Camera Models

Textbook

- If you are interested, this book has it all:

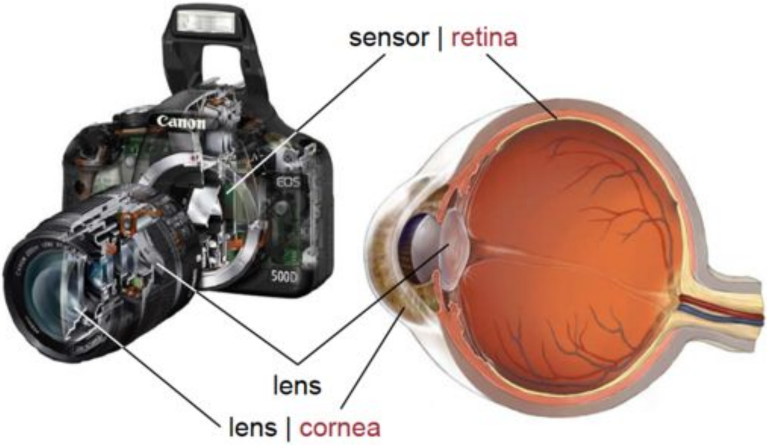
A. Zisserman and R. Hartley

Multiview Geometry

Cambridge University Press, 2003

Camera

- Camera is structurally similar to the eye



[Source: L.W. Kheng]

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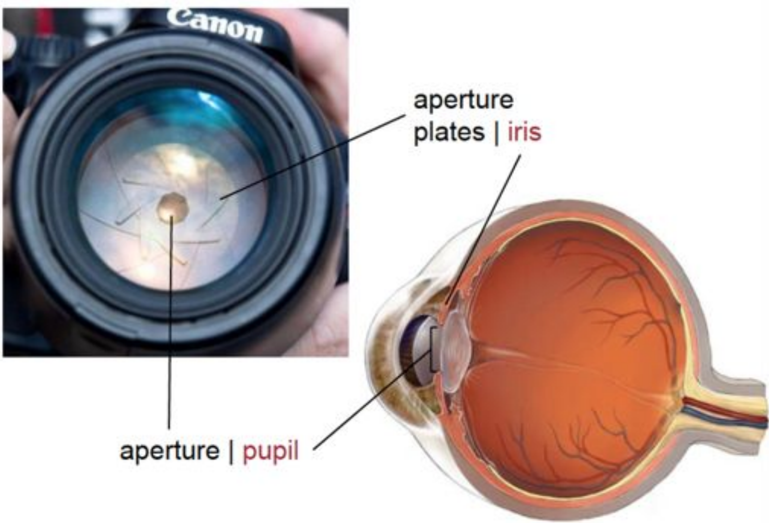
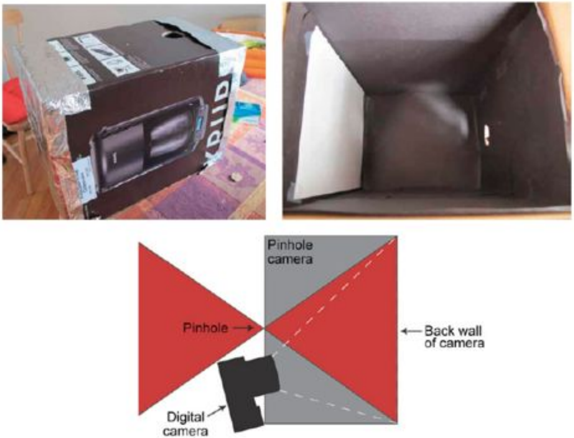


Camera

- Remember the pinhole camera from Lecture 2?



[Source: A. Torralba]

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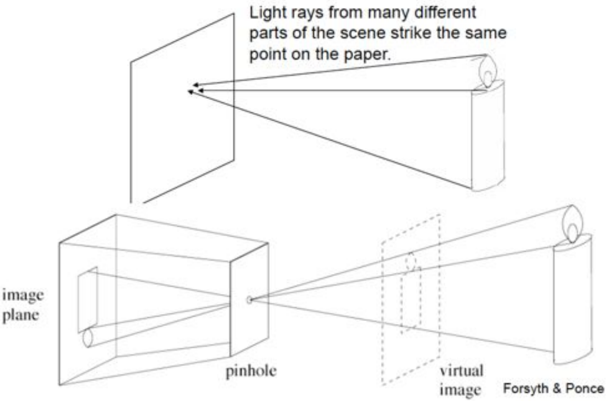

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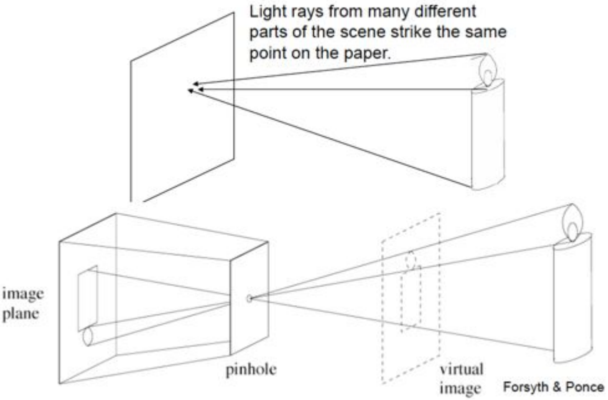
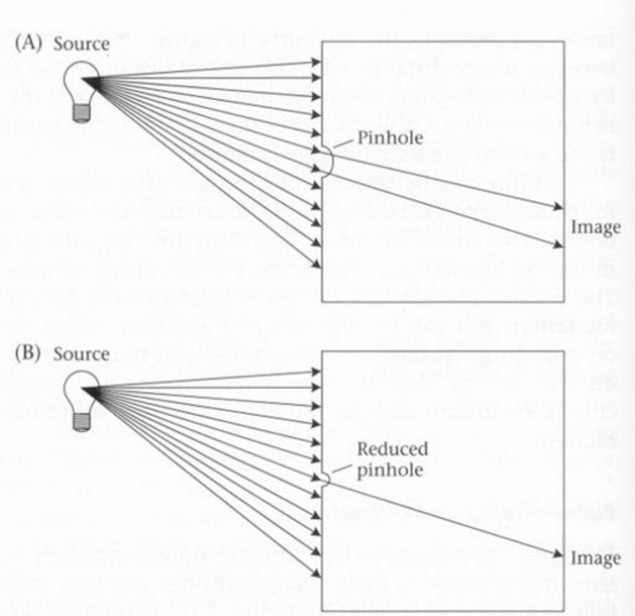
Camera

- Remember the pinhole camera from Lecture 2?
- Size of the pinhole is called **aperture**



[Source: A. Torralba]

Pinhole Camera



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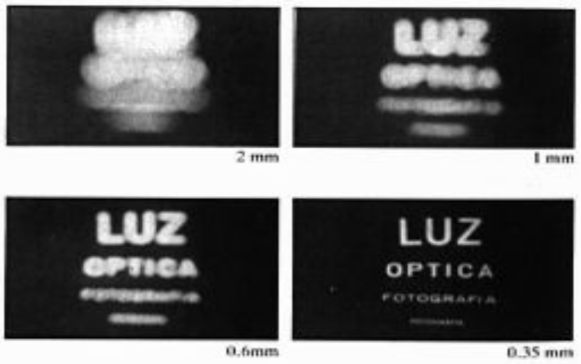
Shrinking the Aperture



Why not make the aperture as small as possible?

- Less light gets through
- Diffraction effects...

[Source: N. Snavely]

Shrinking the Aperture

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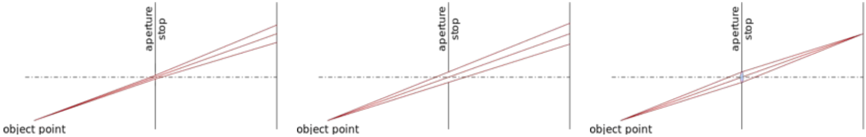
Adding a Lens

[Pic from Wikipedia]



Small pinhole | Big pinhole | Lens

- A lens focuses light onto the film
- There is a specific distance at which objects are **in focus**

Adding a Lens

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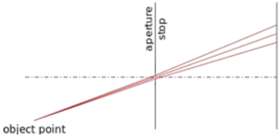

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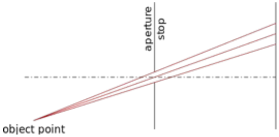

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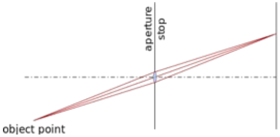

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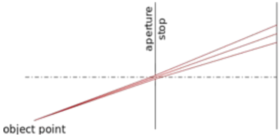

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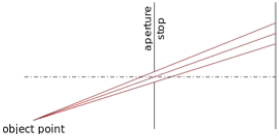

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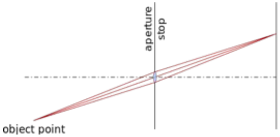

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[Source: N. Snavely]

Some “Cameras” Have Bigger Lenses than Others

http://www.use.com/images/s_2/thick_glasses_13b6941623c255ff400a_1.jpg?

