Introduction to Deep Learning

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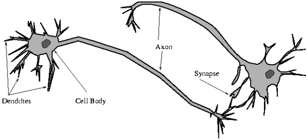
University of Toronto, 2015

Outline

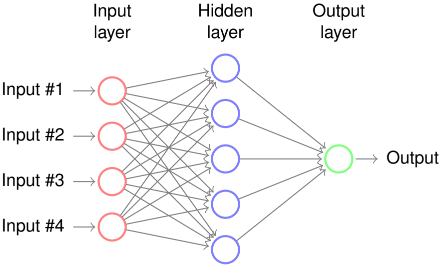
What are neural networks?

Let's ask Google

- Biological



- Computational



What are neural networks?

...**Neural networks** (NNs) are computational models inspired by biological neural networks [...] and **are used to** estimate or **approximate functions**... [Wikipedia]

What are neural networks?

Origins:

- Traced back to threshold logic [W. McCulloch and W. Pitts, 1943]
- Perceptron [F. Rosenblatt, 1958]

What are neural networks? Use cases

- Classification
- Playing video games
- Captcha
- Neural Turing Machine (e.g., learn how to sort) Alex Graves

  http://www.technologyreview.com/view/532156/googles-secretive-deepmind-startup-unveils-a-neural-turing-machine/

What are neural networks?
Example:

- input x
- parameters w_1, w_2, b

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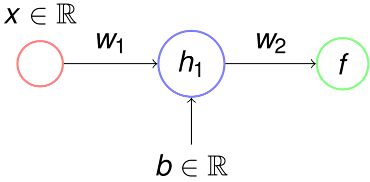
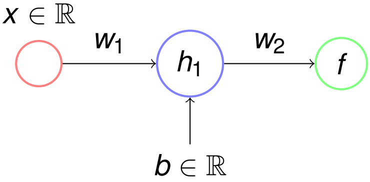


How to compute the function?

Forward propagation/pass, inference, prediction:

- Given input x and parameters w, b
- Compute (latent variables/) intermediate results in a feed-forward manner
- Until we obtain output function f



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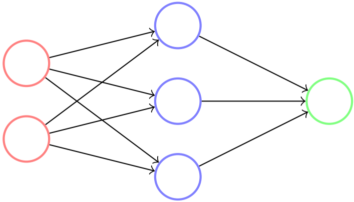

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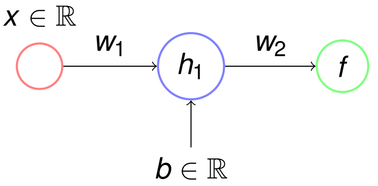

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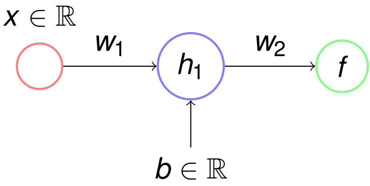

$$
\begin{aligned}
h_1 &= \sigma(w_1 \cdot x + b) \\
f &= w_2 \cdot h_1
\end{aligned}
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Sigmoid function:

$$\sigma(z) = 1/(1 + \exp(-z))$$

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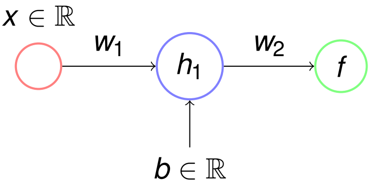

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$x = \ln 2$, $b = \ln 3$, $w_1 = 2$, $w_2 = 2$
$h_1 = ?$
$f = ?$

How to compute the function?

Given parameters, what is f for $x = 0$, $x = 1$, $x = 2$, ...

$$f = w_2 \sigma(w_1 \cdot x + b)$$

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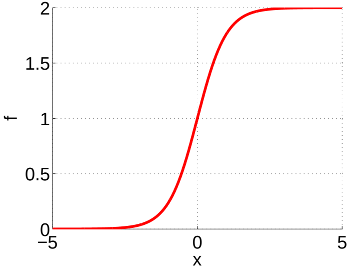

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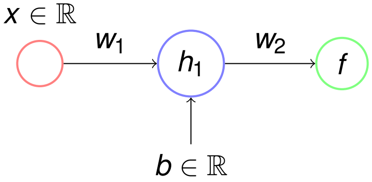

