, and 15, with eight angular bins (except for the central region), for a total of 17 spatial bins and 16 orientation bins.
- The 272D histogram is then projected onto a 128D descriptor using PCA trained on a large database.



(a) image gradients (b) keypoint descriptor

[Source: R. Szeliski]

Other Descriptors

- SURF
- DAISY
- LBP
- HOG
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Local Features

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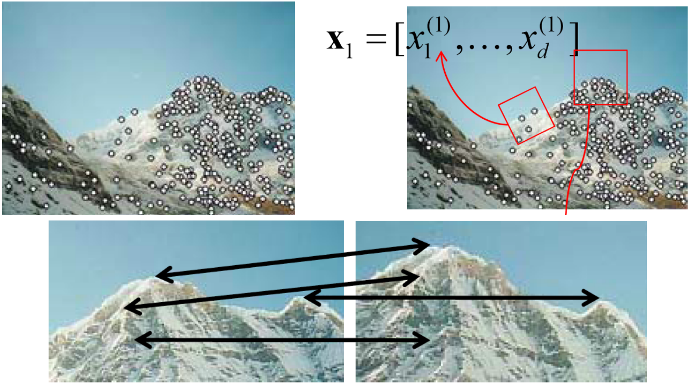

[Source: K. Grauman]

Image Features:

Matching the Local Descriptors

Matching the Local Descriptors

Once we have extracted keypoints and their descriptors, we want to match the features between pairs of images.

- Ideally a match is a correspondence between a local part of the object on one image to the same local part of the object in another image
- How should we compute a match?

Figure: Images from K. Grauman

Matching the Local Descriptors

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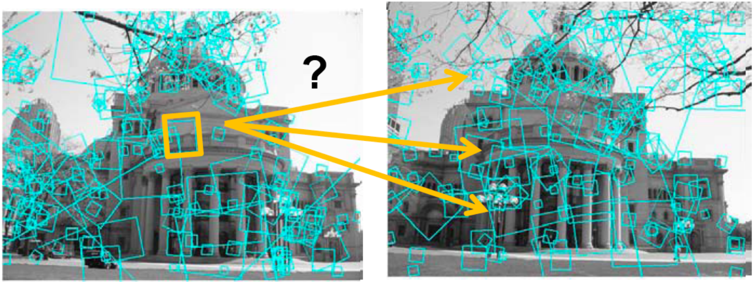
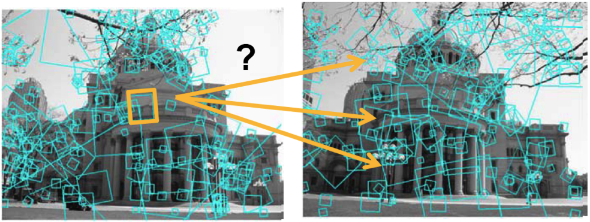
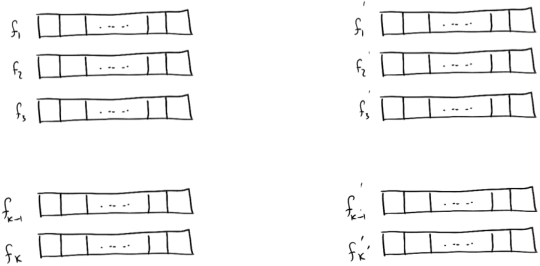


