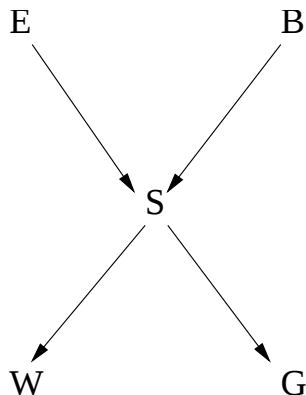


Tutorial Examples Uncertainty

March 24, 2015

Inference in Bayes Nets



$$P(E, S, B, W, G) = P(E)P(B)P(S|E, B)P(W|S)P(G|S)$$

Inference in Bayes Nets

$P(E)$	e	$-e$
	$1/10$	$9/10$

$P(B)$	b	$-b$
	$1/10$	$9/10$

$P(S E,B)$	s	$-s$
$e \wedge b$	$9/10$	$1/10$
$e \wedge -b$	$2/10$	$8/10$
$-e \wedge b$	$8/10$	$2/10$
$-e \wedge -b$	0	1

$P(W S)$	w	$-w$
s	$8/10$	$2/10$
$-s$	$2/10$	$8/10$

$P(G S)$	g	$-g$
s	$1/2$	$1/2$
$-s$	0	1

Inference in Bayes Nets

- ▶ Given the alarm went off (s) what is the probability that Mrs. Gibbons phones you (g)?

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$$P(g|s) = 1/2$$

Inference in Bayes Nets

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Inference in Bayes Nets

- ▶ Given that Mrs. Gibbons phones you (g) what is the probability the alarm went off (s)?
- 1. Bayes Rule says: $P(S|g) = P(g|S) * P(S)/P(g)$
- 2. $P(-s|g) = P(g|-s) * P(-s)/P(g) = 0 * P(-s)/P(g) = 0$.
- 3. Therefore $P(s|g) = 1$ ($P(s|g) + P(-s|g)$ must sum to 1).

$$P(s|g) = 1 \quad P(-s|g) = 0$$

Alternatively: $-s \rightarrow -g$, so $g \rightarrow s$, so $P(s|g) = 1$.

Inference in Bayes Nets

- ▶ Given that Mrs. Gibbons phones you (g) what is the probability the alarm went off (s)?

$$\begin{aligned}P(g) &= P(g \wedge s) + P(g \wedge \neg s) \\ &= P(g|s) \times P(s) + P(g|\neg s) \times P(\neg s)\end{aligned}$$

Inference in Bayes Nets

- Say that there was a burglary (b) and but no earthquake ($\neg e$), what is the expression specifying the posterior probability of Dr. Watson phoning you (w) given the evidence. (You do not need to calculate a numeric answer, just give the probability expression).

Inference in Bayes Nets

- Say that there was a burglary (b) and but no earthquake ($-e$), what is the expression specifying the posterior probability of Dr. Watson phoning you (w) given the evidence. (You do not need to calculate a numeric answer, just give the probability expression).

$$P(w|b, -e)$$

Inference in Bayes Nets

- ▶ What is $P(G|S)$? (i.e., the four probability values) $P(g|s)$, $P(-g|s)$, $P(g|-s)$, $P(-g|-s)$.

Inference in Bayes Nets

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$$\begin{aligned} P(g|s) &= 1/2 & P(-g|s) &= 1/2 \\ P(g|-s) &= 0 & P(-g|-s) &= 1 \end{aligned}$$

Inference in Bayes Nets

- ▶ What is $P(G|S \wedge W)$? (i.e., the 8 probability values $P(g|s \wedge w)$, $P(g|s \wedge \neg w)$, $\dots$, $P(\neg g| \neg s \wedge \neg w)$).