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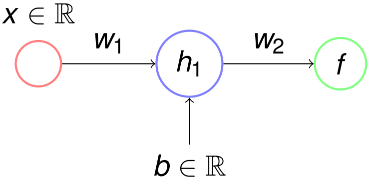

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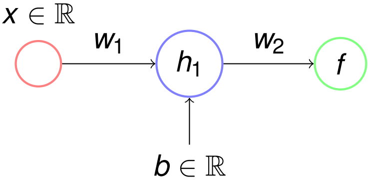

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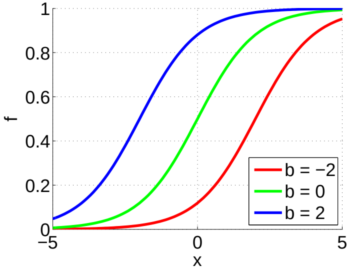

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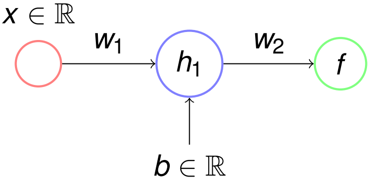

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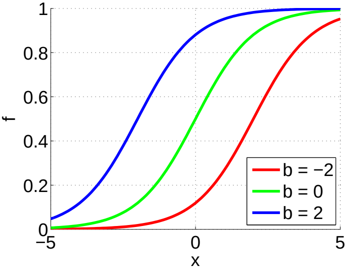

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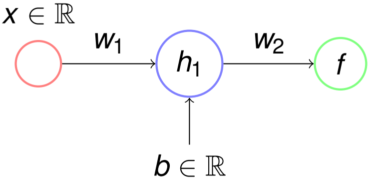

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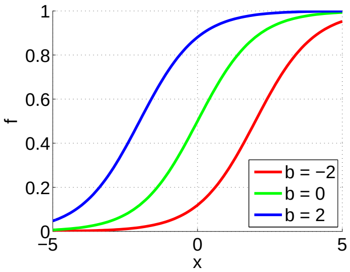

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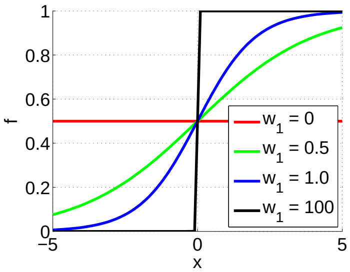

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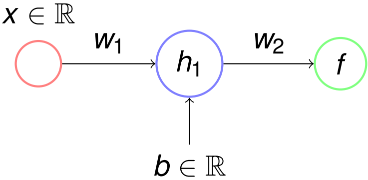

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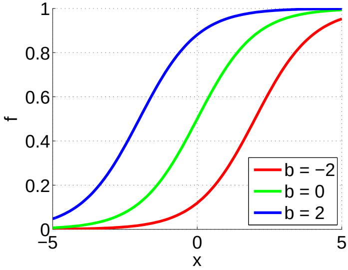

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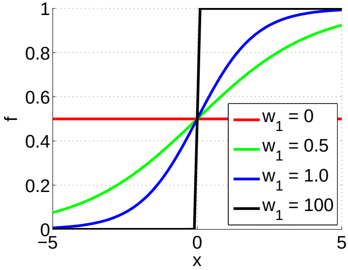
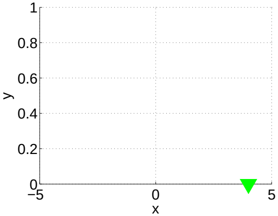


Keep in mind the step function.

How to use Neural Networks for binary classification?

Feature/Measurement: x

Output: How likely is the input to be a cat?