Imaging

- Images are 2D projections of real world scene
- Images capture two kinds of information:
  - Geometric: positions, points, lines, curves, etc.
  - Photometric: intensity, color
- Complex 3D-2D relationships
- Camera models approximate these relationships

[Source: L.W. Kheng]

Projection

[Source: N. Snavely]

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[source: R. Urtasun]

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Modeling Projection

[Pics from: A. Torralba, Forsyth & Ponce]

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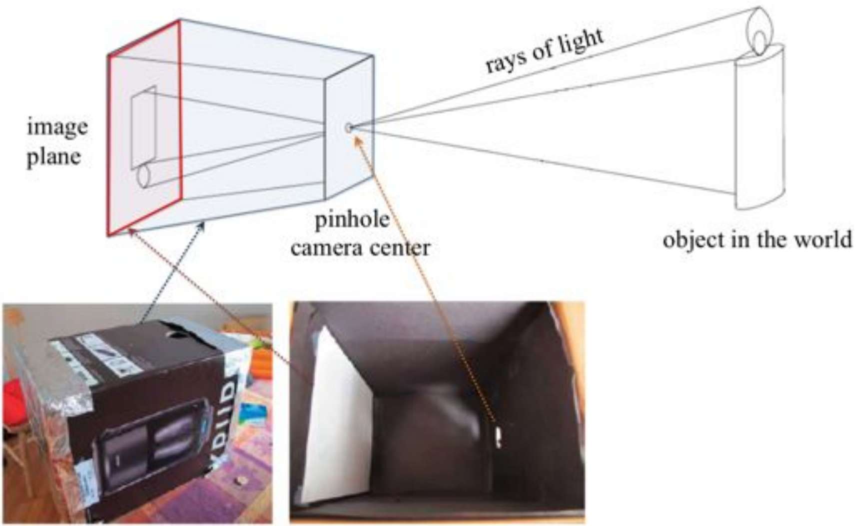

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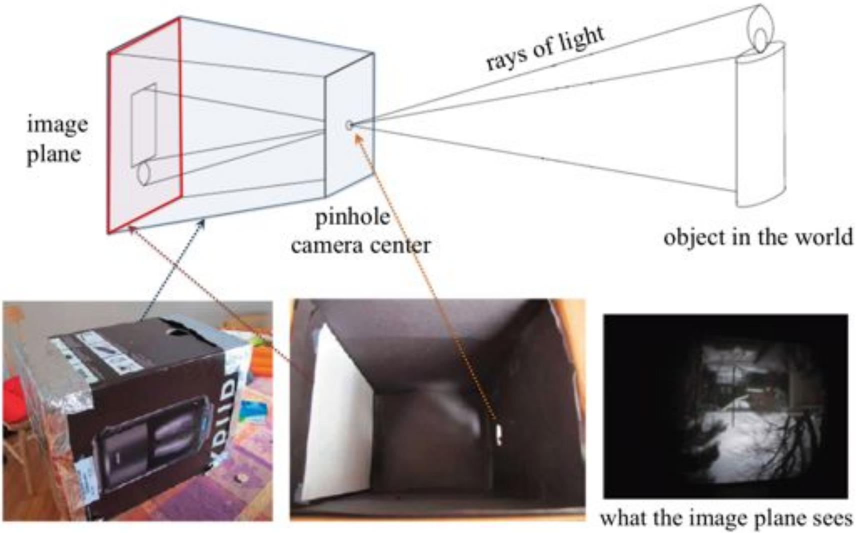

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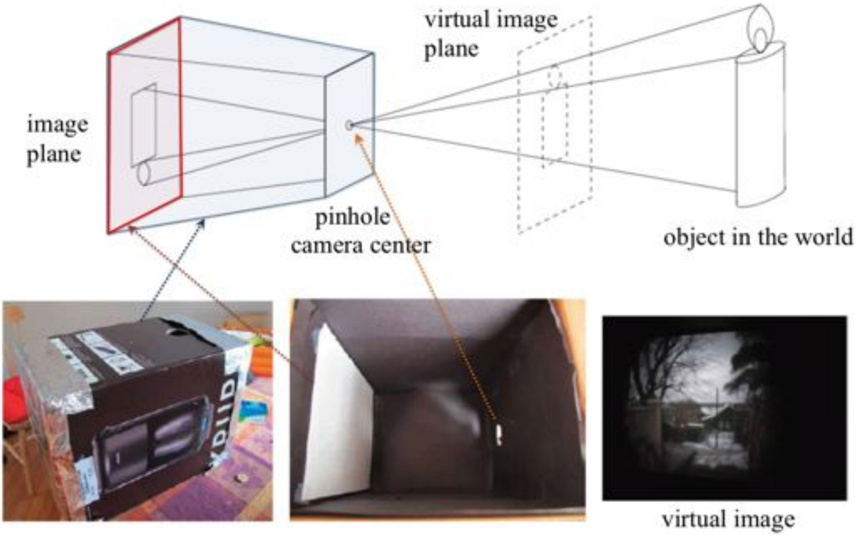

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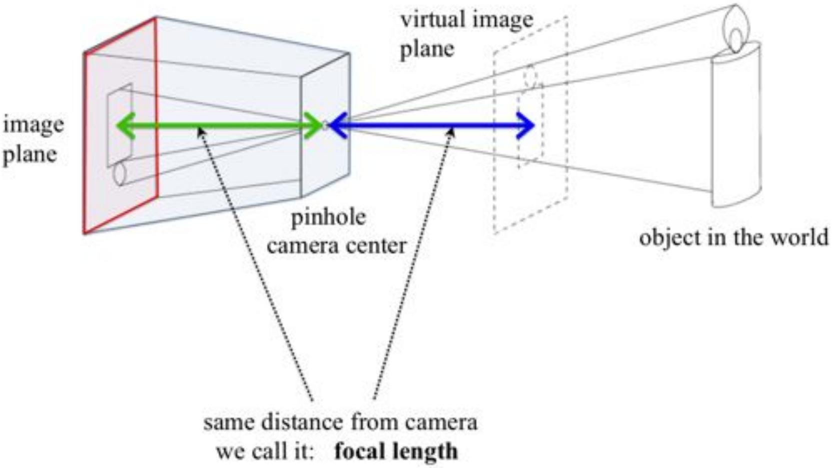

- Can be thought of as **zoom**
- Related to the **field of view**

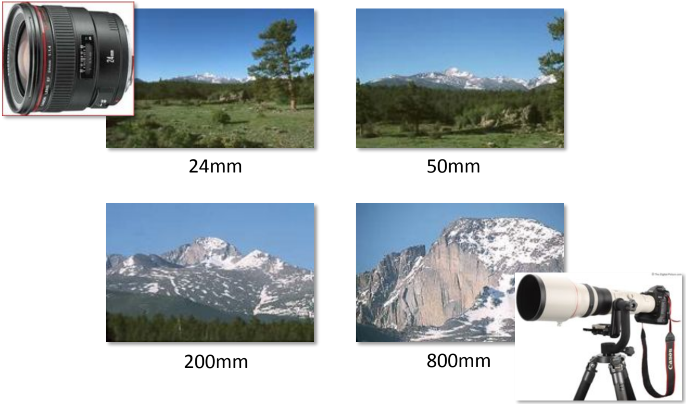

Figure: Image from N. Snavely

[Source: N. Snavely, slide credit: R. Urtasun]

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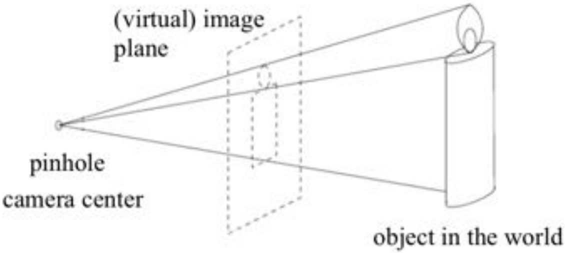
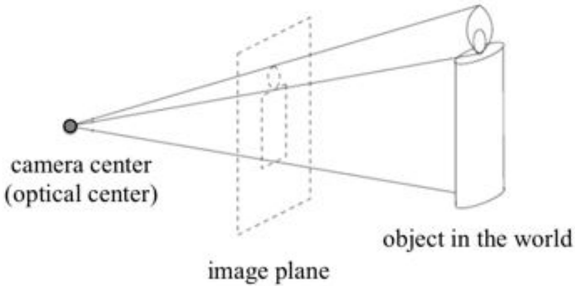


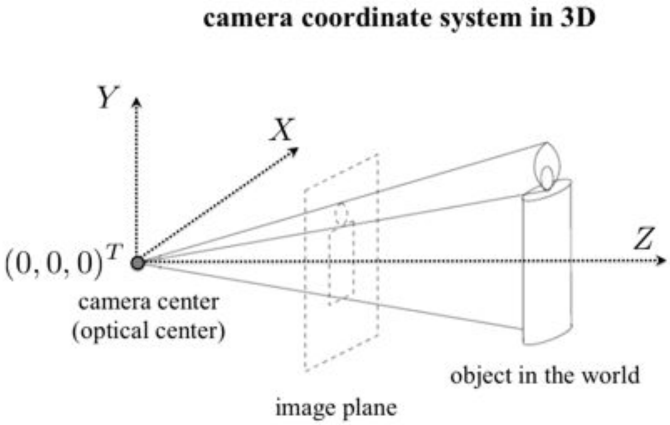
- Since it's easier to think in a non-upsidedown world, we will work with the virtual image plane, and just call it the image plane.
- How do points in 3D project to image plane? If I know a point in 3D, can I compute to which pixel it projects?

Modeling Projection



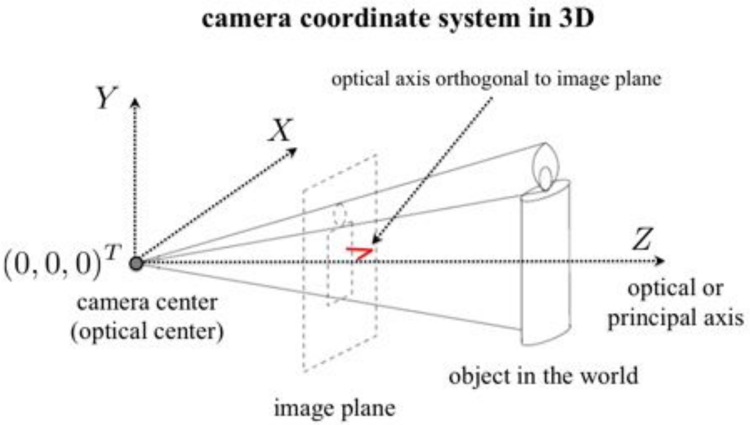
- First some notation which will help us derive the math
- To start with, we need a coordinate system

Modeling Projection



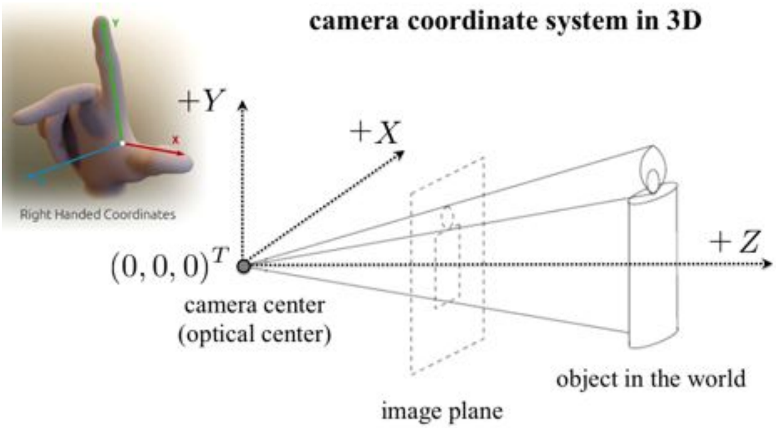
- We place a coordinate system relative to camera: **optical center** or **camera center C** is thus at origin $(0, 0, 0)$.

