- Image features are useful descriptions of local or global image properties designed (or learned!) to accomplish a certain task
- You may want to choose different features for different tasks
- Depending on the problem we need to typically answer three questions:
  - Where to extract image features?
  - What to extract (what's the content of the feature)?
  - How to use them for your task, e.g., how to match them?

• Let's watch a video clip



• Where is the movie taking place?

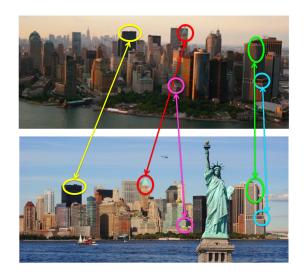


• Where is the movie taking place?





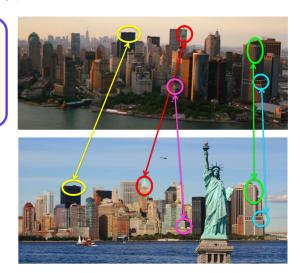
• Where is the movie taking place?



• Where is the movie taking place?

#### We matched in:

- Distinctive locations: keypoints
- Distinctive features: descriptors



• **Tracking**: Where to did the scene/actors move?



Where did it each point originate from the previous frame?

• Tracking: Where to did the scene/actors move?

#### We matched:

- Quite distinctive locations
- Quite distinctive features



Where did it each point originate from the previous frame?

• A shot in a movie is a clip with a coherent camera (no sudden viewpoint

changes)



• A shot in a movie is a clip with a coherent camera (no sudden viewpoint

changes)

#### We matched:

- **Globally** one descriptor for full image
- Descriptor can be simple, e.g. color



• How could we tell which type of scene it is?



What kind of scene is behind the actors? Kitchen? Bedroom? Street? Bar?



• How could we tell which type of scene it is?

#### We matched:

- **Globally** one descriptor for full image (?)
- More complex descriptor: color, gradients, "deep" features (learned), etc



What kind of scene is behind the actors? Kitchen? Bedroom? Street? Bar?



• How would we solve this?



Are these two cups of the same type?

• How would we solve this?

#### We matched:

- One descriptor for full patch
- Descriptor can be simple, e.g. color



Are these two cups of the same type?

• How would we solve this?



Where can I find this pattern? -----

• How would we solve this?

#### We matched:

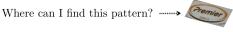
- At each location
- Compared pixel values



Where can I find this pattern? -----

• How would we solve this?



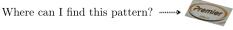


• How would we solve this?

#### We matched:

- Distinctive locations
- Distinctive features
- Affine invariant





• How would we solve this?

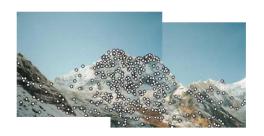


- Detection: Where to extract image features?
  - "Interesting" locations (keypoints, interesting regions)
  - In each location (densely)
- Description: What to extract?
  - What's the spatial scope of the feature?
  - What's the content of the feature?
- Matching: How to match them?

- Detection: Where to extract image features?
  - "Interesting" locations (keypoints)
  - In each location (densely)
- **Description**: What to extract?
  - What's the spatial scope of the feature?
  - What's the content of the feature?
- Matching: How to match them?

Interest Point (Keypoint) Detection

# Application Example: Image Stitching



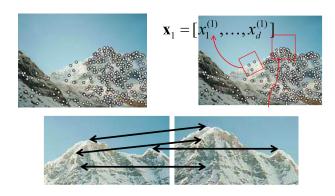




[Source: K. Grauman]

#### Local Features

- **Detection**: Identify the interest points.
- Description: Extract feature vector descriptor around each interest point.
- Matching: Determine correspondence between descriptors in two views.



[Source: K. Grauman]

### Goal: Repeatability of the Interest Point Operator

- Our goal is to detect (at least some of) the same points in both images
- We have to be able to run the detection procedure independently per image
- We need to generate enough points to increase our chances of detecting matching points
- We shouldn't generate too many or our matching algorithm will be too slow

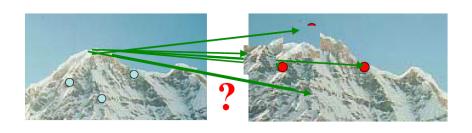


Figure: Too few keypoints  $\rightarrow$  little chance to find the true matches

[Source: K. Grauman, slide credit: R. Urtasun]

# Goal: Distinctiveness of the Keypoints

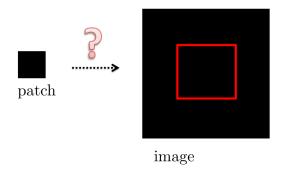
• We want to be able to **reliably** determine which point goes with which.



