Tutorial Examples Logic

March 13, 2015
1. Nobody likes taxes.
2. Some people like anchovies.
3. Emma is a Doberman pincher and a good dog.
English To Logic

1. Nobody likes taxes.
   - $\neg \exists X. \text{likesTaxes}(X)$
   - $\neg \exists X, Y. \text{person}(X) \land \text{tax}(Y) \land \text{likes}(X, Y)$
   - $\forall X. \text{person}(X) \rightarrow \neg \exists Y. \text{tax}(Y) \land \text{likes}(X, Y)$

2. Some people like anchovies.

3. Emma is a Doberman pincher and a good dog.
1. Nobody likes taxes.
   ▶ $\neg \exists X. \text{likesTaxes}(X)$
   Works if every object in the domain is a person.
   ▶ $\neg \exists X, Y. \text{person}(X) \land \text{tax}(Y) \land \text{likes}(X, Y)$
   Works if we want to talk about different kinds of taxes or different things that are liked.
   ▶ $\forall X. \text{person}(X) \rightarrow (\neg \exists Y. \text{tax}(Y) \land \text{likes}(X, Y))$
   Equivalent. $\neg \exists X, Y. \text{person}(X) \land \text{tax}(Y) \land \text{likes}(X, Y) \equiv$
   $\forall X, Y. \neg \text{person}(X) \lor \neg \text{tax}(Y) \lor \neg \text{likes}(X, Y) \equiv$
   $\forall X. \text{person}(X) \rightarrow (\forall Y. \neg \text{tax}(Y) \lor \neg \text{likes}(X, Y)) \equiv$
   $\forall X. \text{person}(X) \rightarrow (\neg \exists Y. \text{tax}(Y) \land \text{likes}(X, Y)) \equiv$

2. Some people like anchovies.

3. Emma is a Doberman pincher and a good dog.
1. Nobody likes taxes.
2. Some people like anchovies.
   ▶ \( \exists X. \text{person}(X) \land \text{likes}(X, \text{anchovies}) \).
   ▶ \( \exists X, Y. \text{person}(X) \land \text{anchovy}(Y) \land \text{likes}(X, Y) \).
3. Emma is a Doberman pincher and a good dog.
1. Nobody likes taxes.
2. Some people like anchovies.
3. Emma is a Doberman pincher and a good dog.
   ▶ $\text{doberman}(\text{Emma}) \land \text{good}(\text{Emma})$. 
Consider a first-order language $\mathcal{L}$ containing the following basic symbols:

- Constants, $A$, $B$, $C$, $D$.
- The binary predicate $R$.
- The unary predicates $P$ and $Q$. 
Let $\mathcal{M}$ be a model for $\mathcal{L}$, with domain $D = \{a, b, c, d\}$, and interpretation function $\sigma$:

1. $A^\sigma = a$, $B^\sigma = b$, $C^\sigma = c$, $D^\sigma = d$.
2. $R^\sigma = \{(b, a), (c, d)\}$.
3. $P^\sigma = \{b, c\}$.
4. $Q^\sigma = \{a, d\}$.

Which of the following formulas are satisfied by $\mathcal{M}$.

1. $R(C, B) \lor R(B, A)$
Let $\mathcal{M}$ be a model for $\mathcal{L}$, with domain $D = \{a, b, c, d\}$, and interpretation function $\sigma$:

1. $A^\sigma = a$, $B^\sigma = b$, $C^\sigma = c$, $D^\sigma = d$.
2. $R^\sigma = \{(b, a), (c, d)\}$.
3. $P^\sigma = \{b, c\}$.
4. $Q^\sigma = \{a, d\}$.

Which of the following formulas are satisfied by $\mathcal{M}$.

1. $\forall x. P(x) \land \neg Q(x)$. 
Let $\mathcal{M}$ be a model for $\mathcal{L}$, with domain $D = \{a, b, c, d\}$, and interpretation function $\sigma$:

1. $A^\sigma = a$, $B^\sigma = b$, $C^\sigma = c$, $D^\sigma = d$.
2. $R^\sigma = \{(b, a), (c, d)\}$.
3. $P^\sigma = \{b, c\}$.
4. $Q^\sigma = \{a, d\}$.

Which of the following formulas are satisfied by $\mathcal{M}$.