Modeling Projection



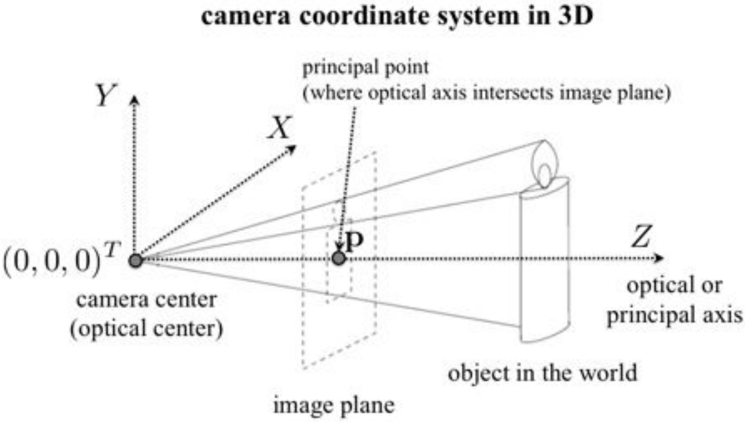
- The Z axis is called the **optical** or **principal axis**. It is orthogonal to the image plane. Axes X and Y are parallel to the image axes.

Modeling Projection



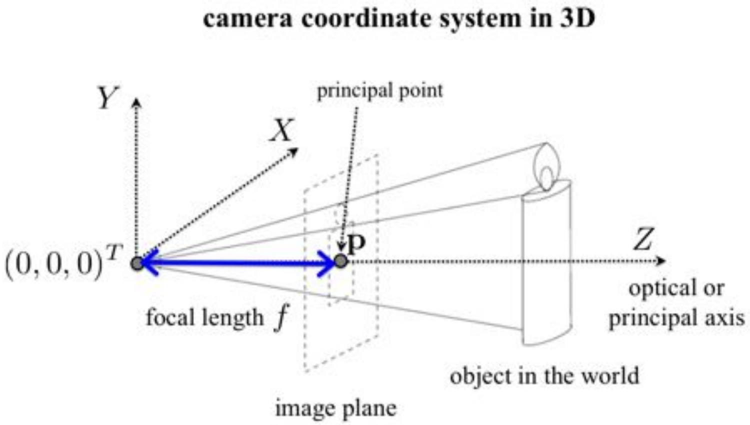
- We will use a **right handed** coordinate system

Modeling Projection



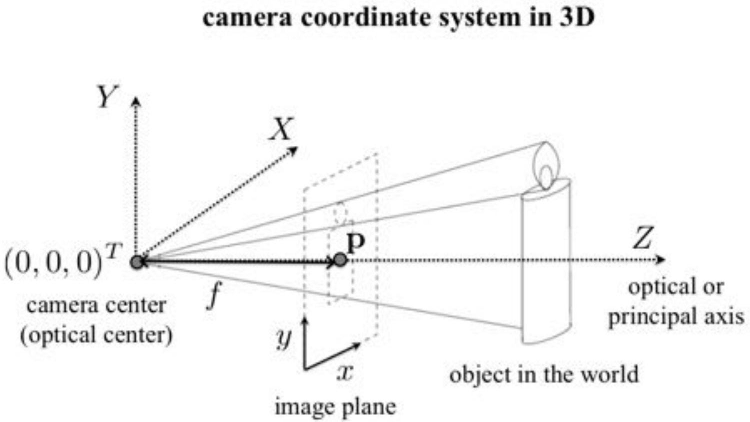
- The optical axis intersects the image plane in a point, **p**. We call this point a **principal point**. It's worth to remember the principal point since it will appear again later in the math.

Modeling Projection



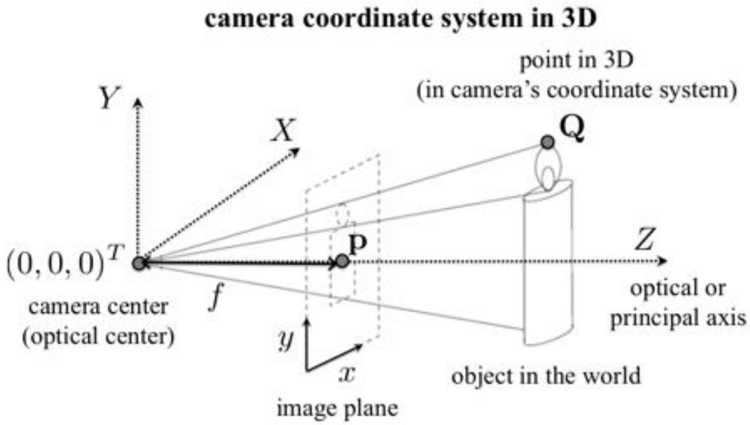
- The distance from the camera center to the principal point is called **focal length**, we will denote it with f. It's worth to remember the focal length since it will appear again later in the math.

Modeling Projection



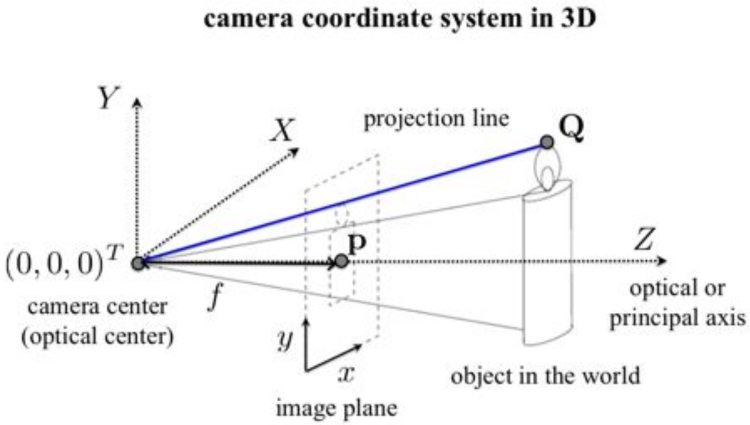
- We'll denote the image axes with x and y. An image we see is of course represented with these axes. We'll call this an **image coordinate system**.
- The tricky part is how to get from the camera's coordinate system (3D) to the image coordinate system (2D).

Modeling Projection



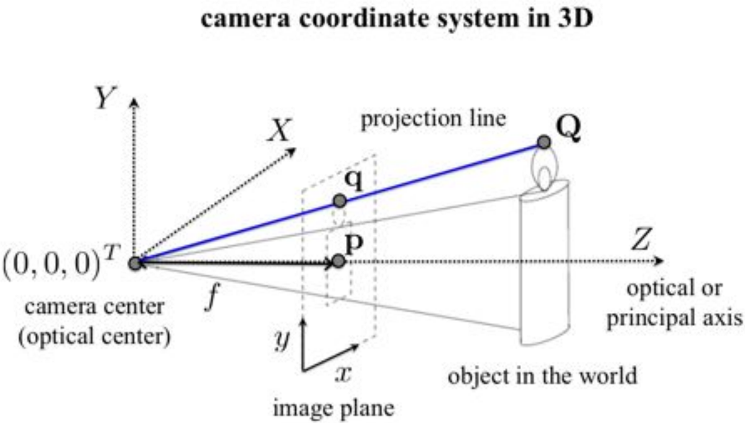
- Let's take some point **Q** in 3D. **Q** "lives" relative to the camera; its coordinates are assumed to be in camera's coordinate system.

Modeling Projection



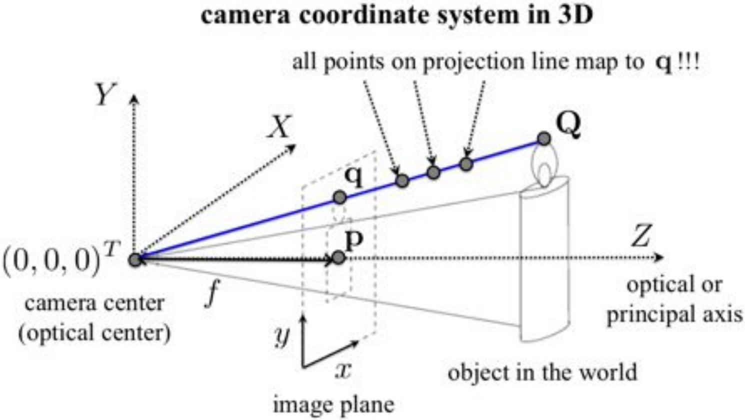
- We call the line from **Q** to camera center a **projection line**.

Modeling Projection



- The projection line intersects the image plane in a point **q**. This is the point we see in our image.

Modeling Projection



- First thing to notice is that all points from **Q**'s projection line project to the same point **q** in the image!
- **Ambiguity**: It's impossible to know how far a 3D point is from the camera along the projection line by looking only at the image (point **q**).

Modeling projection

From the movie Bone Collector

- **Ambiguity**: It's impossible to know how far a 3D point is from the camera along the projection line by looking only at the image (point **q**).
- It's impossible to know the real 3D size of objects just from an image
- Why did the detective put a dollar bill next to the footprint?
- How would you compute the shoe's dimensions?