Figure: Images from K. Grauman

Matching the Local Descriptors

- Simple: **Compare them all**, compute Euclidean distance





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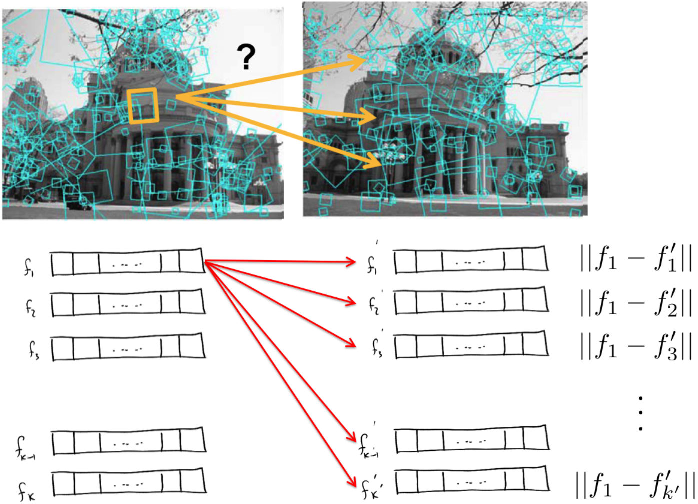
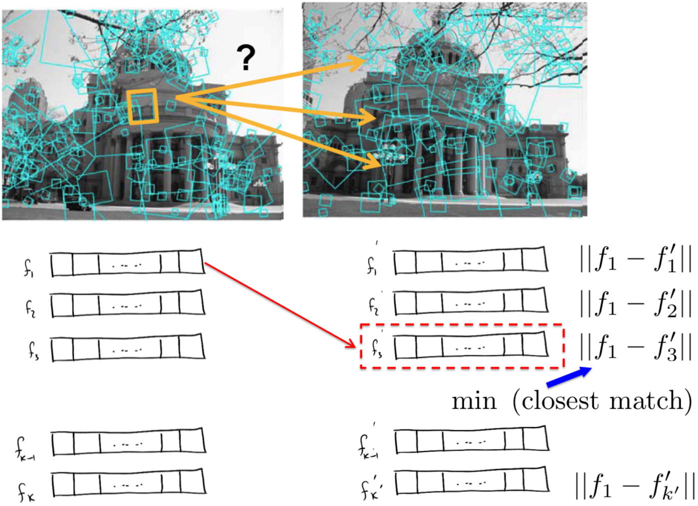


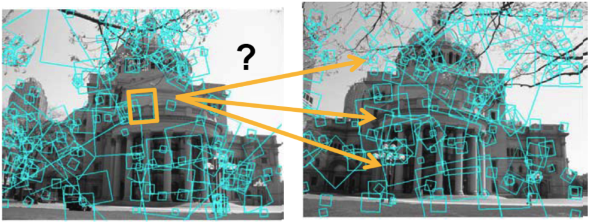
Matching the Local Descriptors

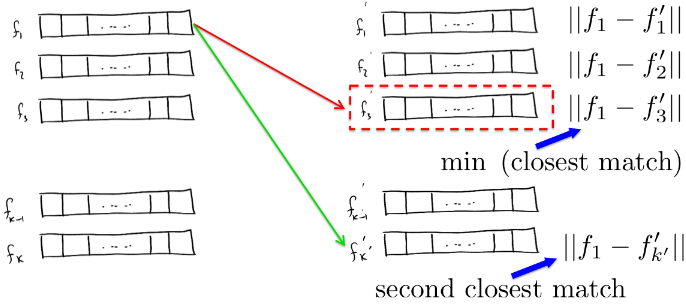
- Find closest match (min distance). How do we know if match is **reliable**?



Matching the Local Descriptors

- Find also the second closest match. Match reliable if first distance "much" smaller than second distance



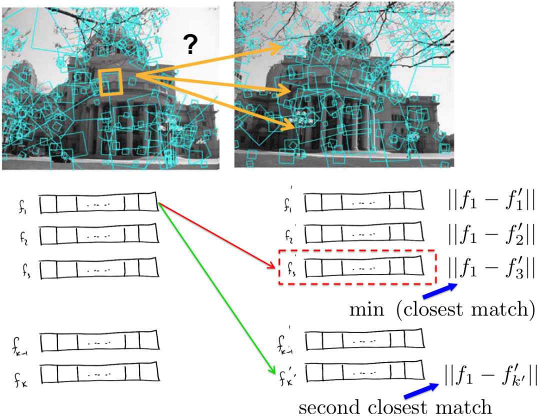


Matching the Local Descriptors

- Compute the ratio:

$$\phi_i = \frac{||f_i - {f'}_i^*||}{||f_i - {f'}_i^{**}||}$$

where ${f'}_i^*$ is the closest and ${f'}_i^{**}$ second closest match to f_i.



Which Threshold to Use?