1. $\forall x. P(x) \rightarrow \neg \exists y. R(y, x)$. 
Let $\mathcal{M}$ be a model for $\mathcal{L}$, with domain $D = \{a, b, c, d\}$, and interpretation function $\sigma$:

1. $A^\sigma = a$, $B^\sigma = b$, $C^\sigma = c$, $D^\sigma = d$.
2. $R^\sigma = \{(b, a), (c, d)\}$.
3. $P^\sigma = \{b, c\}$.
4. $Q^\sigma = \{a, d\}$.

Which of the following formulas are satisfied by $\mathcal{M}$.

1. $\forall x. Q(x) \rightarrow \neg \exists y. R(y, x)$. 

Models

Let $\mathcal{M}$ be a model for $\mathcal{L}$, with domain $D = \{a, b, c, d\}$, and interpretation function $\sigma$:

1. $A^\sigma = a, \; B^\sigma = b, \; C^\sigma = c, \; D^\sigma = d$.
2. $R^\sigma = \{(b, a), (c, d)\}$.
3. $P^\sigma = \{b, c\}$.
4. $Q^\sigma = \{a, d\}$.

Which of the following formulas are satisfied by $\mathcal{M}$.

1. $\forall x. Q(x) \rightarrow \exists y. R(y, x)$. 

Let $\mathcal{M}$ be a model for $\mathcal{L}$, with domain $D = \{a, b, c, d\}$, and interpretation function $\sigma$:

1. $A^\sigma = a$, $B^\sigma = b$, $C^\sigma = c$, $D^\sigma = d$.
2. $R^\sigma = \{(b, a), (c, d)\}$.
3. $P^\sigma = \{b, c\}$.
4. $Q^\sigma = \{a, d\}$.

Which of the following formulas are satisfied by $\mathcal{M}$.

1. $\exists x, y. (P(x) \land Q(y)) \rightarrow R(y, x)$. 
Consider the following sentences:

1. Marcus was a man.
2. Marcus was a Roman.
3. All men are people.
4. Caesar was a ruler.
5. All Romans were either loyal to Caesar or hated him (or both).
6. Everyone is loyal to someone.
7. People only try to assassinate rulers they are not loyal to.
8. Marcus tried to assassinate Caesar.
Covert to First Order Logic

1. Marcus was a man.
   
   \[ man(marcus) \]

2. Marcus was a Roman.
   
   \[ roman(marcus) \]

3. All men are people.
   
   \[ \forall X. man(X) \rightarrow person(X) \]

4. Caesar was a ruler.
   
   \[ ruler(caesar) \]

5. All Romans were either loyal to Caesar or hated him (or both).
   
   \[ \forall X. roman(X) \rightarrow loyal_to(X, caesar) \lor hates(X, caesar) \]

6. Everyone is loyal to someone.
   
   \[ \forall X. \exists Y. person(Y) \land loyal_to(X, Y) \]

7. People only try to assassinate rulers they are not loyal to.
   
   \[ \forall X, Y. tried_to_kill(X, Y) \rightarrow \neg loyal_to(X, Y) \]

8. Marcus tried to assassinate Caesar.
   
   \[ tried_to_kill(marcus, caesar) \]
Covert to Clauses

1. $\text{man}(\text{marcus})$
2. $\text{roman}(\text{marcus})$
3. $\forall X. (\text{man}(X) \rightarrow \text{person}(X))$
   \[ (\neg \text{man}(X) \lor \text{person}(X)) \]
4. $\text{ruler}(\text{caesar})$
5. $\forall X. (\text{roman}(X) \rightarrow (\text{loyal}_\text{to}(X, \text{caesar}) \lor \text{hates}(X, \text{caesar})))$
   \[ (\neg \text{roman}(X) \lor \text{loyal}_\text{to}(X, \text{caesar}) \lor \text{hates}(X, \text{caesar})) \]
6. $\forall X. \exists Y. (\text{person}(Y) \land \text{loyal}_\text{to}(X, Y))$
   \[ \text{person}(f(X)) \]
   \[ \text{loyal}_\text{to}(X, f(X)) \]
7. $\forall X, Y. (\text{tried}_\text{to}_\text{kill}(X, Y) \rightarrow \neg \text{loyal}_\text{to}(X, Y))$
   \[ (\neg \text{tried}_\text{to}_\text{kill}(X, Y) \lor \neg \text{loyal}_\text{to}(X, Y)) \]
8. Marcus tried to assassinate Caesar.
   $\text{tried}_\text{to}_\text{kill}(\text{marcus}, \text{caesar})$
1. “Who hated Caesar?”