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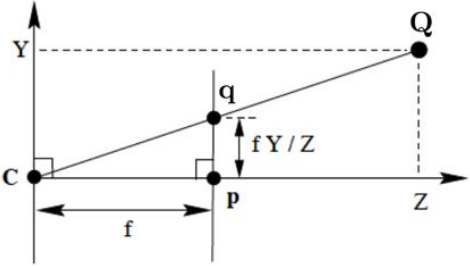
Projection: Ready for Math

[Pic from: Zisserman & Hartley]



Projection Equations

- Using similar triangles:

$$\mathbf{Q} = (X, Y, Z)^T \quad \rightarrow \quad \left(\frac{f \cdot X}{Z}, \frac{f \cdot Y}{Z}, f\right)^T$$

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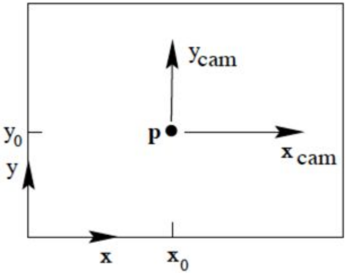

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where $\mathbf{p} = (p_x, p_y)$ is the principal point

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- Get the projection by throwing the last coordinate:

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- This is NOT a linear transformation as a division by Z is non-linear

Homogeneous Coordinates!

- We will use homogeneous coordinates, which simply append a 1 to the vector

Homogeneous coordinates to the rescue!

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous image coordinates

$$(x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

homogeneous scene coordinates

Converting *from* homogeneous coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow (x/w, y/w, z/w)$$

[Source: N. Snavely]

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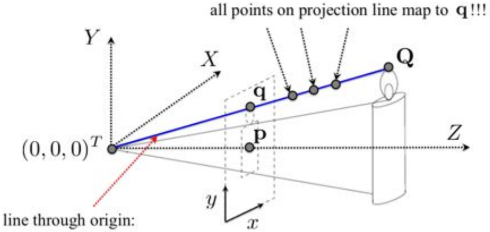

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \lambda \cdot \begin{bmatrix} X_Q \\ Y_Q \\ Z_Q \end{bmatrix} \rightarrow \mathbf{q}$$

- In Projective Geometry, all points are equal under scaling

Useful Trivia about Homogeneous Coordinates

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- We know that a vector orthogonal to two vectors is a cross product between them: $\mathbf{l} = (x_1, y_1, 1)^T \times (x_2, y_2, 1)^T$. And this is easy to compute.

Back to Perspective Projection

- We currently have this (the nasty division by Z):

$$\mathbf{Q} = (X, Y, Z)^T \quad \rightarrow \quad \mathbf{q} = \begin{bmatrix} \frac{f \cdot X}{Z} + p_x \\ \frac{f \cdot Y}{Z} + p_y \end{bmatrix}$$

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- Write this with homogeneous coordinates:

$$\mathbf{Q} = (X, Y, Z)^T \quad \rightarrow \quad \mathbf{q} = \begin{bmatrix} \frac{f \cdot X}{Z} + p_x \\ \frac{f \cdot Y}{Z} + p_y \\ 1 \end{bmatrix} \sim \begin{bmatrix} f \cdot X + Z \cdot p_x \\ f \cdot Y + Z \cdot p_y \\ Z \end{bmatrix}$$

Back to Perspective Projection

- We currently have this (the nasty division by Z):

$$\mathbf{Q} = (X, Y, Z)^T \quad \rightarrow \quad \mathbf{q} = \begin{bmatrix} \frac{f \cdot X}{Z} + p_x \\ \frac{f \cdot Y}{Z} + p_y \end{bmatrix}$$

- Write this with homogeneous coordinates:

$$\mathbf{Q} = (X, Y, Z)^T \quad \rightarrow \quad \mathbf{q} = \begin{bmatrix} \frac{f \cdot X}{Z} + p_x \\ \frac{f \cdot Y}{Z} + p_y \\ 1 \end{bmatrix} \sim \begin{bmatrix} f \cdot X + Z \cdot p_x \\ f \cdot Y + Z \cdot p_y \\ Z \end{bmatrix}$$

- We can now write this as matrix multiplication:

$$\mathbf{Q} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \quad \rightarrow \quad \begin{bmatrix} f \cdot X + Z \cdot p_x \\ f \cdot Y + Z \cdot p_y \\ Z \end{bmatrix} = \begin{bmatrix} f & 0 & p_x \\ 0 & f & p_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

Camera Intrinsics

- From previous slide:

$$\mathbf{Q} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \quad \rightarrow \quad \begin{bmatrix} f \cdot X + Z \cdot p_x \\ f \cdot Y + Z \cdot p_y \\ Z \end{bmatrix} = \begin{bmatrix} f & 0 & p_x \\ 0 & f & p_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

- Write:

$$K = \begin{bmatrix} f & 0 & p_x \\ 0 & f & p_y \\ 0 & 0 & 1 \end{bmatrix}$$

This is called a **camera calibration matrix** or **intrinsic parameter matrix**. The parameters in K are called **internal camera parameters**.

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This is called a **camera calibration matrix** or **intrinsic parameter matrix**. The parameters in K are called **internal camera parameters**.

- Finally: $\begin{bmatrix} w \cdot x \\ w \cdot y \\ w \end{bmatrix} = K \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \quad \rightarrow \quad \mathbf{q} = \begin{bmatrix} x \\ y \end{bmatrix}$

[Source: Zisserman & Hartley]

Camera Intrinsics

- Camera calibration matrix:

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$$K = \begin{bmatrix} f_x & 0 & p_x \\ 0 & f_y & p_y \\ 0 & 0 & 1 \end{bmatrix}$$

- And there might be a skew angle θ between x and y image axis:

$$K = \begin{bmatrix} f_x & -f_x \cot\theta & p_x \\ 0 & f_y / \sin\theta & p_y \\ 0 & 0 & 1 \end{bmatrix}$$

[Source: Zisserman & Hartley]

Camera Intrinsics

- Camera calibration matrix:

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We'll work with this one

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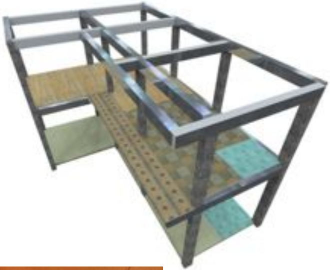
[Source: Zisserman & Hartley]

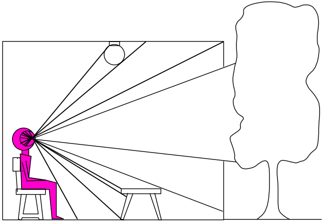
Perspective Projection



[Source: N. Snavely]

Dimensionality Reduction Machine (3D to 2D)

3D world



Point of observation

2D image

What have we lost?

- Angles
- Distances (lengths)

Slide by A. Efros

Projection properties

- **Many-to-one**: any points along same ray map to same point in image

Projection properties

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- Points $\rightarrow$ points

Projection properties

- **Many-to-one**: any points along same ray map to same point in image
- Points $\rightarrow$ points
- Lines $\rightarrow$ lines. Why?

Projection properties

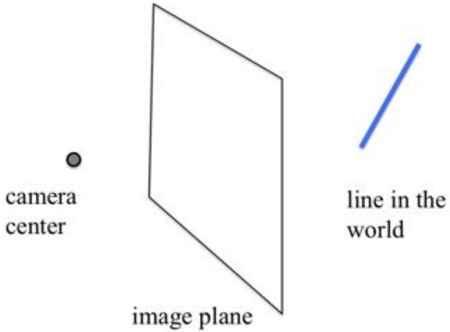

Figure: Proof by drawing

Projection properties

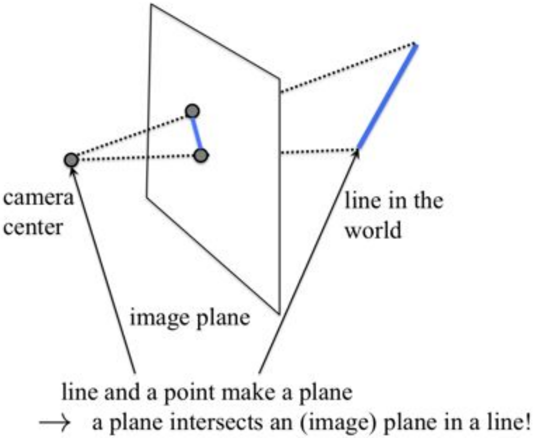

Figure: Proof by drawing

Projection properties

- **Many-to-one**: any points along same ray map to same point in image
- Points → points
- Lines → lines
- But line through principal point projects to a point. Why?

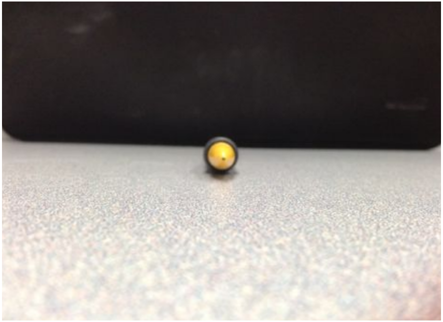

Figure: Can you tell where is the principal point?

Projection properties

- **Many-to-one**: any points along same ray map to same point in image
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- Planes → planes

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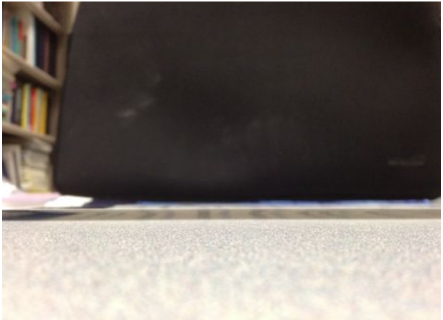
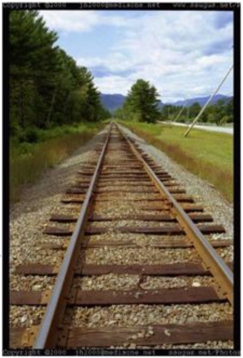
- Planes $\rightarrow$ planes
- But plane through principal point projects to line. Why?