- Setting the threshold too high results in too many false positives, i.e., incorrect matches being returned.
- Setting the threshold too low results in too many false negatives, i.e., too many correct matches being missed

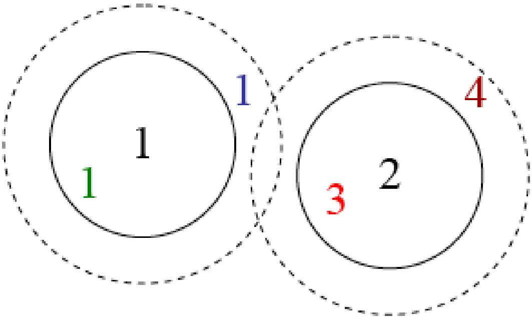


Figure: Images from R. Szeliski

Which Threshold to Use?

- Threshold ratio of nearest to 2nd nearest descriptor
- Typically: $\phi_i < 0.8$

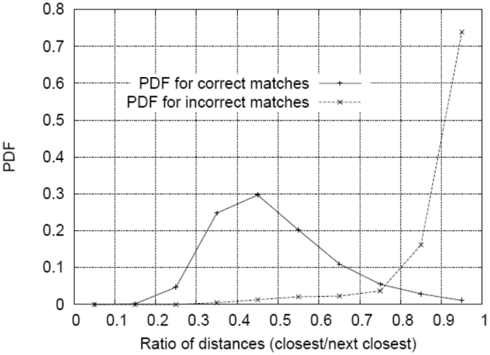


Figure: Images from D. Lowe

[Source: K. Grauman]

Applications of Local Invariant Features

- Wide baseline stereo
- Motion tracking
- Panorama stitching
- Mobile robot navigation
- 3D reconstruction
- Recognition
- Retrieval

[Source: K. Grauman]

Wide Baseline Stereo

[Source: T. Tuytelaars]

Recognizing the Same Object

Schmid and Mohr 1997

Sivic and Zisserman, 2003

Rothganger et al. 2003

Lowe 2002

[Source: K. Grauman]

Motion Tracking

Figure: Images from J. Pilet

Now What

- Now we know how to extract scale and rotation invariant features
- We even know how to match features across images
- Can we use this to find Waldo in an even more sneaky scenario?

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Waldo on the road

template

Now What

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- Can we use this to find Waldo in an even more sneaky scenario?

He comes closer... We know how to solve this

template

Now What

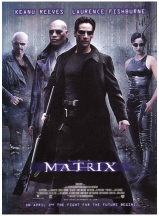
- Now we know how to extract scale and rotation invariant features
- We even know how to match features across images
- Can we use this to find Waldo in an even more sneaky scenario?

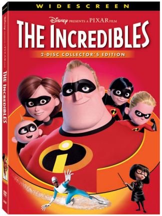
Someone takes a (weird) picture of him!

template

Find My DVD!

- More interesting: If we have DVD covers (e.g., from Amazon), can we match them to DVDs in real scenes?

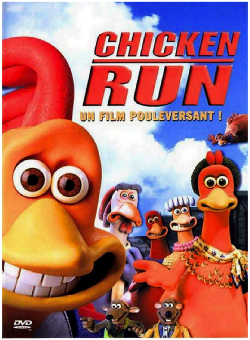






Matching Planar Objects In New Viewpoints

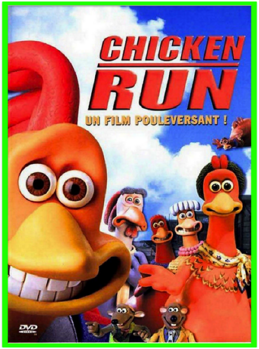
What Kind of Transformation Happened To My DVD?



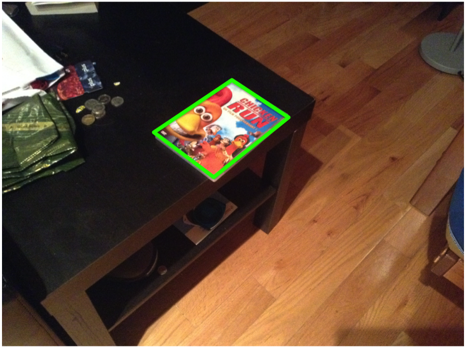
$\xrightarrow{T?}$

What Kind of Transformation Happened To My DVD?