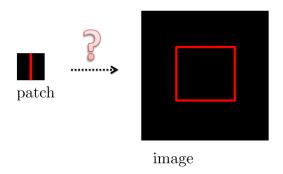
[Source: K. Grauman, slide credit: R. Urtasun]



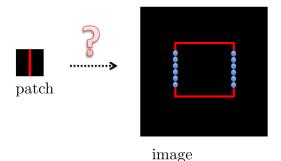
[Source: K. Grauman]



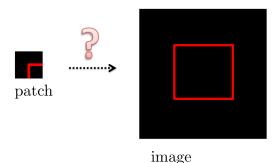
Textureless patches are nearly impossible to localize.



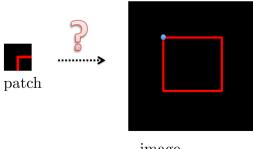
- Textureless patches are nearly impossible to localize.
- Patches with large contrast changes (gradients) are easier to localize.



- Textureless patches are nearly impossible to localize.
- Patches with large contrast changes (gradients) are easier to localize.
- But straight line segments cannot be localized on lines segments with the same orientation (aperture problem)



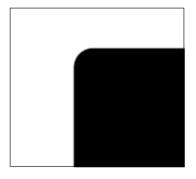
- Textureless patches are nearly impossible to localize.
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- Gradients in at least two different orientations are easiest, e.g., corners!



- image
- Textureless patches are nearly impossible to localize.
- Patches with large contrast changes (gradients) are easier to localize.
- But straight line segments cannot be localized on lines segments with the same orientation (aperture problem)
- Gradients in at least two different orientations are easiest, e.g., corners!

#### Interest Points: Corners

• How can we find corners in an image?



#### Interest Points: Corners

- We should easily recognize the point by looking through a small window.
- Shifting a window in any direction should give a large change in intensity.

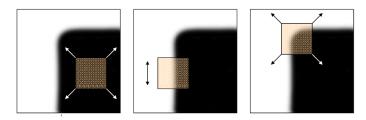
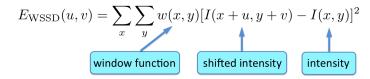


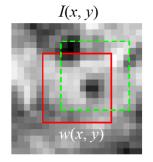
Figure: (left) flat region: no change in all directions, (center) edge: no change along the edge direction, (right) corner: significant change in all directions

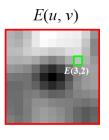
[Source: Alyosha Efros, Darya Frolova, Denis Simakov]

#### Interest Points: Corners

- Compare two image patches using (weighted) summed square difference
- Measures change in appearance of window w(x, y) for the shift

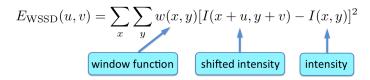


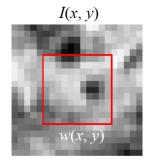


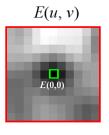


[Source: J. Hays]

- Compare two image patches using (weighted) summed square difference
- Measures change in appearance of window w(x, y) for the shift





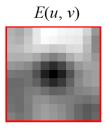


- Compare two image patches using (weighted) summed square difference
- Measures change in appearance of window w(x, y) for the shift

$$E_{\rm WSSD}(u,v) = \sum_x \sum_y w(x,y) [I(x+u,y+v) - I(x,y)]^2$$
 window function shifted intensity intensity



- ullet Let's look at  $E_{
  m WSSD}$
- We want to find out how this function behaves for small shifts



• Remember our goal to detect corners:







• Using a simple first-order Taylor Series expansion:

$$I(x+u,y+v) \approx I(x,y) + u \cdot \frac{\partial I}{\partial x}(x,y) + v \cdot \frac{\partial I}{\partial y}(x,y)$$