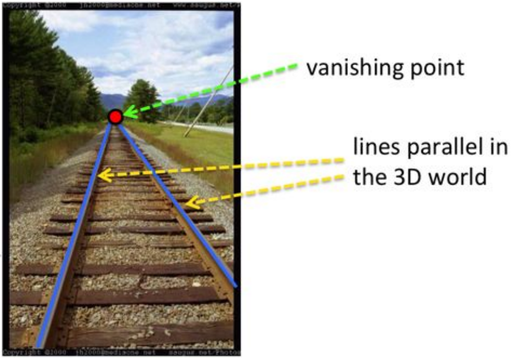


Projection Properties: Cool Facts

Parallel lines converge at a **vanishing point**

- Each different direction in the world **has its own vanishing point**



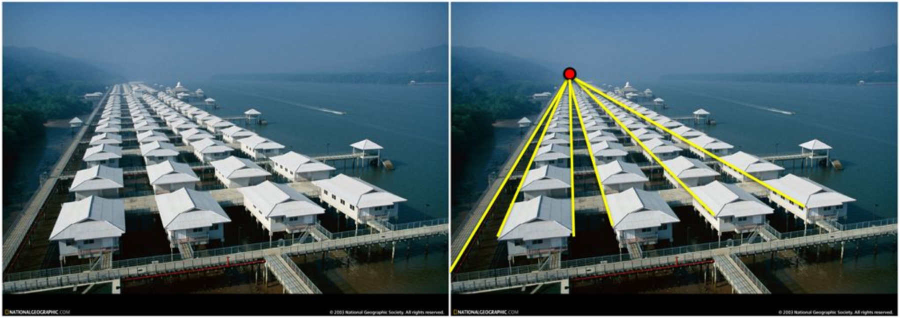


[Adopted from: N. Snavely, R. Urtasun]

Projection Properties: Cool Facts

Parallel lines converge at a **vanishing point**

- Each different direction in the world **has its own vanishing point**
- All lines with the same 3D direction intersect at the **same vanishing point**



[Pic: R. Szeliski]

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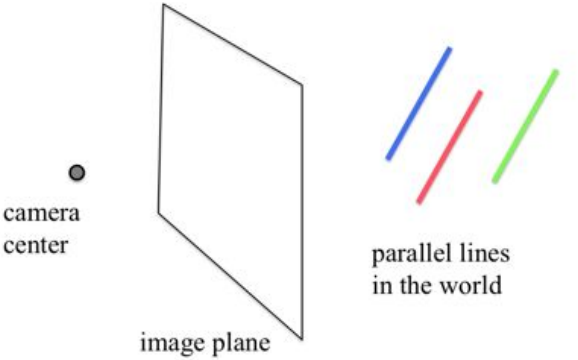

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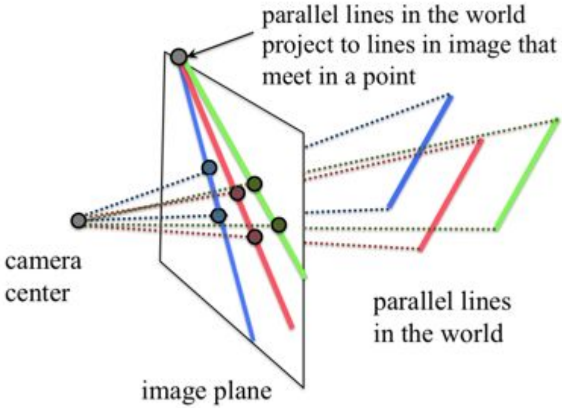

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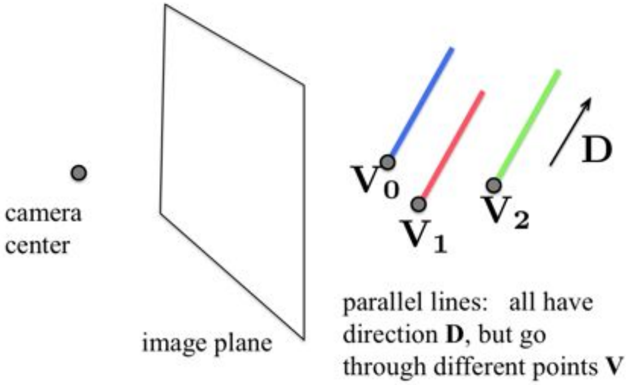

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- Line that passes through **V** with direction **D**: $\mathbf{X} = \mathbf{V} + t\mathbf{D}$.

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- Line that passes through $\mathbf{V}$ with direction $\mathbf{D}$: $\mathbf{X} = \mathbf{V} + t\mathbf{D}$.
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$$\begin{bmatrix} wx \\ wy \\ w \end{bmatrix} = K\mathbf{X} = \begin{bmatrix} f & 0 & p_x \\ 0 & f & p_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_x + tD_x \\ V_y + tD_y \\ V_z + tD_z \end{bmatrix} = \begin{bmatrix} fV_x + ftD_x + p_x V_z + tp_x D_z \\ fV_y + ftD_y + p_y V_z + tp_y D_z \\ V_z + tD_z \end{bmatrix}$$

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- Send $t \rightarrow \infty$ and compute x and y:

$$x = \lim_{t \to \infty} \frac{fV_x + ftD_x + p_x V_z + tp_x D_z}{V_z + tD_z} = \frac{fD_x + p_x D_z}{D_z}$$

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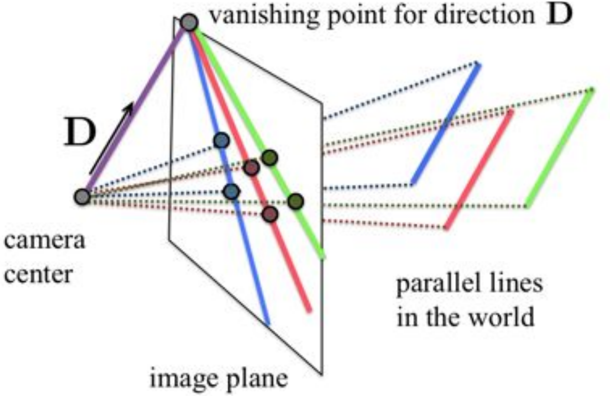
- This doesn't depend on $\mathbf{V}$! So all lines with direction $\mathbf{D}$ go to this point!

Projection Properties: Vanishing Point

- All lines with the same 3D direction intersect at the **same vanishing point**.

Projection Properties: Vanishing Point

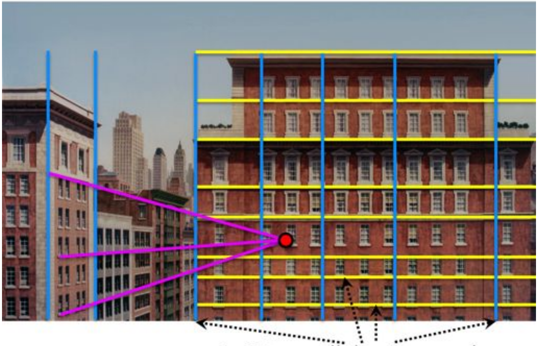
- All lines with the same 3D direction intersect at the **same vanishing point**.
- The easiest way to find this point: Translate line with direction **D** to the camera center. This line intersects the image plane in the vanishing point corresponding to direction **D**! Why?



Projection Properties: Cool Facts

Parallel lines converge at a **vanishing point**

- Each different direction in the world **has its own vanishing point**
- Lines parallel to image plane are also parallel in the image (we say that they intersect at infinity). Why?



In 3D: parallel to image plane
In 2D: parallel (no VP)

Projection Properties: Cool Facts

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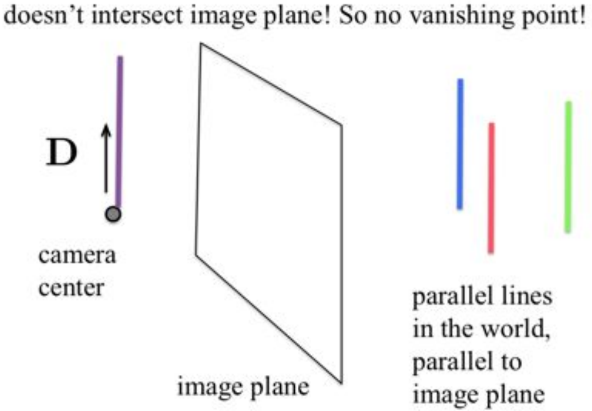

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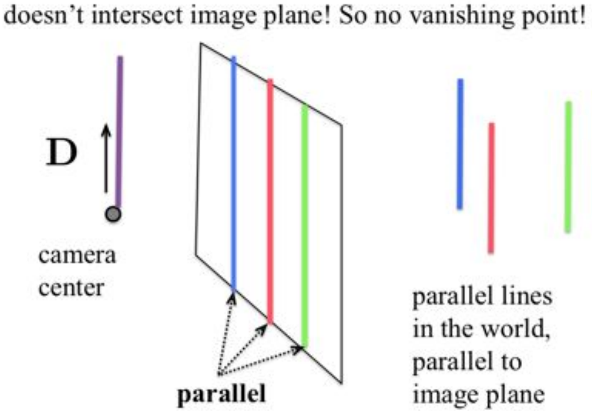


Projection Properties: Cool Tricks

- This picture has been recorded from a car with a camera on top. We know the camera intrinsic matrix K.
- Can we tell the incline of the hill we are driving on?
- How?

Projection Properties: Cool Tricks

- Can we tell the incline of the hill we are driving on?

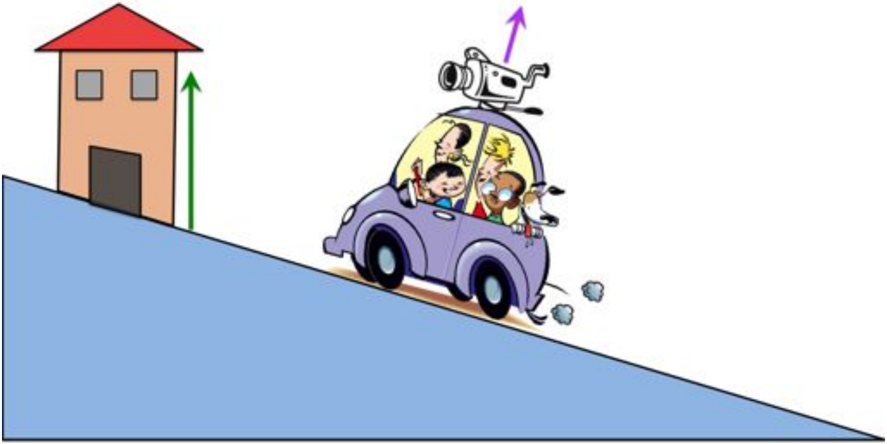


Figure: This is the 3D world behind the picture.

