

CSC384h: Intro to Artificial Intelligence

▶ Reasoning Under Uncertainty

▶ D-Separation

More generally...

- ▶ Many conditional independencies hold in a given BN.
 - ▶ These independencies are useful in computation, explanation, etc.
- ▶ How do we determine if two variables X , Y are independent given a set of variables E ?

Simple graphical property: **D-separation**

- ▶ A set of variables E *d-separates* X and Y if it *blocks every undirected path* in the BN between X and Y . (We'll define *blocks* next.)
- ▶ X and Y are **conditionally independent** given evidence E if E d-separates X and Y
 - ▶ thus BN gives us an easy way to tell if two variables are independent (set $E = \emptyset$) or cond. independent given E .

Blocking in D-Separation

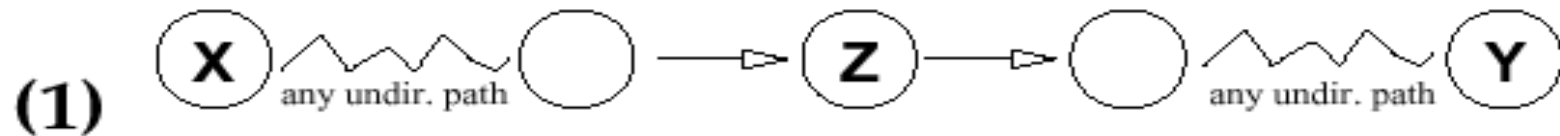
- ▶ Let P be an **undirected path** from X to Y in a BN. Let \mathbf{E} (evidence) be a set of variables.

We say \mathbf{E} blocks path P

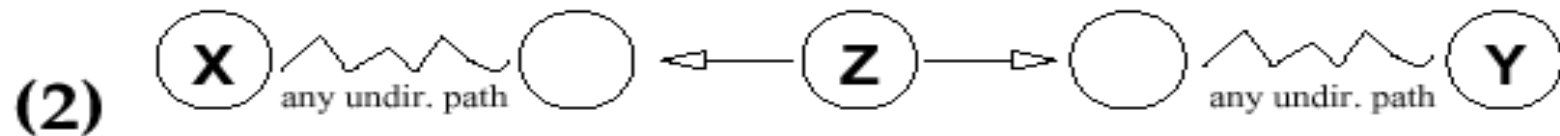
iff **there is some** node Z on the path P such that:

- ▶ **Case 1:** $Z \in \mathbf{E}$ and one arc on P enters (goes into) Z and one leaves (goes out of) Z ; or
- ▶ **Case 2:** $Z \in \mathbf{E}$ and both arcs on P leave Z ; or
- ▶ **Case 3:** both arcs on P enter Z and *neither Z , nor any of its descendants, are in \mathbf{E} .*