Inference in Bayes Nets

- ▶ What is $P(G|S \wedge W)$? (i.e., the 8 probability values $P(g|s \wedge w)$, $P(g|s \wedge \neg w)$, $\dots$, $P(\neg g|\neg s \wedge \neg w)$).

$$P(g|s, \neg w) = P(g|s, w) = P(g|s) = 1/2$$

$$P(\neg g|s, \neg w) = P(\neg g|s, w) = P(\neg g|s) = 1/2$$

$$P(g|\neg s, \neg w) = P(g|\neg s, w) = P(g|\neg s) = 0$$

$$P(\neg g|\neg s, \neg w) = P(\neg g|\neg s, w) = P(\neg g|\neg s) = 1$$

Inference in Bayes Nets

- ▶ What do these values tell us about the relationship between G , W and S ?

G is conditionally independent of W given S

Inference in Bayes Nets

- ▶ What do these values tell us about the relationship between G , W and S ?

Inference in Bayes Nets

- ▶ What is $P(G|W)$? (i.e., the four probability values $P(g|w)$, $P(-g|w)$, $P(g|-w)$, and $P(-g|-w)$).

Inference in Bayes Nets

- ▶ What is $P(G|W)$? (i.e., the four probability values $P(g|w)$, $P(-g|w)$, $P(g|-w)$, and $P(-g|-w)$).

Must do variable elimination.

Inference in Bayes Nets

- ▶ What is $P(G|W)$? (i.e., the four probability values $P(g|w)$, $P(-g|w)$, $P(g|-w)$, and $P(-g|-w)$).
- ▶ Query variable is G .
- ▶ First run of VE, evidence is $W = w$.
- ▶ Second run of VE, evidence is $W = -w$.
- ▶ Use same ordering for both runs of VE: E, B, S, G .
- ▶ With same ordering some factors can be reused between the two runs of VE.

Inference in Bayes Nets

- ▶ What is $P(G|W)$? (i.e., the four probability values $P(g|w)$, $P(-g|w)$, $P(g|-w)$, and $P(-g|-w)$).
 1. E : $P(E)$, $P(S|E, B)$
 2. B : $P(B)$,
 3. S : $P(w|S)$, $P(S|G)$
 4. G :

Inference in Bayes Nets

- ▶ What is $P(G|W)$? (i.e., the four probability values $P(g|w)$, $P(-g|w)$, $P(g|-w)$, and $P(-g|-w)$).

1. E : $P(E)$, $P(S|E, B)$

2. B : $P(B)$,

3. S : $P(w|S)$, $P(S|G)$

4. G :

$$\begin{aligned} F_1(S, B) &= \sum_E P(E) \times P(S|E, B) \\ &= P(e) \times P(S|e, B) + P(-e) \times P(S|-e, B) \end{aligned}$$

$$\begin{aligned} F_1(-s, -b) &= P(e)P(-s, e, -b) + P(-e)P(-s, -e, -b) \\ &= 0.1 \times 0.8 + 0.9 \times 1 = 0.98 \end{aligned}$$

$$\begin{aligned} F_1(-s, b) &= P(e)P(-s, e, b) + P(-e)P(-s, -e, b) \\ &= 0.1 \times 0.1 + 0.9 \times 0.2 = 0.19 \end{aligned}$$

$$\begin{aligned} F_1(s, -b) &= P(e)P(s, e, -b) + P(-e)P(s, -e, -b) \\ &= 0.1 \times 0.2 + 0.9 \times 0 = 0.02 \end{aligned}$$

$$\begin{aligned} F_1(s, b) &= P(e)P(s, e, b) + P(-e)P(s, -e, b) \\ &= 0.1 \times 0.9 + 0.9 \times 0.8 = 0.81 \end{aligned}$$

Inference in Bayes Nets

1. E : $P(E), P(S|E, B)$
2. B : $P(B), F_1(S, B)$
3. S : $P(w|S), P(S|G)$
4. G :

$$\begin{aligned} F_2(S) &= \sum_B P(B) \times F_1(S, B) \\ &= P(b)F_1(S, b) + P(-b)F_1(S, -b) \end{aligned}$$

$$\begin{aligned} F_2(-s) &= P(b)F_1(-s, b) + P(-b)F_1(-s, -b) \\ &= 0.1 \times 0.19 + 0.9 \times 0.98 = \mathbf{0.901} \end{aligned}$$

$$\begin{aligned} F_2(s) &= P(b)F_1(s, b) + P(-b)F_1(s, -b) \\ &= 0.1 \times 0.81 + 0.9 \times 0.02 = \mathbf{0.099} \end{aligned}$$