• And plugging it in our expression for  $E_{WSSD}$ :

$$E_{\text{WSSD}}(u, v) = \sum_{x} \sum_{y} w(x, y) \Big( I(x + u, y + v) - I(x, y) \Big)^{2}$$

$$\approx \sum_{x} \sum_{y} w(x, y) \Big( I(x, y) + u \cdot I_{x} + v \cdot I_{y} - I(x, y) \Big)^{2}$$

$$= \sum_{x} \sum_{y} w(x, y) \Big( u^{2} I_{x}^{2} + 2u \cdot v \cdot I_{x} \cdot I_{y} + v^{2} I_{y}^{2} \Big)$$

$$= \sum_{x} \sum_{y} w(x, y) \cdot \begin{bmatrix} u & v \end{bmatrix} \begin{bmatrix} I_{x}^{2} & I_{x} \cdot I_{y} \\ I_{x} \cdot I_{y} & I_{y}^{2} \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

• Since (u, v) doesn't depend on (x, y) we can rewriting it slightly:

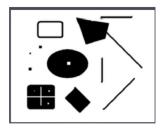
$$E_{\text{WSSD}}(u, v) = \sum_{x} \sum_{y} w(x, y) \begin{bmatrix} u & v \end{bmatrix} \begin{bmatrix} I_{x}^{2} & I_{x} \cdot I_{y} \\ I_{x} \cdot I_{y} & I_{y}^{2} \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

$$= \begin{bmatrix} u & v \end{bmatrix} \underbrace{\left(\sum_{x} \sum_{y} w(x, y) \begin{bmatrix} I_{x}^{2} & I_{x} \cdot I_{y} \\ I_{x} \cdot I_{y} & I_{y}^{2} \end{bmatrix}\right) \begin{bmatrix} u \\ v \end{bmatrix}}_{\text{Let's denotes this with } M}$$

$$= \begin{bmatrix} u & v \end{bmatrix} M \begin{bmatrix} u \\ v \end{bmatrix}$$

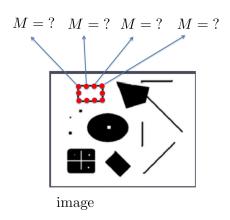
• M is a 2 × 2 second moment matrix computed from image gradients:

$$M = \sum_{x} \sum_{y} w(x, y) \begin{bmatrix} I_{x}^{2} & I_{x} \cdot I_{y} \\ I_{x} \cdot I_{y} & I_{y}^{2} \end{bmatrix}$$

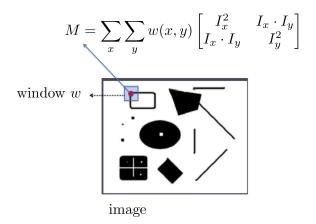


image

 $\bullet$  Let's say I have this image



- Let's say I have this image
- $\bullet$  I need to compute a  $2\times 2$  second moment matrix in each image location



- Let's say I have this image
- ullet I need to compute a 2  $\times$  2 second moment matrix in each image location
- ullet In a particular location I need to compute M as a weighted average of gradients in a window

$$M = \sum_{x} \sum_{y} w(x, y) \begin{bmatrix} I_x^2 & I_x \cdot I_y \\ I_x \cdot I_y & I_y^2 \end{bmatrix}$$









$$I_x = rac{\partial I}{\partial x}$$

 $I_x = \frac{\partial I}{\partial x} \qquad I_y = \frac{\partial I}{\partial y}$ 

 $I_x \cdot I_y$ 

- Let's say I have this image
- $\bullet$  I need to compute a  $2\times 2$  second moment matrix in each image location
- In a particular location I need to compute M as a weighted average of gradients in a window

I can do this efficiently by computing three matrices,  $I_x^2$ ,  $I_y^2$  and  $I_x \cdot I_y$ , and convolving each one with a filter, e.g. a box or Gaussian filter

- We now have *M* computed in each image location
- Our  $E_{WSSD}$  is a quadratic function where M implies its shape

$$E_{\text{WSSD}}(u, v) = \begin{bmatrix} u & v \end{bmatrix} M \begin{bmatrix} u \\ v \end{bmatrix}$$

$$M = \sum_{x} \sum_{y} w(x, y) \begin{bmatrix} I_{x}^{2} & I_{x} \cdot I_{y} \\ I_{x} \cdot I_{y} & I_{y}^{2} \end{bmatrix}$$

• Let's take a horizontal "slice" of  $E_{WSSD}(u, v)$ :

$$\begin{bmatrix} u & v \end{bmatrix} M \begin{bmatrix} u \\ v \end{bmatrix} = \text{const}$$