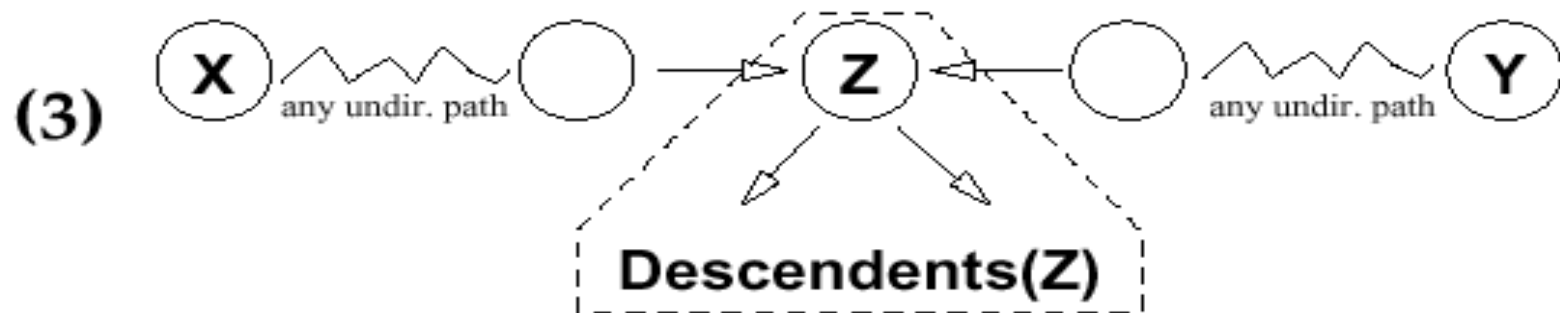
Blocking: Graphical View



If Z in evidence, the path between X and Y blocked



If Z in evidence, the path between X and Y blocked



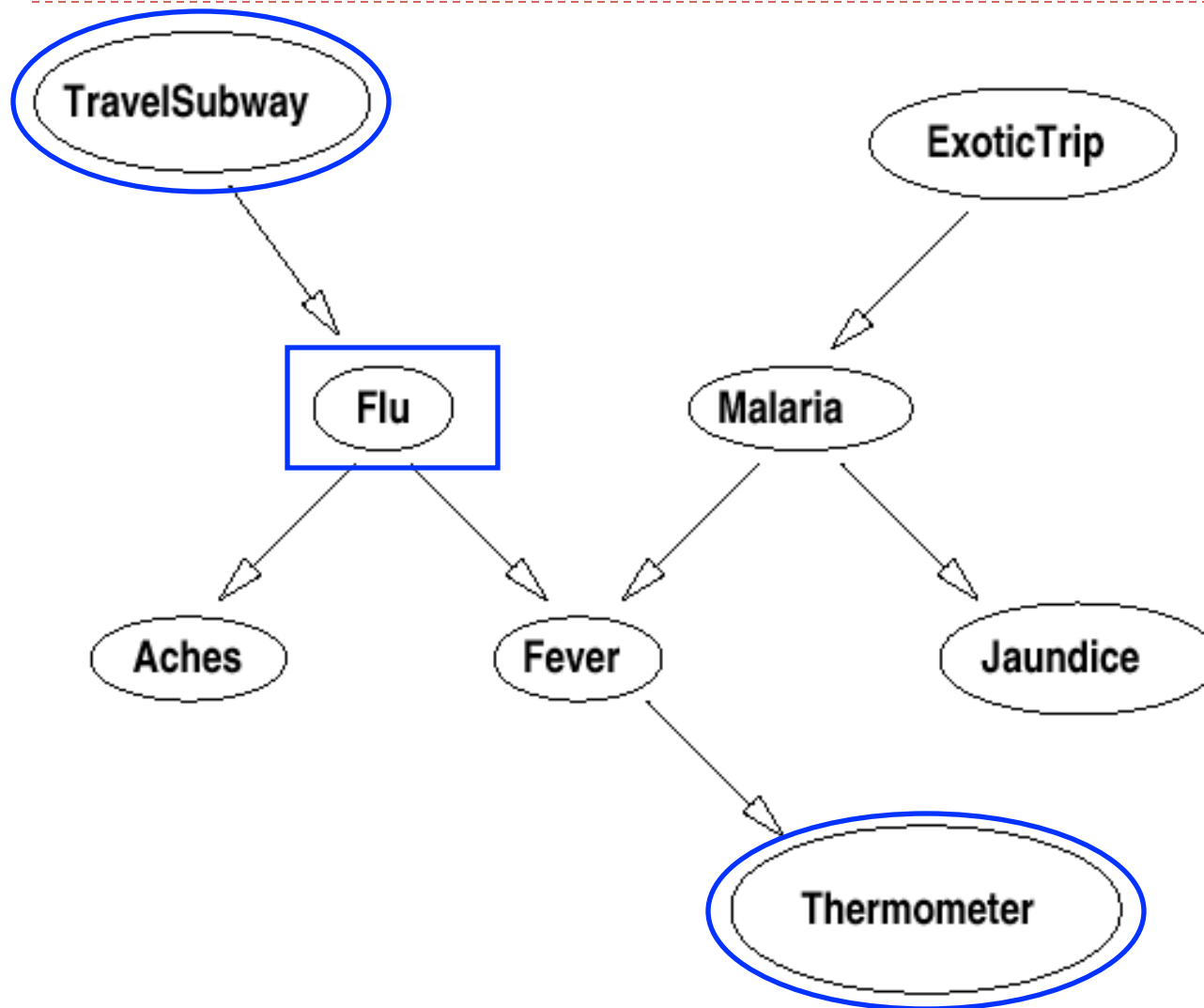
If Z is **not** in evidence and **no** descendent of Z is in evidence, then the path between X and Y is blocked