Inference in Bayes Nets

1. E : $P(E), P(S|E, B)$
2. B : $P(B), F_1(S, B)$
3. S : $P(w|S), P(S|G), F_2(S)$
4. G :

$$\begin{aligned} F_3(G) &= \sum_S P(w|S) \times P(S|G) \times F_2(S) \\ &= P(w|s)P(s|G)F_2(s) + P(w|-s)P(-s|G)F_2(-s) \end{aligned}$$

$$\begin{aligned} F_3(-g) &= P(w|s)P(s|-g)F_2(s) + P(w|-s)P(-s|-g)F_2(-s) \\ &= 0.8 \times 0.5 \times 0.099 + 0.2 \times 1 \times 0.901 = 0.2198 \end{aligned}$$

$$\begin{aligned} F_3(g) &= P(w|s)P(s|g)F_2(s) + P(w|-s)P(-s|g)F_2(-s) \\ &= 0.8 \times 0.5 \times 0.099 + 0.2 \times 0 \times 0.901 = 0.0396 \end{aligned}$$

Inference in Bayes Nets

1. E : $P(E), P(S|E, B)$
2. B : $P(B), F_1(S, B)$
3. S : $P(w|S), P(S|G), F_2(S)$
4. G : $F_3(G)$

Normalize $F_3(G)$:

$$P(-g|w) = \frac{0.2198}{0.2198+0.0396} = 0.8473$$

$$P(g|w) = \frac{0.0396}{0.2198+0.0396} = 0.1527$$

Inference in Bayes Nets

► Now $P(G | -w)$?

1. E : $P(E), P(S|E, B)$
2. B : $P(B),$
3. S : $P(-w|S), P(S|G)$
4. G :

Already computed as $F_1(S, B)$

Inference in Bayes Nets

1. E : $P(E), P(S|E, B)$
2. B : $P(B), F_1(S, B)$
3. S : $P(-w|S), P(S|G)$
4. G :

Already computed as $F_2(S)$

Inference in Bayes Nets

1. E : $P(E), P(S|E, B)$
2. B : $P(B), F_1(S, B)$
3. S : $P(-w|S), P(S|G), F_2(S)$
4. G :

$$\begin{aligned} F_3(G) &= \sum_S P(-w|S) \times P(S|G) \times F_2(S) \\ &= P(-w|s)P(s|G)F_2(s) + P(-w|-s)P(-s|G)F_2(-s) \end{aligned}$$

$$\begin{aligned} F_3(-g) &= P(-w|s)P(s|-g)F_2(s) + P(-w|-s)P(-s|-g)F_2(-s) \\ &= 0.2 \times 0.5 \times 0.099 + 0.8 \times 1 \times 0.901 = \mathbf{0.7307} \end{aligned}$$

$$\begin{aligned} F_3(g) &= P(-w|s)P(s|g)F_2(s) + P(-w|-s)P(-s|g)F_2(-s) \\ &= 0.2 \times 0.5 \times 0.099 + 0.8 \times 0 \times 0.901 = \mathbf{0.0099} \end{aligned}$$

Inference in Bayes Nets

1. E : $P(E), P(S|E, B)$
2. B : $P(B), F_1(S, B)$
3. S : $P(-w|S), P(S|G), F_2(S)$
4. G : $F_3(G)$

Normalize $F_3(G)$:

$$P(-g|w) = \frac{0.7307}{0.7307+0.0099} = 0.9866$$

$$P(g|w) = \frac{0.0099}{0.2198+0.00099} = 0.0134$$

Inference in Bayes Nets

- ▶ What do these values tell us about the relationship between G and W , and why does this relationship differ when we know S ?

Inference in Bayes Nets

- ▶ What do these values tell us about the relationship between G and W , and why does this relationship differ when we know S ?

G and W are not independent of each other. But when S is known they become independent.