Projection Properties: Cool Tricks

- Can we tell the incline of the hill we are driving on?

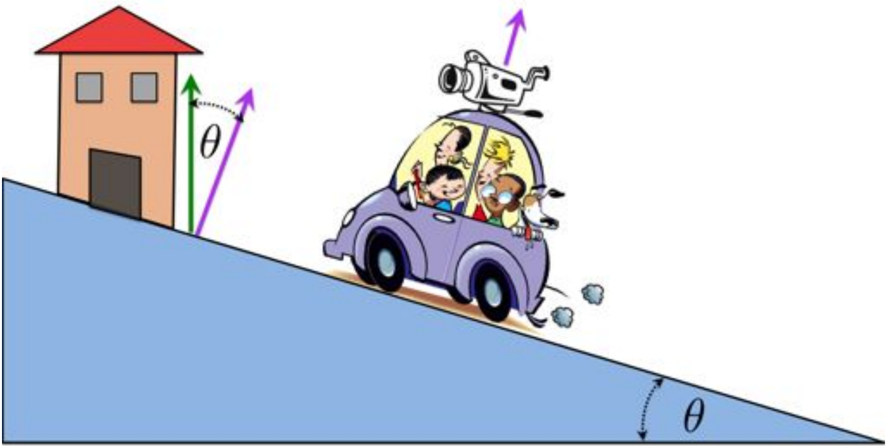


Figure: If we compute the 3D direction of the house's vertical lines relative to camera, we have the incline! How can we do that?

Projection Properties: Cool Tricks

- Can we tell the incline of the hill we are driving on?

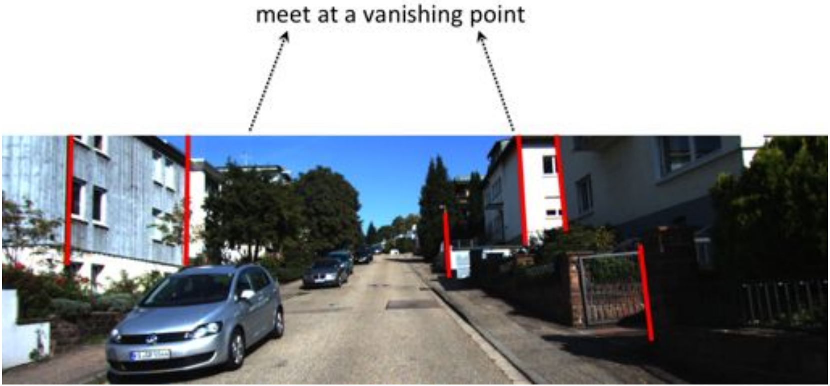


Figure: Extract "vertical" lines and compute vanishing point. How can we compute direction in 3D from vanishing point (if we have K)?

Projection Properties: Cool Tricks

- Can we tell the incline of the hill we are driving on?

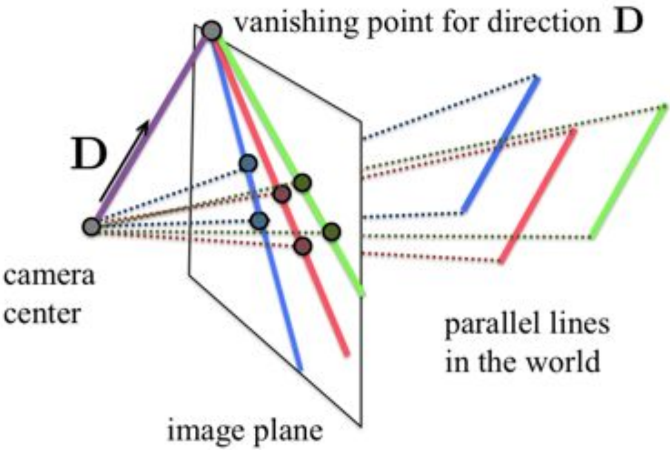
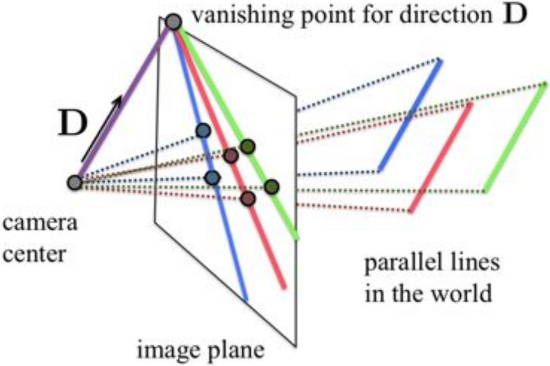


Figure: This picture should help.

Projection Properties: Cool Tricks

- Can we tell the incline of the hill we are driving on?

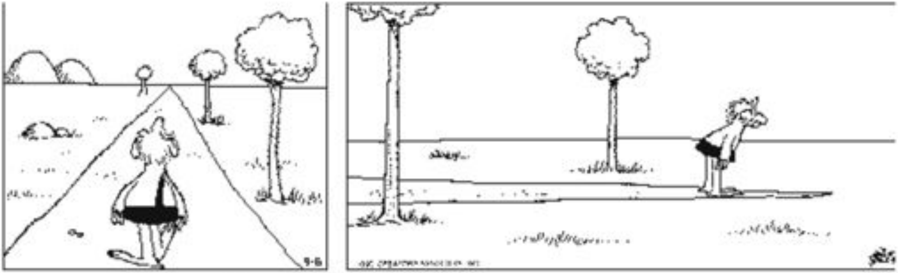


- We have:

$$\begin{bmatrix} w \cdot vp_x \\ w \cdot vp_y \\ w \end{bmatrix} = K\mathbf{D} \quad \rightarrow \quad \mathbf{D} = wK^{-1} \begin{bmatrix} vp_x \\ vp_y \\ 1 \end{bmatrix} \quad \rightarrow \quad \text{normalize } \mathbf{D} \text{ to norm } 1$$

Vanishing Points Can be Deceiving

- Parallel lines converge at a **vanishing point**.
- But intersecting lines in 2D are not necessary parallel in 3D.

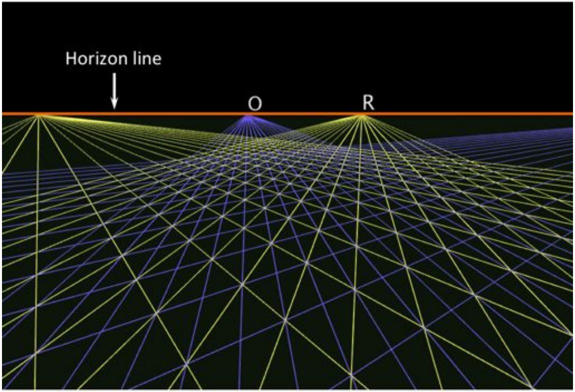


[Source: A. Jepson]

Projection Properties: Cool Facts

Parallel lines converge at a **vanishing point**

- Each different direction in the world **has its own vanishing point**
- For lines on the same 3D plane, the vanishing points lie on a **line**. We call it a **vanishing line**. Vanishing line for the ground plane is a **horizon line**.



http://4.bp.blogspot.com/-0Jm9d9j35Tc/T5ESbVpKl7I/AAAAAAAACEk/nVAiTxBuiyc/s1600/perspectiveGrid-01.png

Projection Properties: Cool Facts

Parallel lines converge at a **vanishing point**

- For lines on the same 3D plane, the vanishing points lie on a **line**. We call it a **vanishing line**. Vanishing line for the ground plane is a **horizon line**.
- Some horizon lines are nicer than others ;)

Punta Cana

Projection Properties: Cool Facts

Parallel lines converge at a **vanishing point**

- For lines on the same 3D plane, the vanishing points lie on a **line**. We call it a **vanishing line** or a **horizon line**.
- Parallel planes in 3D have the **same horizon line** in the image.

Projection Properties: Cool Facts

- Can I tell how much above ground this picture was taken?

Projection Properties: Cool Facts

- Can I tell how much above ground this picture was taken?

Projection Properties: Cool Facts

- Same distance as where the horizon intersects a building

Projection Properties: Cool Facts

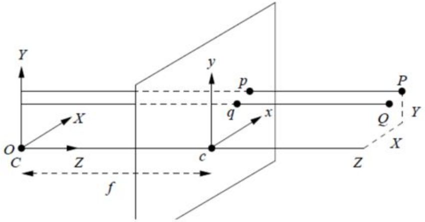
- Same distance as where the horizon intersects a building: 50 floors up

Projection Properties: Cool Facts

- This is only true when the camera (image plane) is orthogonal to the ground plane. And the ground plane is flat.
- A very nice explanation of this phenomena can be find by Derek Hoiem here: `https://courses.engr.illinois.edu/cs543/sp2011/materials/3dscene_book_svg.pdf`

Orthographic Projection

[Source: R. Urtasun]



Orthographic Projection

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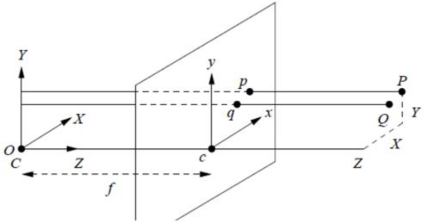
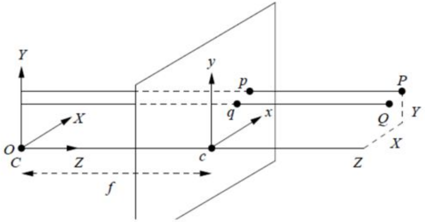


- Requires no division and simply drops the Z coordinate.