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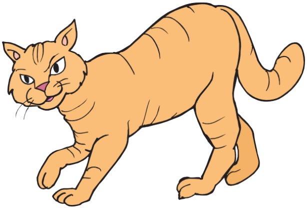

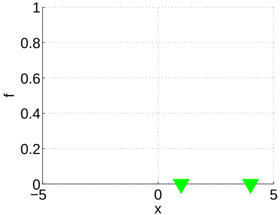

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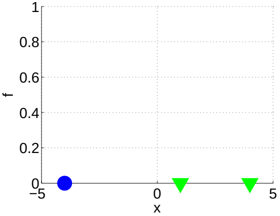

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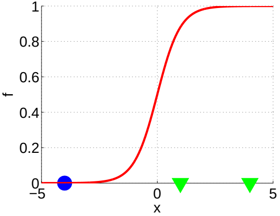

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Shifted feature/measurement: x

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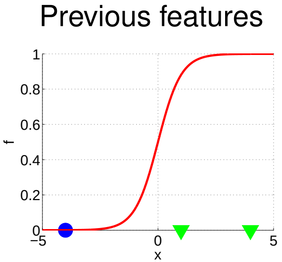

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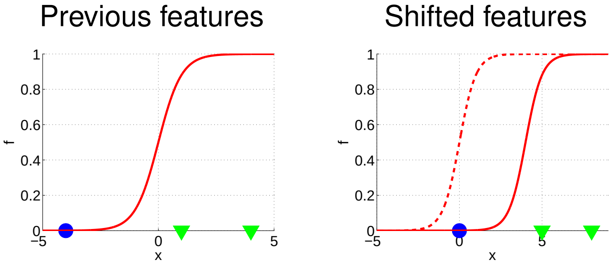

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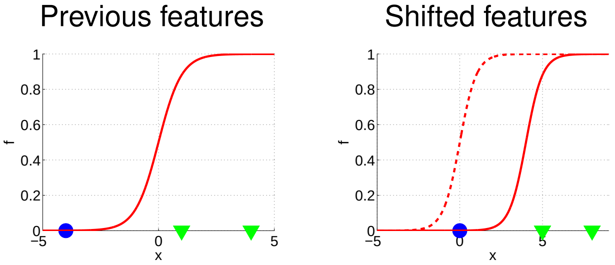
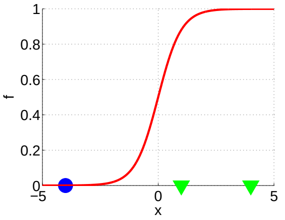


Learning/Training means finding the right parameters.

So far we are able to scale and translate sigmoids.

- How well can we approximate an arbitrary function?
- With the simple model we are obviously not going very far.

Features are good
Simple classifier


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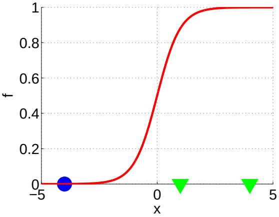
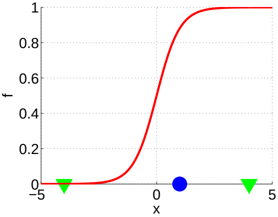
Features are good	Features are noisy
Simple classifier	More complex classifier
	

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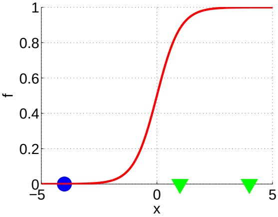
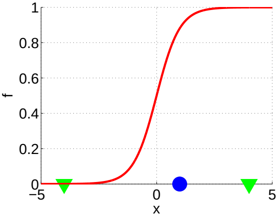
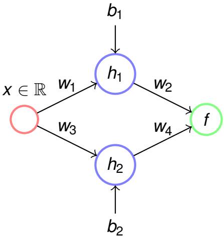
	

- How can we generalize?

Let's use more hidden variables:



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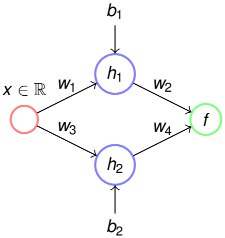

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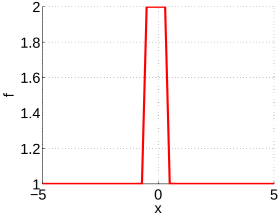
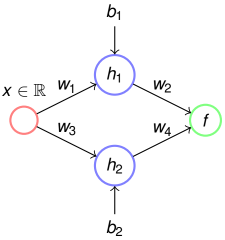
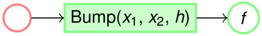
Combining two step functions gives a bump.



$$w_1 = -100,\ b_1 = 40,\ w_3 = 100,\ b_2 = 60,\ w_2 = 1,\ w_4 = 1$$

So let's simplify:





We simplify a pair of hidden nodes to a "bump" function:

- Starts at x_1
- Ends at x_2
- Has height h

Now we can represent "bumps" very well. How can we generalize?