- Rectangle goes to a parallelogram (almost but not really, but let's believe that for now)



$\xrightarrow{T?}$



All 2D Linear Transformations

Linear transformations are combinations of

- Scale,
- Rotation
- Shear
- Mirror

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

[Source: N. Snavely]

All 2D Linear Transformations

Properties of linear transformations:

- Origin maps to origin
- Lines map to lines

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What about the translation?

[Source: N. Snavely]

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Affine Transformations

Affine transformations are combinations of

- Linear transformations, and
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$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} e \\ f \end{bmatrix}$$

same as:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b & e \\ c & d & f \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

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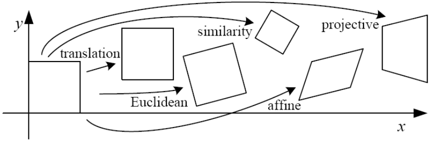
Properties of affine transformations:

- Origin does not necessarily map to origin
- Lines map to lines
- Parallel lines remain parallel
- Ratios are preserved
- Closed under composition
- Rectangles go to parallelograms

[Source: N. Snavely]

2D Image Tranformations

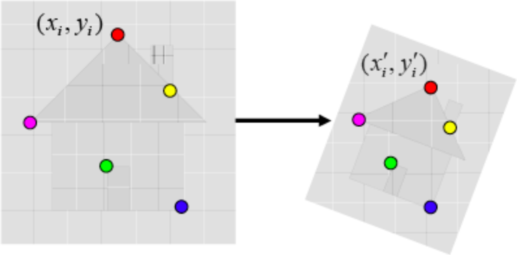


Transformation	Matrix	# DoF	Preserves	Icon
translation	$\begin{bmatrix} \boldsymbol{I} & \vert & \boldsymbol{t} \end{bmatrix}_{2\times3}$	2	orientation	
rigid (Euclidean)	$\begin{bmatrix} \boldsymbol{R} & \vert & \boldsymbol{t} \end{bmatrix}_{2\times3}$	3	lengths	
similarity	$\begin{bmatrix} s\boldsymbol{R} & \vert & \boldsymbol{t} \end{bmatrix}_{2\times3}$	4	angles	
affine	$\begin{bmatrix} \boldsymbol{A} \end{bmatrix}_{2\times3}$	6	parallelism	
projective	$\begin{bmatrix} \tilde{\boldsymbol{H}} \end{bmatrix}_{3\times3}$	8	straight lines	

- These transformations are a nested set of groups
- Closed under composition and inverse is a member

What Transformation Happened to My DVD?

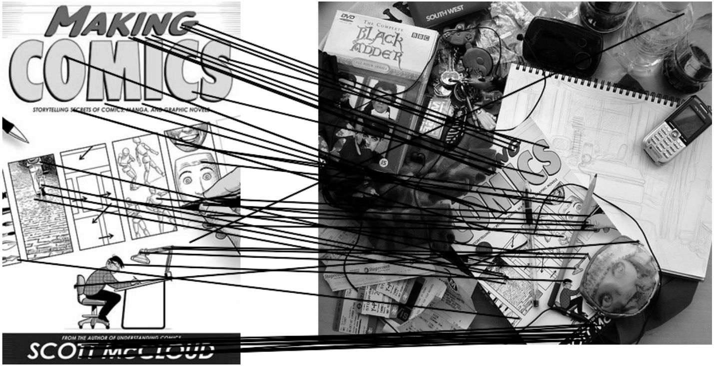
- Affine transformation approximates viewpoint changes for roughly **planar objects** and roughly **orthographic cameras** (more about these later in class)
- DVD went affine!



Computing the (Affine) Transformation

Given a set of matches between images I and J

- How can we compute the affine transformation A from I to J?
- Find transform A that best agrees with the matches



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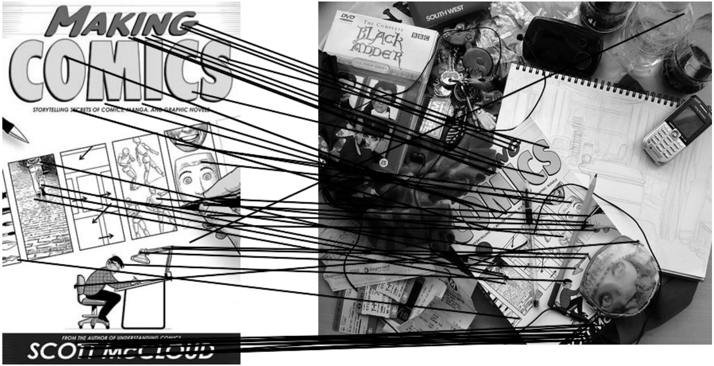