2. First order logic:
\[ \exists Z. \text{hates}(Z, \text{caesar}) \]

3. Negate:
\[ \forall Z. \neg \text{hates}(Z, \text{caesar}) \]

4. Clausal form.
\[ \neg \text{hates}(Z, \text{caesar}) \lor \text{answer}(Z) \]
1. man(marcus)  
2. roman(marcus)  
3. (¬man(X) ∨ person(X))  
4. ruler(caesar)  
5. (¬roman(X) ∨ loyal_to(X, caesar) ∨ hates(X, caesar))  
6. person(f(X))  
7. loyal_to(X, f(X))  
8. (¬tried_to_kill(X, Y) ∨ ¬loyal_to(X, Y))  
9. tried_to_kill(marcus, caesar)  
10. ¬hates(Z, caesar) ∨ answer(Z)  
11. R[9, 8a]{X=marcus, Y=caesar} ¬loyal_to(marcus, caesar)
Resolution Proof

1. $man(marcus)$
2. $roman(marcus)$
3. $(\neg man(X) \lor person(X))$
4. $ruler(caesar)$
5. $(\neg roman(X) \lor loyal_to(X, caesar) \lor hates(X, caesar))$
6. $person(f(X))$
7. $loyal_to(X, f(X))$
8. $(\neg tried_to_kill(X, Y) \lor \neg loyal_to(X, Y))$
9. $tried_to_kill(marcus, caesar)$
10. $\neg hates(Z, caesar) \lor answer(Z)$
11. $R[9, 8a]\{X=marcus, Y=caesar\} \neg loyal_to(marcus, caesar)$
12. $R[11, 5c]\{X=marcus\} \neg roman(marcus) \lor hates(marcus, caesar)$
Resolution Proof

1. $\text{man}(\text{marcus})$
2. $\text{roman}(\text{marcus})$
3. $(\neg \text{man}(X) \lor \text{person}(X))$
4. $\text{ruler}(\text{caesar})$
5. $(\neg \text{roman}(X) \lor \text{loyal\_to}(X, \text{caesar}) \lor \text{hates}(X, \text{caesar}))$
6. $\text{person}(f(X))$
7. $\text{loyal\_to}(X, f(X))$
8. $(\neg \text{tried\_to\_kill}(X, Y) \lor \neg \text{loyal\_to}(X, Y))$
9. $\text{tried\_to\_kill}(\text{marcus}, \text{caesar})$
10. $(\neg \text{hates}(Z, \text{caesar}) \lor \text{answer}(Z))$
11. $R[9, 8a]\{X=\text{marcus}, Y=\text{caesar}\} \neg \text{loyal\_to}(\text{marcus}, \text{caesar})$
12. $R[11, 5c]\{X=\text{marcus}\} \neg \text{roman}(\text{marcus}) \lor \text{hates}(\text{marcus}, \text{caesar})$
13. $R[12a, 2]\{} \text{hates}(\text{marcus}, \text{caesar})$
Resolution Proof

1. $\text{man}(\text{marcus})$
2. $\text{roman}(\text{marcus})$
3. $(\neg \text{man}(X) \lor \text{person}(X))$
4. $\text{ruler}(\text{caesar})$
5. $(\neg \text{roman}(X) \lor \text{loyal}_\text{to}(X, \text{caesar}) \lor \text{hates}(X, \text{caesar}))$
6. $\text{person}(f(X))$
7. $\text{loyal}_\text{to}(X, f(X))$
8. $(\neg \text{tried}_\text{to}_\text{kill}(X, Y) \lor \neg \text{loyal}_\text{to}(X, Y))$
9. $\text{tried}_\text{to}_\text{kill}(\text{marcus}, \text{caesar})$
10. $\neg \text{hates}(Z, \text{caesar}) \lor \text{answer}(Z)$
11. $R[9, 8a]\{X = \text{marcus}, Y = \text{caesar}\} \neg \text{loyal}_\text{to}(\text{marcus}, \text{caesar})$
12. $R[11, 5c]\{X = \text{marcus}\} \neg \text{roman}(\text{marcus}) \lor \text{hates}(\text{marcus}, \text{caesar})$
13. $R[12a, 2]\{} \text{hates}(\text{marcus}, \text{caesar})$
14. $R[13, 10]\{Z = \text{marcus}\} \text{answer}(\text{marcus})$