Orthographic Projection

[Source: R. Urtasun]

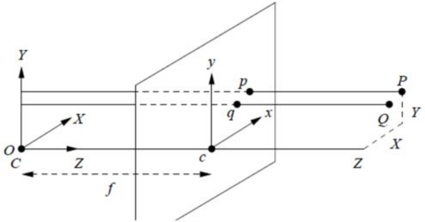


- Requires no division and simply drops the Z coordinate.
- Orthographic projection:

$$\mathbf{Q} = \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \quad \rightarrow \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Orthographic Projection

[Source: R. Urtasun]



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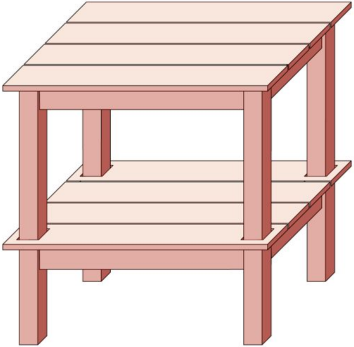
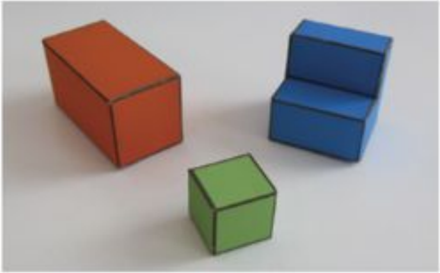
- Special case of perspective projection where the distance from the camera center to the image plane is infinity

Orthographic Projection

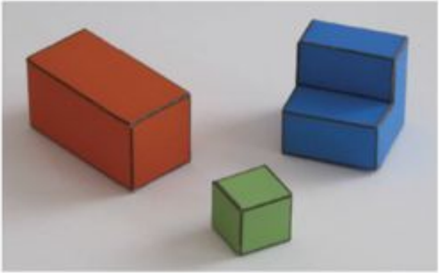


[Source: N. Snavely]

Orthographic Projection



Perspective projection



Parallel (orthographic) projection

- For perspective projection lines parallel in 3D **are not** parallel in the image.
- For orthographic projection lines parallel in 3D **are** parallel in the image.

[Source: A. Torralba]

Camera Parameters

We are not yet done with projection. To fully specify projection, we need to:

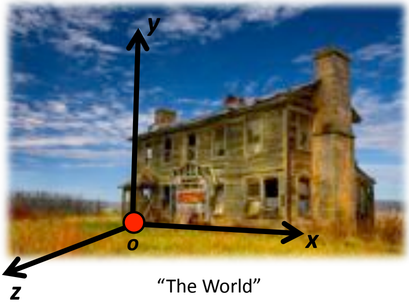
- Describe its **internal parameters** (we know this, this is our **K**)

Camera Parameters

We are not yet done with projection. To fully specify projection, we need to:

- Describe its **internal parameters** (we know this, this is our **K**)
- Describe its **pose in the world**. Two important coordinate systems:
  - World coordinate system
  - Camera coordinate system

Camera



"The World"

[Source: N. Snavely, slide credit: R. Urtasun]

Camera Parameters

- Why two coordinate systems?

Figure: Imagine this is your room.

Camera Parameters

- Why two coordinate systems?

Figure: When you were furnishing you measured everything in detail.

Camera Parameters

- Why two coordinate systems?

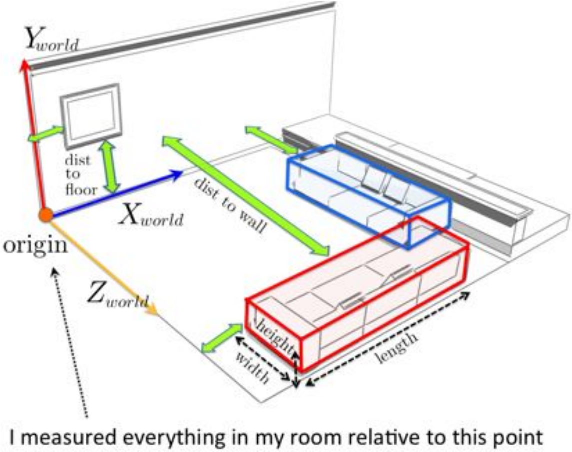


Figure: Thus you know all coordinates relative to a special point (origin) and coordinate system in the room. This is your **room's (world) coordinate system**.

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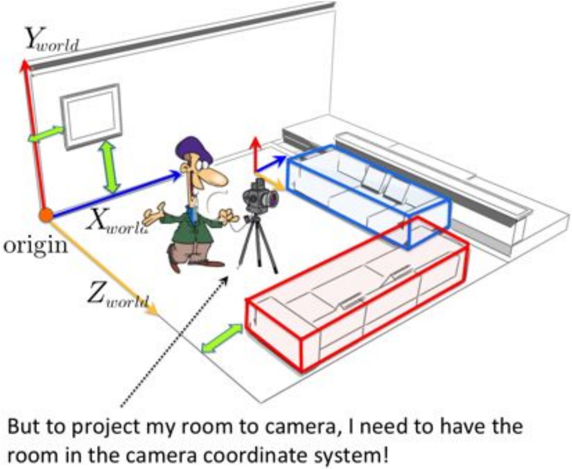

Figure: Now you take a picture and you wonder how points project to camera. In order to project, you need all points in **camera's coordinate system**.

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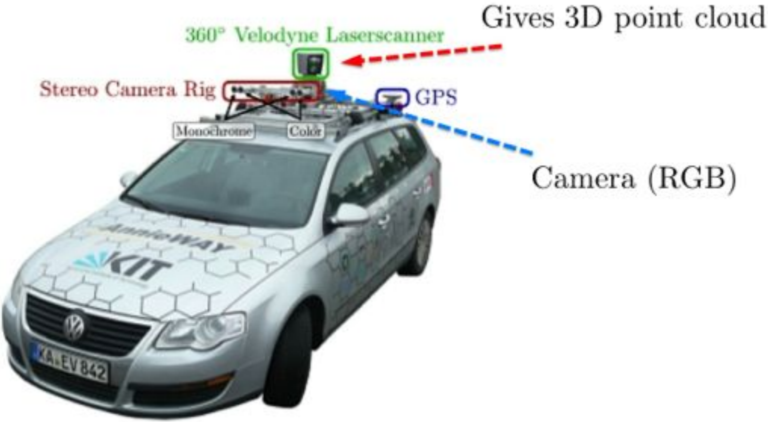

Figure: For e.g. self-driving cars, 3D points are typically measured with Velodyne.

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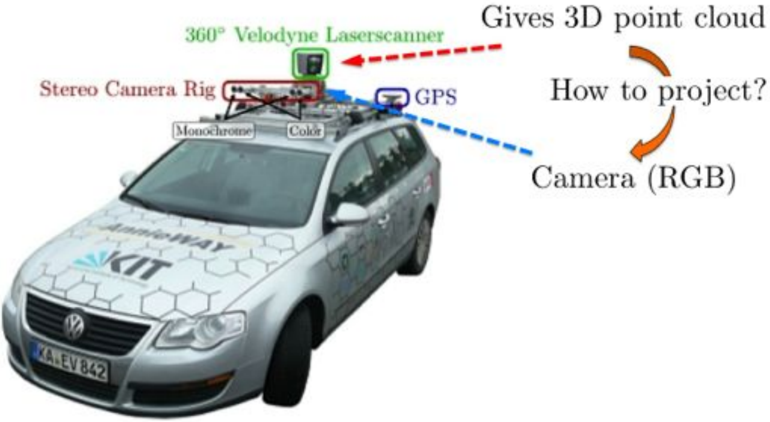

Figure: We want to be able to project the 3D points in Velodyne's coordinate system onto an image captured by a camera.

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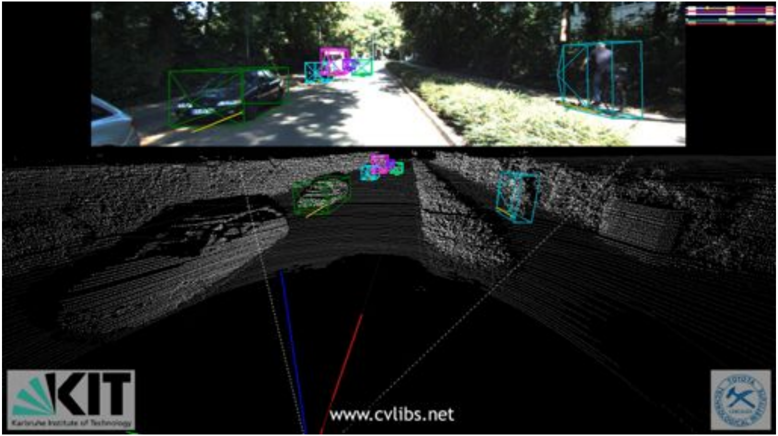

Figure: We want to be able to project the 3D points in Velodyne's coordinate system onto an image captured by a camera.

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To project a point (X, Y, Z) in world coordinates on the image plane, we need to:

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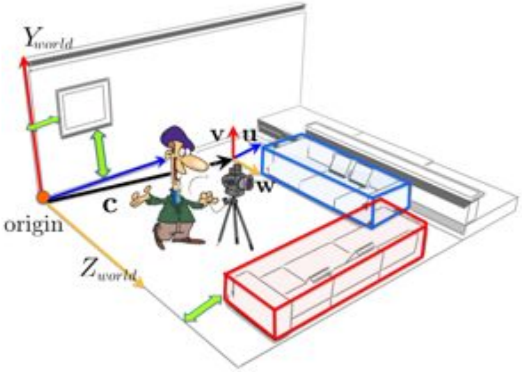
  - Need to know **camera intrinsics**
- These can all be described with matrices!

[Source: N. Snavely, slide credit: R. Urtasun]

Camera Extrinsics



c	...	camera position in room coordinate system
u, v, w	...	3 orthogonal directions of camera in room coordinate system

Figure: We first need our camera position and orientation in the room's world.

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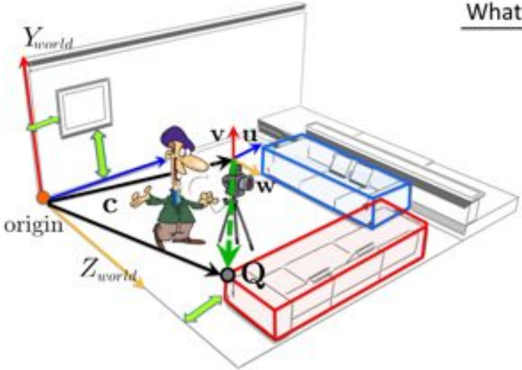

What is Q in camera's coordinate system??