Recall: D-Separation

D-separation:

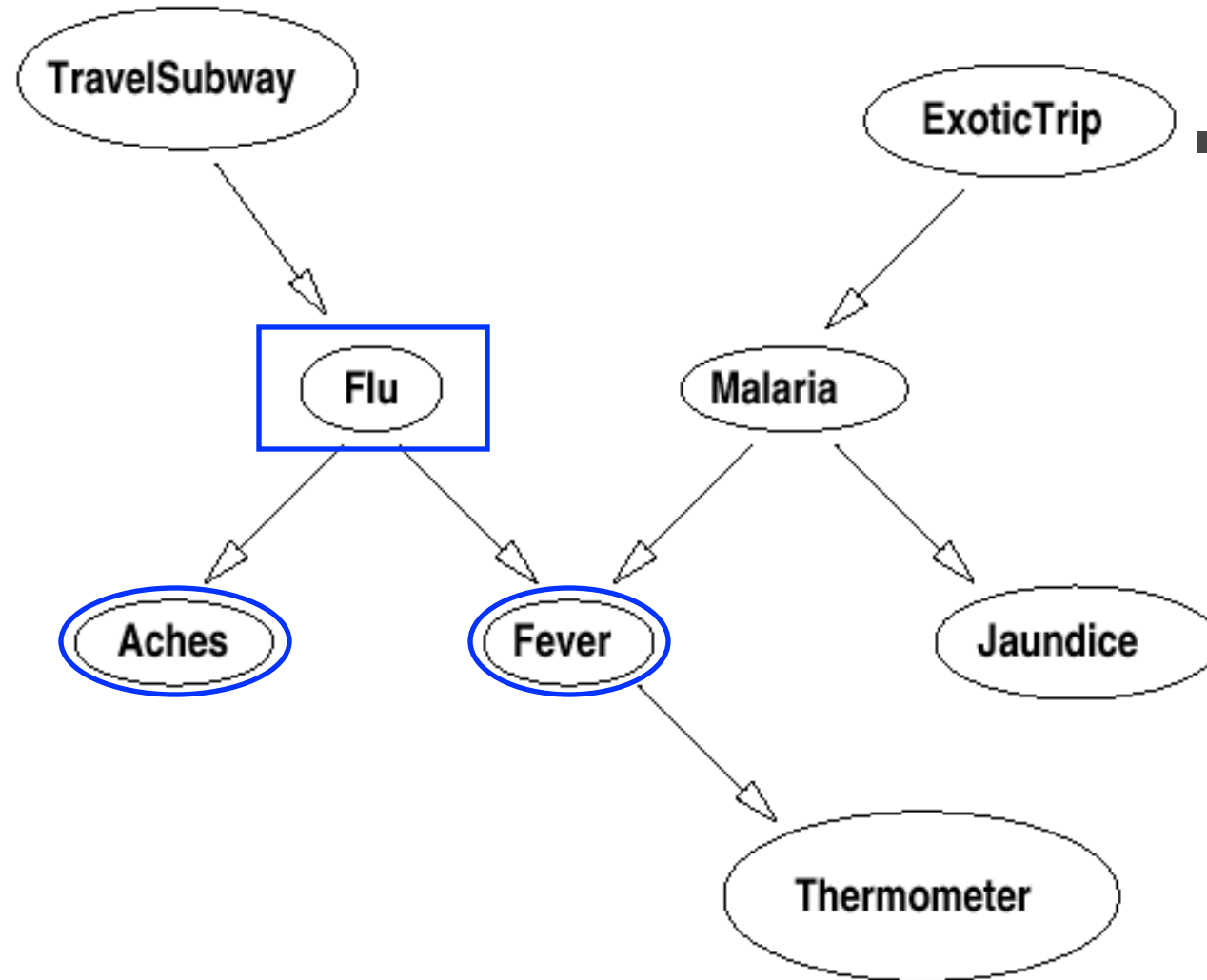
A set of variables **E** *d-separates* X and Y if it *blocks every undirected path* in the BN between X and Y.