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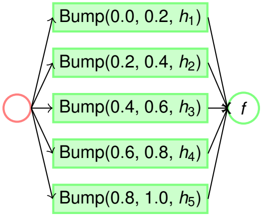

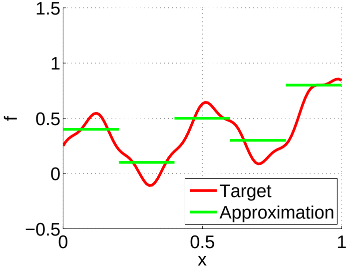

More bumps gives more accurate approximation.

Corresponds to a single layer network.

- Universality: theoretically we can approximate an arbitrary function
- So we can learn a really complex cat classifier
- Where is the catch?

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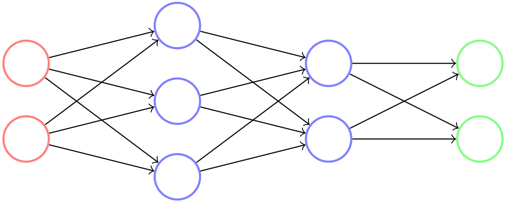
- So we can learn a really complex cat classifier
- Where is the catch?

- Complexity, we might need quite a few hidden units
- Overfitting, memorize the training data

Generalizations are possible to

Generalizations are possible to

- include more input dimensions
- capture more output dimensions
- employ multiple layers for more efficient representations



See ‘http://neuralnetworksanddeeplearning.com/chap4.html’ for a great read!

How do we find the parameters to obtain a good approximation? How do we tell a computer to do that?

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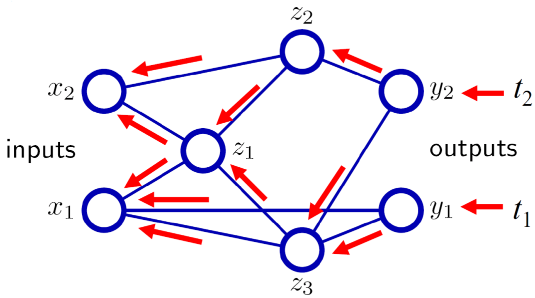
Intuitive explanation:

- Compute approximation error at the output
- Propagate error back by computing individual contributions of parameters to error



[Fig. from H. Lee]

Example for backpropagation of error:

- Target function: $5x^2$
- Approximation: $f(x, w)$
- Domain of interest: $x \in \{0, 1, 2, 3\}$
- Error:

$$e(w) = \sum_{x \in \{0,1,2,3\}} (5x^2 - f(x, w))^2$$

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$$\min_w e(w) = \min_w \sum_{x \in \{0,1,2,3\}} \underbrace{(5x^2 - f(x, w))^2}_{\ell(x,w)}$$

How to optimize?

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How to optimize? **Gradient descent**

Gradient descent

$$\min_w e(w)$$

Gradient descent

$$\min_{w} e(w)$$

Algorithm: start with w_0, $t = 0$

1. Compute gradient $g_t = \frac{\partial e}{\partial w}\Big|_{w=w_t}$
2. Update $w_{t+1} = w_t - \eta g_t$
3. Set $t \leftarrow t + 1$

Chain rule is important to compute gradients:

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- Log loss
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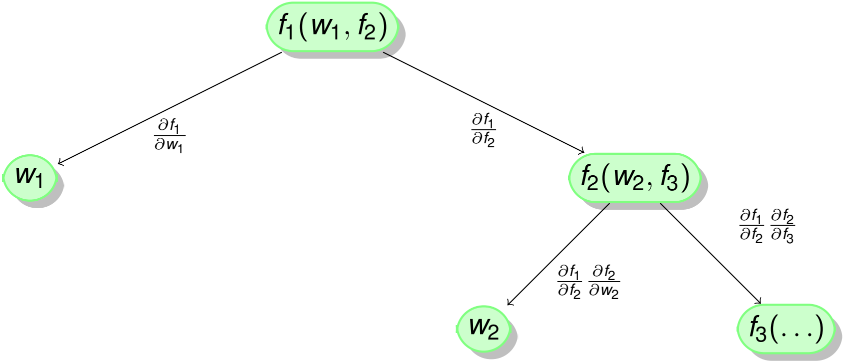
Slightly more complex example:
Composite function represented as a directed a-cyclic graph

$$\ell(x, w) = f_1(w_1, f_2(w_2, f_3(\ldots)))$$



Repeated application of chain rule for efficient computation of all gradients

Back propagation doesn't work well for deep sigmoid networks:

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- Attractivity of many local minima (random initialization is very far from good points)
- Requires a lot of training samples
- Need for significant computational power

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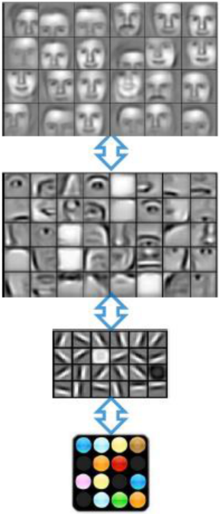
Solution: 2 step approach

- Greedy layerwise pre-training
- Perform full fine tuning at the end

Why go deep?

- Representation efficiency (fewer computational units for the same function)
- Hierarchical representation (non-local generalization)
- Combinatorial sharing (re-use of earlier computation)
- Works very well



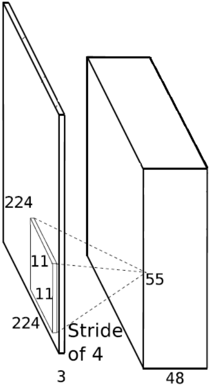
[Fig. from H. Lee]

To obtain more flexibility/non-linearity we use additional function prototypes:

To obtain more flexibility/non-linearity we use additional function prototypes:

- Sigmoid
- Rectified linear unit (ReLU)
- Pooling
- Dropout
- Convolutions

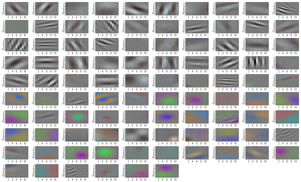
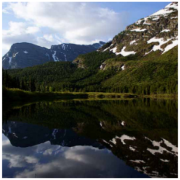
Convolutions

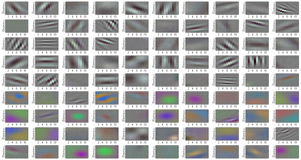
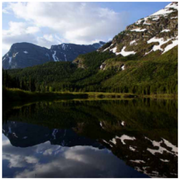


What do the numbers mean?

See Sanja's lecture 14 for the answers...

[Fig. adapted from A. Krizhevsky]

$f_{\text{conv}}($  ,  $)$

$f_{\text{conv}}($  $,$  $)$

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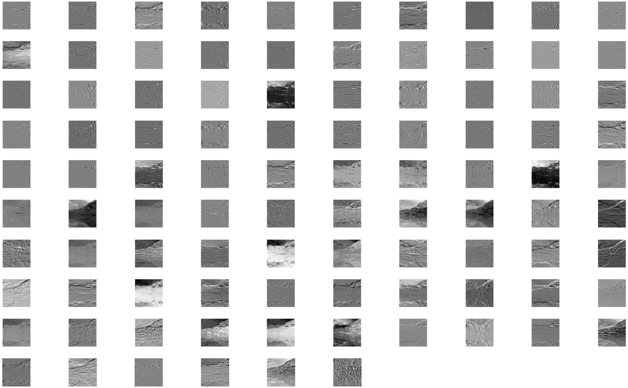
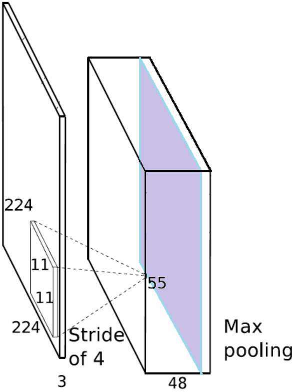
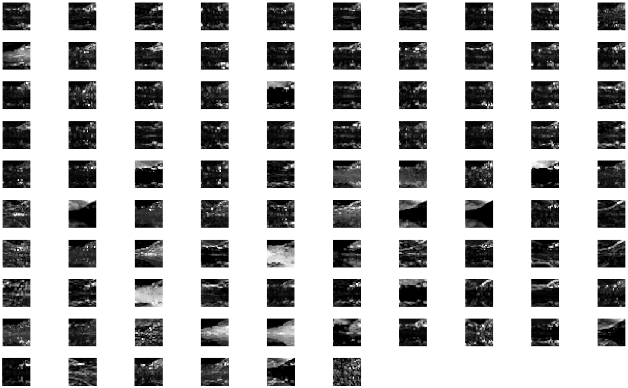