• This is the equation of an ellipse

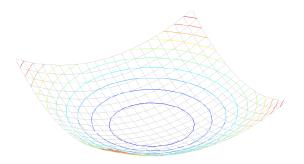


Figure: Different ellipses obtain by different horizontal "slices"

• Let's take a horizontal "slice" of  $E_{WSSD}(u, v)$ :

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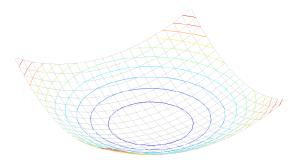


Figure: Different ellipses obtain by different horizontal "slices"

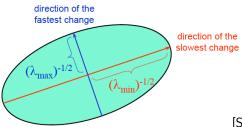
• Our matrix *M* is symmetric:

$$M = \sum_{x} \sum_{y} w(x, y) \begin{bmatrix} I_{x}^{2} & I_{x} \cdot I_{y} \\ I_{x} \cdot I_{y} & I_{y}^{2} \end{bmatrix}$$

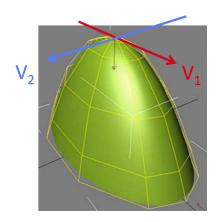
• And thus we can diagonalize it (in Matlab: [V,D] = EIG(M)):

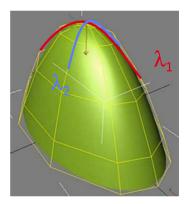
$$M = V \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} V^{-1}$$

ullet Columns of V are major and minor axes of ellipse,  $\lambda^{-1/2}$  are radius



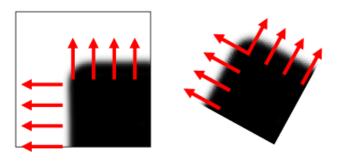
- Columns of V are principal directions
- $\lambda_1$ ,  $\lambda_2$  are principal curvatures





[Source: F. Flores-Mangas]

• The eigenvalues of M ( $\lambda_1$ ,  $\lambda_2$ ) reveal the amount of intensity change in the two principal orthogonal gradient directions in the window

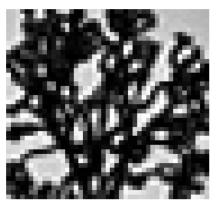


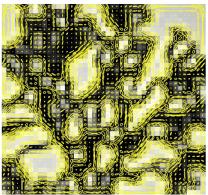
[Source: R. Szeliski, slide credit: R. Urtasun]

• How do the ellipses look like for this image?

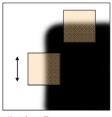


• How do the ellipses look like for this image?



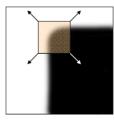


[Source: J. Hays]



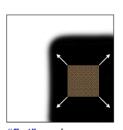
"edge":

$$\lambda_1 >> \lambda_2$$
 $\lambda_2 >> \lambda_1$ 



"corner":

 $\lambda_1$  and  $\lambda_2$  are large,  $\lambda_1 \sim \lambda_2$ ;



"flat" region

 $\lambda_1$  and  $\lambda_2$  are small;

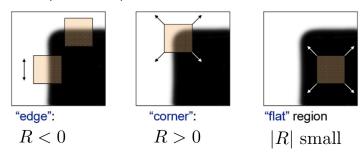
[Source: K. Grauman, slide credit: R. Urtasun]

#### Interest Points: Criteria to Find Corners

• Harris and Stephens, '88, is rotationally invariant and downweighs edge-like features where  $\lambda_1\gg\lambda_0$ 

$$R = \det(M) - \alpha \cdot \operatorname{trace}(M)^2 = \lambda_0 \lambda_1 - \alpha (\lambda_0 + \lambda_1)^2$$

- Why go via det and trace and not use a criteria with  $\lambda$ ?
- $\alpha$  a constant (0.04 to 0.06)



• The corresponding detector is called Harris corner detector

#### Interest Points: Criteria to Find Corners

• Harris and Stephens, 88 is rotationally invariant and downweighs edge-like features where  $\lambda_1 \gg \lambda_0$ 

$$R = \det(M) - \alpha \operatorname{trace}(M)^2 = \lambda_0 \lambda_1 - \alpha (\lambda_0 + \lambda_1)^2$$