D-Separation: Intuitions



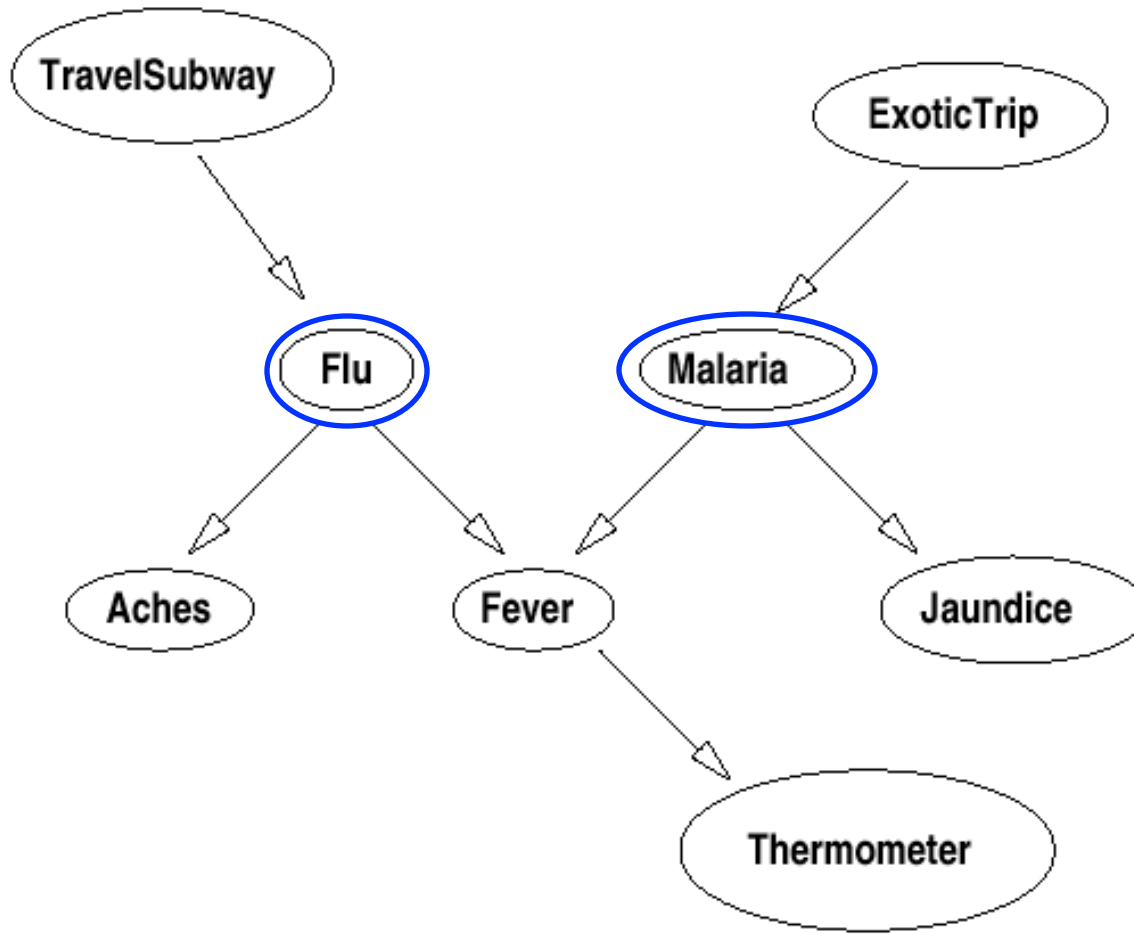
- Subway and Thermometer are **dependent**; but are **independent given Flu** (since Flu blocks the only path)