Max Pooling



What is happening here?

[Fig. adapted from A. Krizhevsky]



Rectified Linear Unit (ReLU)

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- Drop information if smaller than zero
- **Fixes the problem of vanishing gradients to some degree**

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Dropout

Rectified Linear Unit (ReLU)

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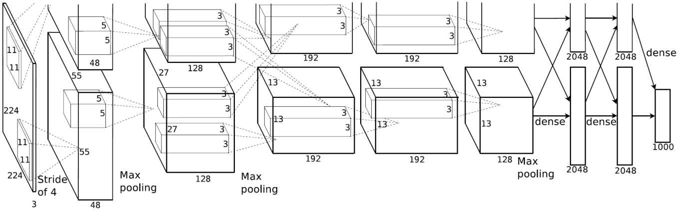
- **Fixes the problem of vanishing gradients to some degree**

Dropout

- Drop information at random
- Kind of a regularization, enforcing redundancy

A famous deep learning network called “AlexNet.”



- The network won the ImageNet competition in 2012.
- How many parameters?
- Given an image, what is happening?
- Inference Time: about 2ms per image when processing many images in parallel on the GPU
- Training Time: a few days given a single recent GPU

[Fig. adapted from A. Krizhevsky]

Demo

Neural networks have been used for many applications:

- Classification and Recognition in Computer Vision
- Text Parsing in Natural Language Processing
- Playing Video Games
- Stock Market Prediction
- Captcha

Demos:

- Russ website
- Antonio Places website

Classification in Computer Vision: ImageNet Challenge

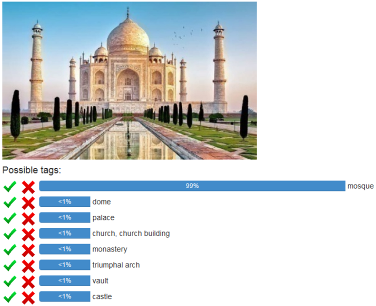
http://deeplearning.cs.toronto.edu/

Since it's the end of the semester, let's find the beach...

Classification in Computer Vision: ImageNet Challenge

http://deeplearning.cs.toronto.edu/

A place to maybe prepare for exams...



Links:

- Tutorials: http://deeplearning.net/tutorial/deeplearning.pdf
- Toronto Demo by Russ and students: http://deeplearning.cs.toronto.edu/
- MIT Demo by Antonio and students: http://places.csail.mit.edu/demo.html
- Honglak Lee: http://deeplearningworkshopnips2010.files.wordpress.com/2010/09/r workshop-tutorial-final.pdf
- Yann LeCun: http://www.cs.nyu.edu/ yann/talks/lecun-ranzato-icml2013.pdf
- Richard Socher: http://lxmls.it.pt/2014/socher-lxmls.pdf

Videos:

- Video games: https://www.youtube.com/watch?v=mARt-xPablE
- Captcha: http://singularityhub.com/2013/10/29/tiny-ai-startup-vicarious-says-its-solved-captcha/
- https://www.youtube.com/watch?v=lge-dl2JUAM#t=27
- Stock exchange: http://cs.stanford.edu/people/eroberts/courses/soco/projects/neural-networks/Applications/stocks.html