- Shi and Tomasi, 94 proposed the smallest eigenvalue of **A**, i.e.,  $\lambda_0^{-1/2}$ .
- Triggs, 04 suggested

$$\lambda_0 - \alpha \lambda_1$$

also reduces the response at 1D edges, where aliasing errors sometimes inflate the smaller eigenvalue

• Brown et al, 05 use the harmonic mean

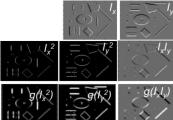
$$\frac{\det(\mathbf{A})}{\operatorname{trace}(\mathbf{A})} = \frac{\lambda_0 \lambda_1}{\lambda_0 + \lambda_1}$$

[Source R. Urtasun]

#### Harris Corner detector

- **1** Compute gradients  $I_x$  and  $I_y$
- 2 Compute  $I_x^2$ ,  $I_y^2$ ,  $I_x \cdot I_y$
- **3** Average (Gaussian)  $\rightarrow$  gives M
- Compute  $R = \det(M) \alpha \operatorname{trace}(M)^2$  for each image window (*cornerness* score)
- **5** Find points with large R (R >threshold).
- Take only points of local maxima, i.e., perform non-maximum suppression





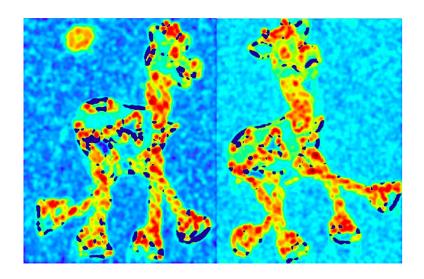


# Example

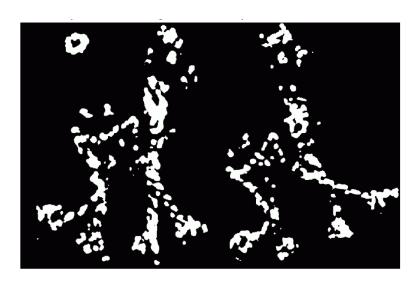


[Source: K. Grauman]

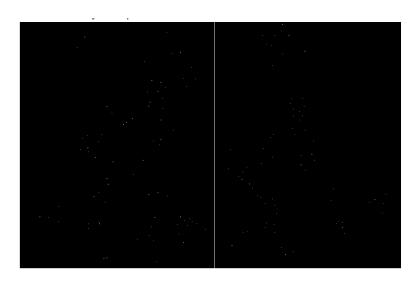
# 1) Compute Cornerness



# 2) Find High Response



# 3) Non-maxima Suppresion



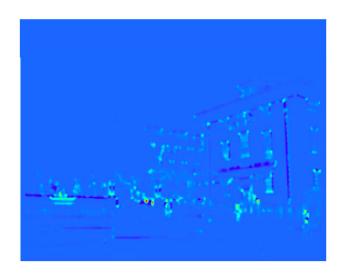
## Results



## Another Example



## Cornerness



### Interest Points



### Interest Points – Ideal Properties?

 We want corner locations to be invariant to photometric transformations and covariant to geometric transformations

Invariance: Image is transformed and corner locations do not change

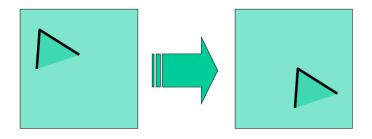
Covariance : If we have two transformed versions of the same image, features

should be detected in corresponding locations



# Properties of Harris Corner Detector

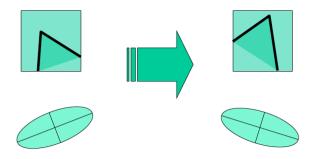
Shift?



Harris corner detector is shift-covariant (our window functions shift)

## Properties of Harris Corner Detector

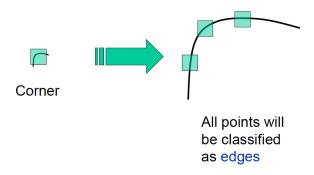
Rotation?



- Second moment ellipse rotates but its shape (i.e. eigenvalues) remains the same
- Harris corner detector is rotation-covariant

## Properties of Harris Corner Detector

Scale?



Corner location is not scale invariant/covariant!

#### Next Time

- Can we also define keypoints that are shift, rotation and scale invariant/covariant?
- What should be our **description** around keypoint?