D-Separation: Intuitions



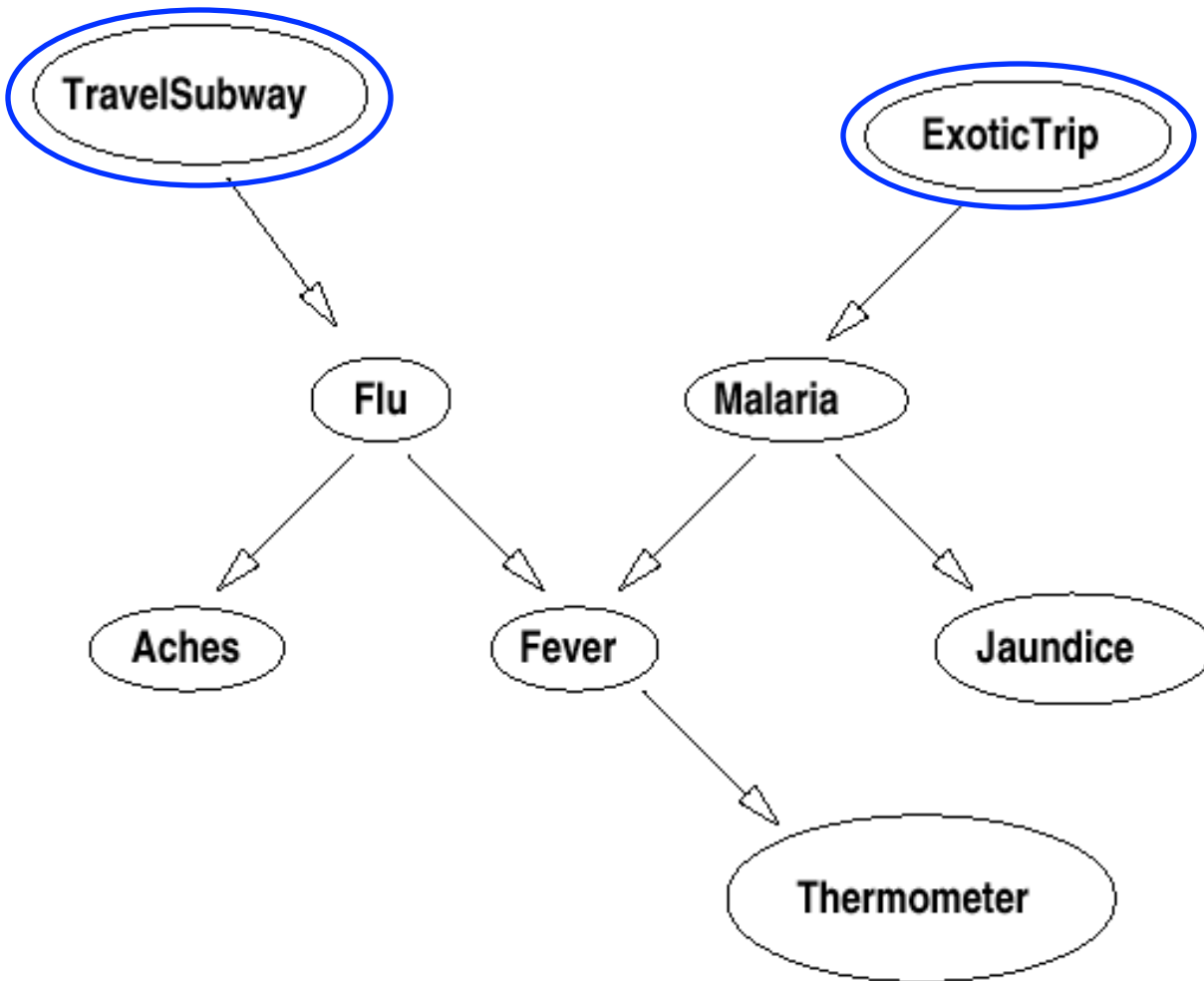
- Aches and Fever are **dependent**; but are **independent given Flu** (since Flu blocks the only path). Similarly for Aches and Therm (dependent, but indep. given Flu).

