Paths in the State Space ordered by cost

Path	Cost
<\$>	0
<s, a=""></s,>	1.0
<s,b></s,b>	1.0
<s,a,c> <s,d></s,d></s,a,c>	1.5
<s,d></s,d>	2.0
•••	

Note cost is non-decreasing

 Uniform Cost Search expands paths in non-decreasing order of cost. So it only goes down this list.
 LEMMA 1

• It does not miss any paths on this list. LEMMA 2

Path	Cost
<s></s>	0
<s, a=""></s,>	1.0
<s,a,b> <s,a,b,c></s,a,b,c></s,a,b>	1.0
<s,a,b,c></s,a,b,c>	3.0
<s,a,b,d,e></s,a,b,d,e>	4.0
•••	

2

It does not miss any paths on this list. LEMMA 2

Path	Cost
<\$>	0
<s, a=""></s,>	1.0
<s,a,b></s,a,b>	1.0
<s,a,b,c></s,a,b,c>	3.0
<s,a,b,d,e></s,a,b,d,e>	4.0
•••	

If <S,a,b,d,e> is expanded next, <S,a,b,c> must already been expanded. If not it or <S,a,b,c> or <S,a,b> or <S,a> or <S> must be available for expansion (on OPEN) and would be expanded next as they all have lower cost than <S,a,b,d,e>

Thus working its way down such a list of paths in this order the first path achieving the goal that Uniform cost search finds will be the cheapest way of achieving the goal.