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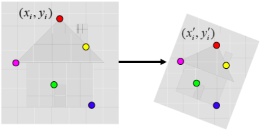

- Let (x_i, y_i) be a point on the reference (model) image, and (x_i', y_i') its match in the test image
- An affine transformation A maps (x_i, y_i) to (x_i', y_i'):

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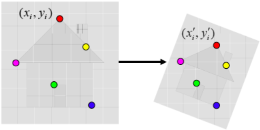

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$$\begin{bmatrix} x'_i \\ y'_i \end{bmatrix} = \begin{bmatrix} a & b & e \\ c & d & f \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}$$

- We can rewrite this into a simple linear system:

$$\begin{bmatrix} x_i & y_i & 0 & 0 & 1 & 0 \\ 0 & 0 & x_i & y_i & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix} = \begin{bmatrix} x'_i \\ y'_i \end{bmatrix}$$

Computing the Affine Transformation

- But we have many matches:

$$\underbrace{\begin{bmatrix} & & \vdots & & & \\ x_i & y_i & 0 & 0 & 1 & 0 \\ 0 & 0 & x_i & y_i & 0 & 1 \\ & & \vdots & & & \end{bmatrix}}_{\mathbf{P}} \underbrace{\begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix}}_{\mathbf{a}} = \underbrace{\begin{bmatrix} \vdots \\ x'_i \\ y'_i \\ \vdots \end{bmatrix}}_{\mathbf{P}'}$$

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- For each match we have two more equations
- How many matches do we need to compute A?
- 6 parameters $\rightarrow$ 3 matches
- But the more, the better (more reliable)
- How do we compute A?

Computing the Affine Transformation

$$\underbrace{\begin{bmatrix} & & \vdots & & & \\ x_i & y_i & 0 & 0 & 1 & 0 \\ 0 & 0 & x_i & y_i & 0 & 1 \\ & & \vdots & & & \end{bmatrix}}_{\mathbf{P}} \underbrace{\begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix}}_{\mathbf{a}} = \underbrace{\begin{bmatrix} \vdots \\ x_i' \\ y_i' \\ \vdots \end{bmatrix}}_{\mathbf{P}'}$$

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$$\min_{a,b,\cdots,f} ||\mathbf{Pa} - \mathbf{P'}||_2^2$$

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- Which has a closed form solution:

$$\mathbf{a} = (\mathbf{P}^T\mathbf{P})^{-1}\mathbf{P}^T\mathbf{P}'$$

Image Alignment Algorithm: Affine Case

Given images I and J

1. Compute image features for I and J
2. Match features between I and J
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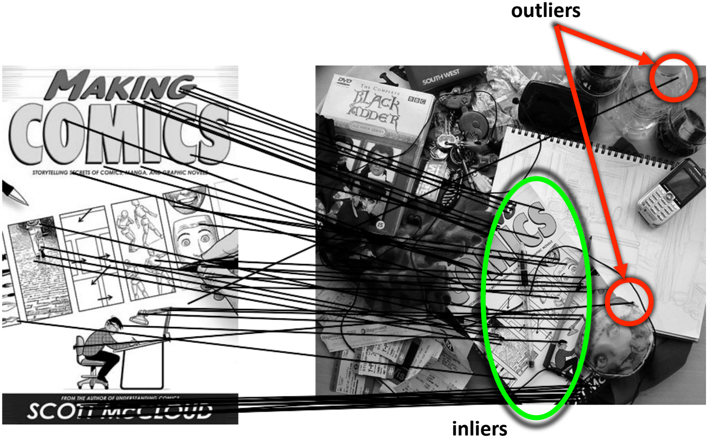
Is there a problem with this?