D-Separation: Intuitions



- Flu and Mal are **independent (given no evidence)**: Fever blocks the path, since it is *not in evidence*, nor is its descendant Therm.
- Flu and Mal are **dependent given Fever (or given Therm)**: nothing blocks path now. **What's the intuition?**

D-Separation: Intuitions

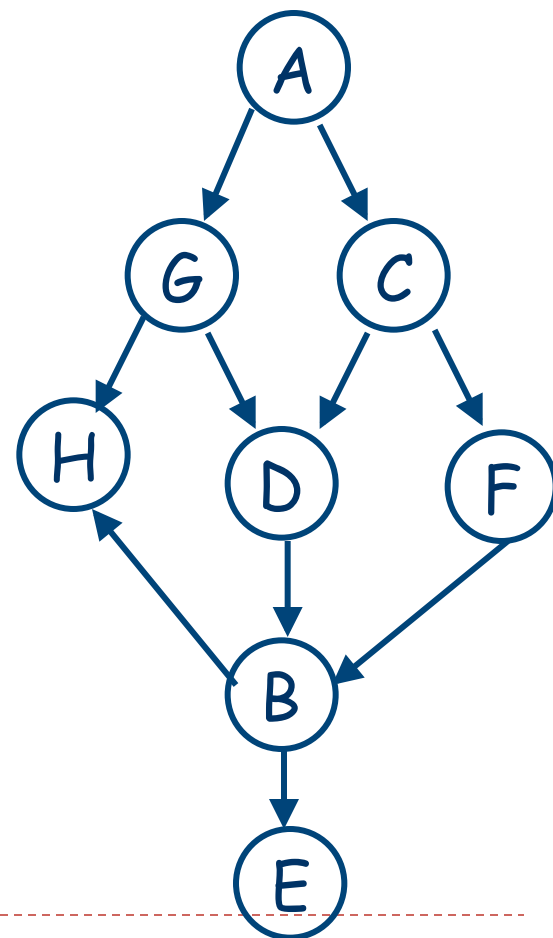


- Subway, ExoticTrip are independent;
- They are dependent given Therm;
- They are independent given Therm and Malaria. This for exactly the same reasons for Flu/Mal above.

D-Separation Example

► In the following network determine if A and E are independent given the evidence:

1. A and E given no evidence?
2. A and E given {C}?
3. A and E given {G,C}?
4. A and E given {G,C,H}?
5. A and E given {G,F}?
6. A and E given {F,D}?
7. A and E given {F,D,H}?
8. A and E given {B}?
9. A and E given {H,B}?
10. A and E given {G,C,D,H,D,F,B}?



D-Separation Example

► In the following network determine if A and E are independent given the evidence:

1. A and E given no evidence? **No**
2. A and E given {C}? **No**
3. A and E given {G,C}? **Yes**
4. A and E given {G,C,H}? **Yes**
5. A and E given {G,F}? **No**
6. A and E given {F,D}? **Yes**
7. A and E given {F,D,H}? **No**
8. A and E given {B}? **Yes**
9. A and E given {H,B}? **Yes**
10. A and E given {G,C,D,H,D,F,B}? **Yes**