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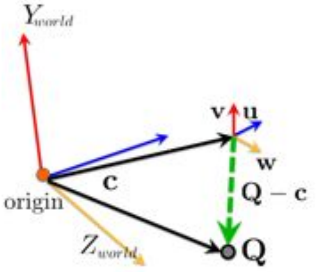

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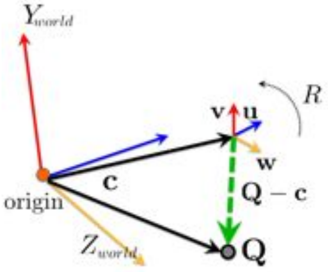

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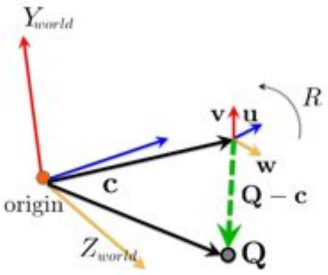

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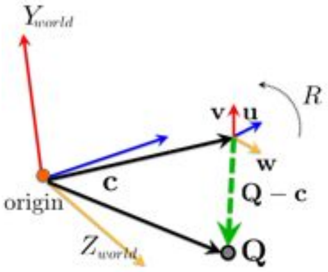

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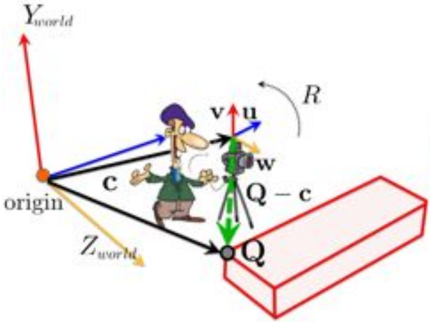

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$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = R\left(\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} - \mathbf{c}\right) = \begin{bmatrix} R & -Rc \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

camera coordinates room (world) coordinates

Figure: Final Transformation

Projection Equations

- **Projection matrix** P models the cumulative effect of all intrinsic and extrinsic parameters. We use homogeneous coordinates for 2D and 3D:

$$\mathbf{q} = \begin{bmatrix} ax \\ ay \\ a \end{bmatrix} = \mathbf{P} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

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- It can be computed as

$$\mathbf{P} = \underbrace{\begin{bmatrix} f & 0 & p_x \\ 0 & f & p_y \\ 0 & 0 & 1 \end{bmatrix}}_{\text{intrinsics } K} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{\text{projection}} \underbrace{\begin{bmatrix} \mathbf{R}_{3\times 3} & 0_{3\times 1} \\ 0_{1\times 3} & 1 \end{bmatrix}}_{\text{rotation}} \underbrace{\begin{bmatrix} \mathbf{I}_{3\times 3} & \mathbf{T}_{3\times 1} \\ 0_{1\times 3} & 1 \end{bmatrix}}_{\text{translation}}$$

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- To get a point $\mathbf{q}$ in the image plane, I need to compute $\mathbf{P}(X, Y, Z, 1)^T$, where $\mathbf{P}$ is a 3×4 matrix. This gives me a 3×1 vector. Now I divide all coordinates with the third coordinate (making the third coordinate equal to 1), and then drop the last coordinate. As simple as that.

The Projection Matrix

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- More compactly

$$\mathbf{P} = \mathbf{K} \begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix}$$

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$$\mathbf{P} = \mathbf{K} [\mathbf{R} \mid \mathbf{t}]$$

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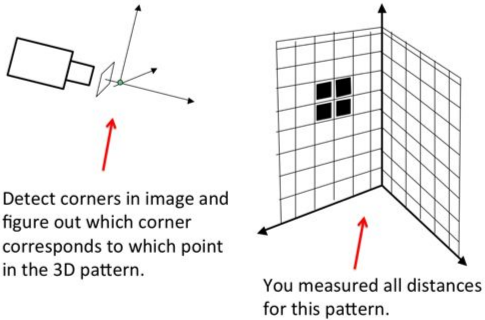
  It's the same thing.

- This might look complicated. Truth is, in most cases you don't have **P** at all, so you can't really compute any projections. When you have a calibrated camera, then someone typically gives you **P**. And then projection is easy.

A Short Note on Camera Calibration



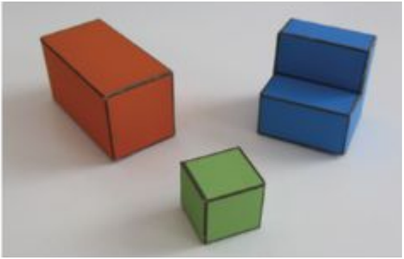
The general procedure:

- Place a 3D pattern (for which you know all distances) in front of camera.
- Take a picture. Detect corners in image and find correspondences with the points in the pattern.
- Go to the internet and check out the math that tells you how to compute K from these 2D-3D correspondences. ;) We won't cover in class.

[Pic from: R. Duraiswami]

Camera Calibration: Interesting Fact

- Let's say you have an image but you don't know **anything** about the camera (for example, image downloaded from the web).
- For images where you see lines corresponding to 3 orthogonal directions, like cubes or rooms, you can compute the camera matrix K as well as R and t!



- How to do this is explained in the Zisserman & Hartley book.

Projection Properties: Cool Facts

- As a consequence, for scenes with lots of lines (e.g. man-made scenes) one can reconstruct the scene in 3D from a **single image**!

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Projection Properties: Cool Facts

- As a consequence, for scenes with lots of lines (e.g. man-made scenes) one can reconstruct the scene in 3D from a **single image**!
- For those interested, check out the math here:

A. Criminisi, I. Reid, and A. Zisserman

Single View Metrology

International Journal of Computer Vision, vol 40, num 2, 2000

http://www.cs.cmu.edu/ ph/869/papers/Criminisi99.pdf

Inserting Objects

[Karsch et al., 2011]

K. Karsch, V. Hedau, D. Forsyth, D. Hoiem, Rendering synthetic objects into legacy photographs, SIGGRAPH'11

link to video   link to paper/code

Camera Calibration: Another Interesting Fact

- From a longer **video** in which the sun travels across the sky you can compute the camera intrinsic matrix, as well as extrinsic, i.e., the GPS location where you are! Well, up to a 100km accuracy...

Camera Calibration: Another Interesting Fact

J.-F. Lalonde, S. G. Narasimhan, and A. A. Efros

What Do the Sun and Sky Tell Us About the Camera?

International Journal on Computer Vision, 88(1), May 2010

Paper: `http://vision.gel.ulaval.ca/~jflalonde/projects/sky/index.html`

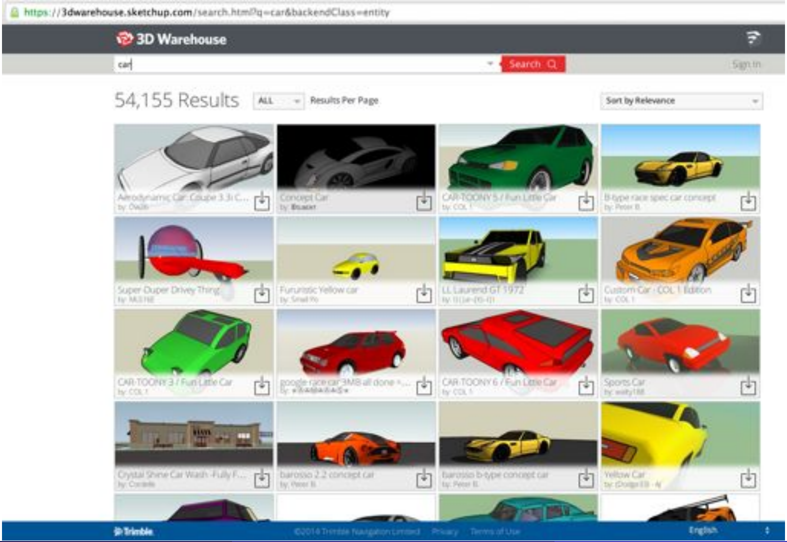
Code: `https://github.com/jflalonde/webcamCalibration`

- From a longer **video** in which the sun travels across the sky you can compute the camera intrinsic matrix, as well as extrinsic, i.e., the GPS location where you are! Well, up to a 100km accuracy...
- Is this useful? Maybe, to catch terrorists that record their videos outside.

Exercise (Not Very Easy, But Fun)

- We want to render (project) a 3D CAD model of a car to this image in a realistic way
- How?

- First get a CAD model. There are tones of them, e.g. 3D Warehouse (free)



Exercise

- We downloaded this model. Now what?

Figure: A CAD model is a collection of 3D vertices and faces that connect the vertices. Each face represents a small triangle. It typically has color.

Exercise

- Our image was collected with a car on the road:
  - A camera was on top of the car, approximately 1.7m above ground
  - Image plane is orthogonal to the ground
  - We have the internal parameters of the camera, **K**.

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- We can now "place" our CAD model to this point (compute R and $\mathbf{t}$)
- Rendering:

  Compute $[ax, ay, a]^T = K\left[R \mid \mathbf{t}\right][X, Y, Z, 1]^T$ for each CAD vertex$[X, Y, Z]^T$. Divide $[ax, ay, a]^T$ with third coord and drop it.

Exercise

- That's it. Make a video for more fun

(click on image to play video)

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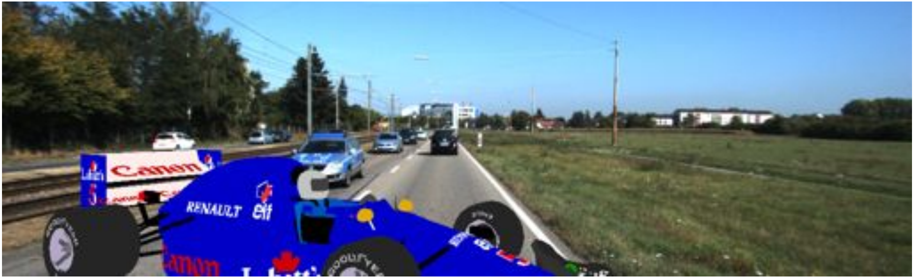

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A Little More on Camera Models