[Source: N. Snavely]

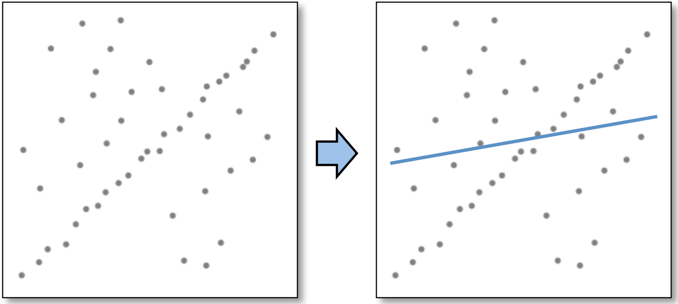
Robustness



[Source: N. Snavely]

Simple Case

- Lets consider a simpler example ... Fit a line to the points below!



Problem: Fit a line to these datapoints

Least squares fit

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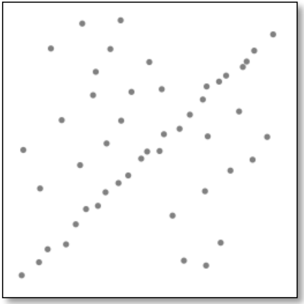

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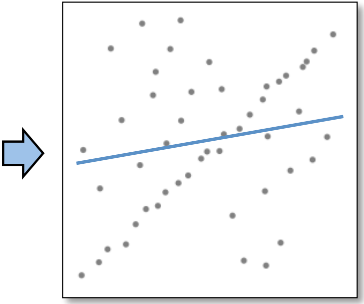

Least squares fit

- How can we fix this?

[Source: N. Snavely]

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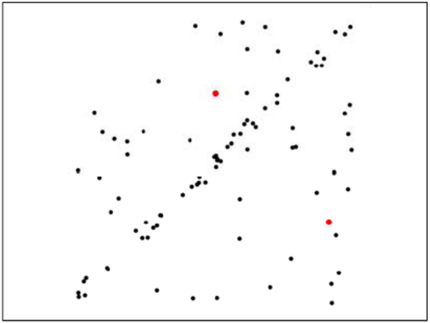
- "Agree" = within a small distance of the line
- Repeat this many times, remember the number of inliers for each trial
- Among several trials, select the one with the largest number of inliers

This procedure is called **RA**ndom **SA**mple **C**onsensus

RANSAC for Line Fitting Example

1. Randomly select minimal subset of points



[Source: R. Raguram]

RANSAC for Line Fitting Example

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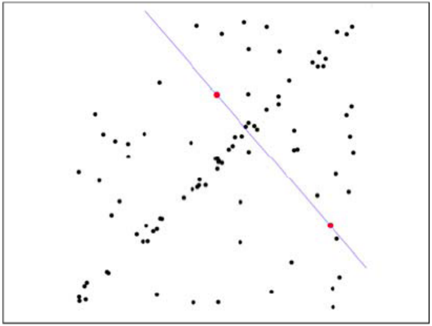

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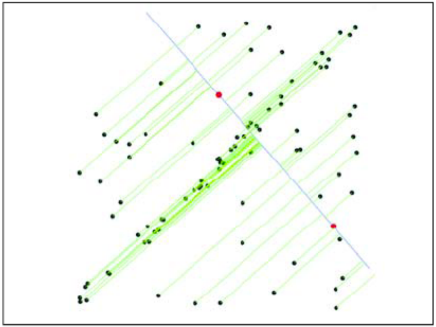

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1. Randomly select minimal subset of points
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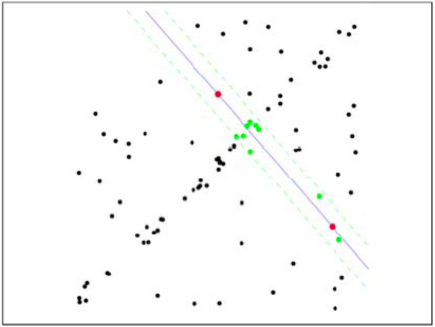
4. Select points consistent with model



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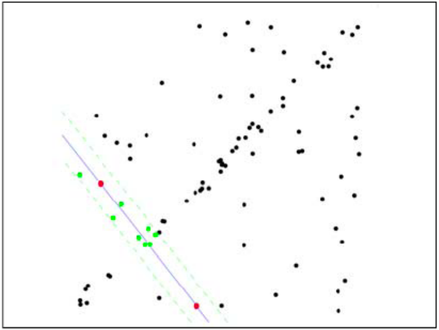

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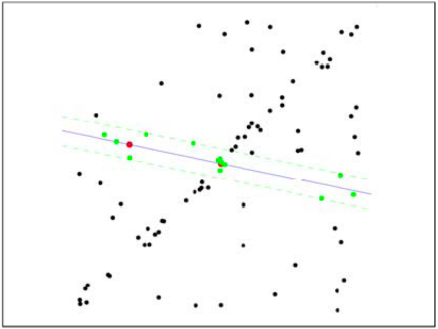

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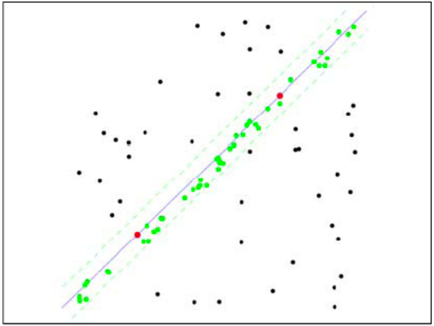

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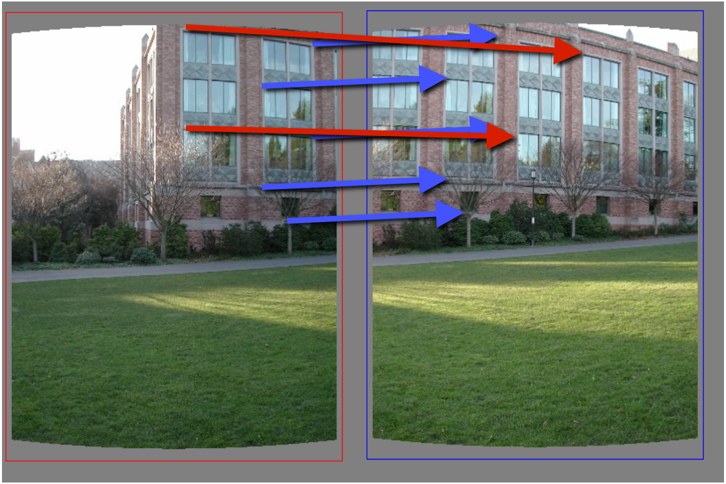
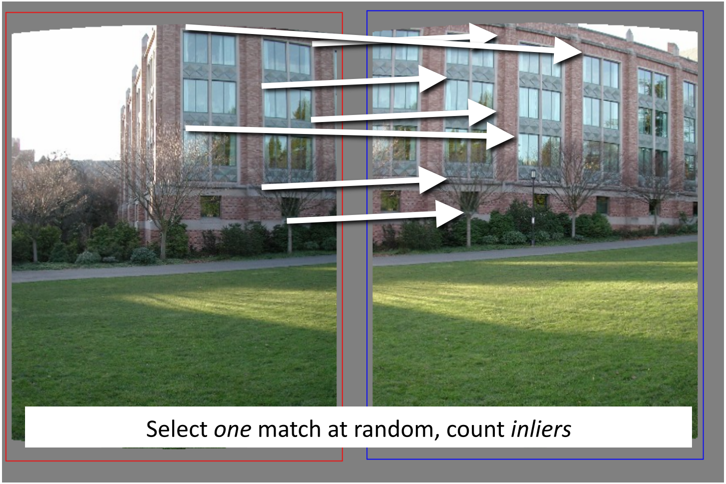
Translations



[Source: N. Snavely]

RAndom SAmple Consensus



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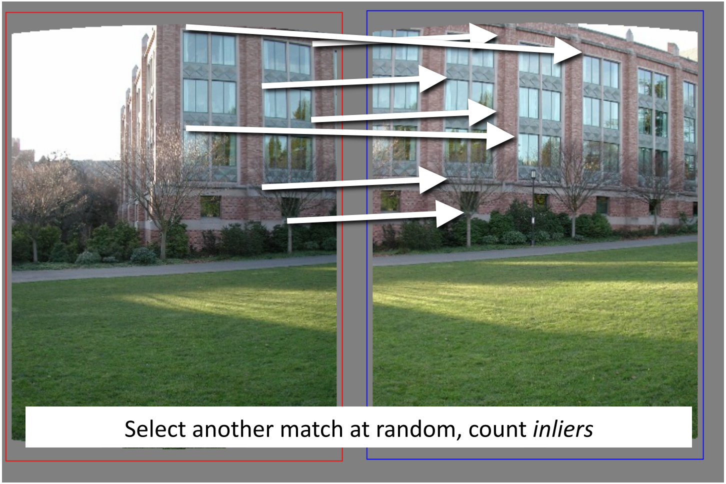

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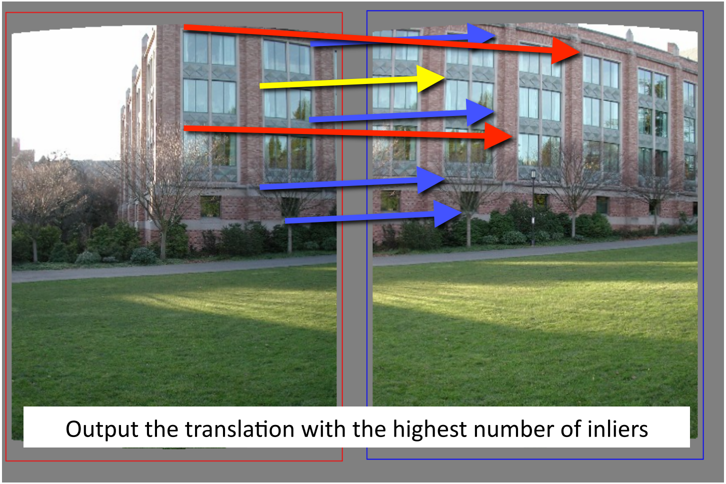

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- RANSAC only has guarantees if there are $< 50\%$ outliers
- "All good matches are alike; every bad match is bad in its own way." – [Tolstoy via Alyosha Efros]

[Source: N. Snavely]

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- How many rounds do we need?

[Source: R. Urtasun]

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RANSAC pros and cons

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- Applicable to many different problems
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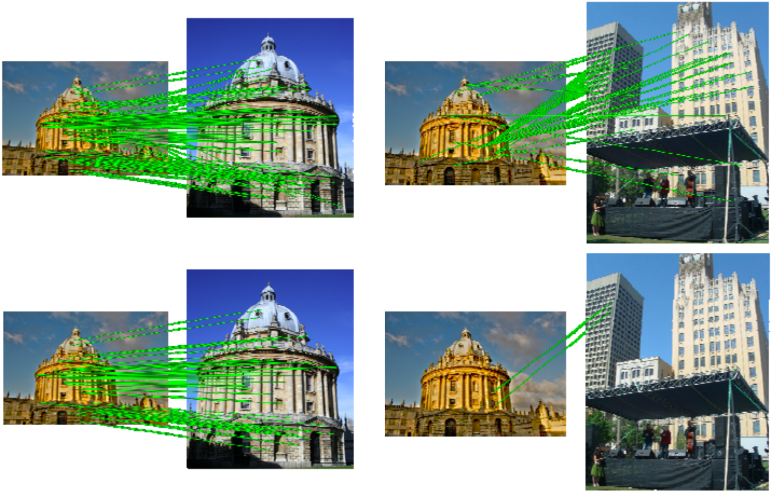
- Parameters to tune
- Sometimes too many iterations are required
- Can fail for extremely low inlier ratios
- We can often do better than brute-force sampling

[Source: N. Snavely, slide credit: R. Urtasun]

Ransac Verification



[Source: K. Grauman, slide credit: R. Urtasun]

Summary – Stuff You Need To Know

To match image I and J under affine transformation:

- Compute scale and rotation invariant keypoints in both images
- Compute a (rotation invariant) feature vector in each keypoint (e.g., SIFT)
- Match all features in I to all features in J
- For each feature in reference image I find closest match in J
- If ratio between closest and second closest match is < 0.8, keep match
- Do RANSAC to compute affine transformation A:
    - Select 3 matches at random
    - Compute A
    - Compute the number of inliers
    - Repeat
    - Find A that gave the most inliers