

# Tutorial Examples Logic

March 13, 2015

# English To Logic

1. Nobody likes taxes.
2. Some people like anchovies.
3. Emma is a Doberman pincher and a good dog.

# English To Logic

1. Nobody likes taxes.

- ▶  $\neg \exists X. \text{likesTaxes}(X)$
- ▶  $\neg \exists X, Y. \text{person}(X) \wedge \text{tax}(Y) \wedge \text{likes}(X, Y)$
- ▶  $\forall X. \text{person}(X) \rightarrow \neg \exists Y. \text{tax}(Y) \wedge \text{likes}(X, Y)$

2. Some people like anchovies.

3. Emma is a Doberman pincher and a good dog.

# English To Logic

## 1. Nobody likes taxes.

- ▶  $\neg \exists X. \text{likesTaxes}(X)$

Works if every object in the in the domain is a person.

- ▶  $\neg \exists X, Y. \text{person}(X) \wedge \text{tax}(Y) \wedge \text{likes}(X, Y)$

Works if we want talk about different kinds of taxes or different things that are liked.

- ▶  $\forall X. \text{person}(X) \rightarrow (\neg \exists Y. \text{tax}(Y) \wedge \text{likes}(X, Y))$

Equivalent.  $\neg \exists X, Y. \text{person}(X) \wedge \text{tax}(Y) \wedge \text{likes}(X, Y) \equiv$

$\forall X, Y. \neg \text{person}(X) \vee \neg \text{tax}(Y) \vee \neg \text{likes}(X, Y) \equiv$

$\forall X. \text{person}(X) \rightarrow (\forall Y. \neg \text{tax}(Y) \vee \neg \text{likes}(X, Y)) \equiv$

$\forall X. \text{person}(X) \rightarrow (\neg \exists Y. \text{tax}(Y) \wedge \text{likes}(X, Y)) \equiv$

## 2. Some people like anchovies.

## 3. Emma is a Doberman pincher and a good dog.

# English To Logic

1. Nobody likes taxes.
2. Some people like anchovies.
  - ▶  $\exists X. person(X) \wedge likes(X, anchovies).$
  - ▶  $\exists X, Y. person(X) \wedge anchovy(Y) \wedge likes(X, Y).$
3. Emma is a Doberman pincher and a good dog.

# English To Logic

1. Nobody likes taxes.
2. Some people like anchovies.
3. Emma is a Doberman pincher and a good dog.
  - ▶  $\text{doberman}(\text{Emma}) \wedge \text{good}(\text{Emma})$ .

# Models

Consider a first-order language  $\mathcal{L}$  containing the following basic symbols:

- ▶ Constants,  $A$ ,  $B$ ,  $C$ ,  $D$ .
- ▶ The binary predicate  $R$ .
- ▶ The unary predicates  $P$  and  $Q$ .

# Models

Let  $\mathcal{M}$  be a model for  $\mathcal{L}$ , with domain  $D = \{a, b, c, d\}$ , and interpretation function  $\sigma$ :

1.  $A^\sigma = a, B^\sigma = b, C^\sigma = c, D^\sigma = d$ .
2.  $R^\sigma = \{(b, a), (c, d)\}$ .
3.  $P^\sigma = \{b, c\}$ .
4.  $Q^\sigma = \{a, d\}$ .

Which of the following formulas are satisfied by  $\mathcal{M}$ .

1.  $R(C, B) \vee R(B, A)$



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2.  $R^\sigma = \{(b, a), (c, d)\}$ .
3.  $P^\sigma = \{b, c\}$ .
4.  $Q^\sigma = \{a, d\}$ .

Which of the following formulas are satisfied by  $\mathcal{M}$ .

1.  $\forall x. P(x) \wedge \neg Q(x)$ .

# Models

Let  $\mathcal{M}$  be a model for  $\mathcal{L}$ , with domain  $D = \{a, b, c, d\}$ , and interpretation function  $\sigma$ :

1.  $A^\sigma = a, B^\sigma = b, C^\sigma = c, D^\sigma = d$ .
2.  $R^\sigma = \{(b, a), (c, d)\}$ .
3.  $P^\sigma = \{b, c\}$ .
4.  $Q^\sigma = \{a, d\}$ .

Which of the following formulas are satisfied by  $\mathcal{M}$ .

1.  $\forall x. P(x) \rightarrow \neg \exists y. R(y, x)$ .

# Models

Let  $\mathcal{M}$  be a model for  $\mathcal{L}$ , with domain  $D = \{a, b, c, d\}$ , and interpretation function  $\sigma$ :

1.  $A^\sigma = a, B^\sigma = b, C^\sigma = c, D^\sigma = d$ .
2.  $R^\sigma = \{(b, a), (c, d)\}$ .
3.  $P^\sigma = \{b, c\}$ .
4.  $Q^\sigma = \{a, d\}$ .

Which of the following formulas are satisfied by  $\mathcal{M}$ .

1.  $\forall x. Q(x) \rightarrow \neg \exists y. R(y, x)$ .

# Models

Let  $\mathcal{M}$  be a model for  $\mathcal{L}$ , with domain  $D = \{a, b, c, d\}$ , and interpretation function  $\sigma$ :

1.  $A^\sigma = a, B^\sigma = b, C^\sigma = c, D^\sigma = d$ .
2.  $R^\sigma = \{(b, a), (c, d)\}$ .
3.  $P^\sigma = \{b, c\}$ .
4.  $Q^\sigma = \{a, d\}$ .

Which of the following formulas are satisfied by  $\mathcal{M}$ .

1.  $\forall x. Q(x) \rightarrow \exists y. R(y, x)$ .

# Models

Let  $\mathcal{M}$  be a model for  $\mathcal{L}$ , with domain  $D = \{a, b, c, d\}$ , and interpretation function  $\sigma$ :

1.  $A^\sigma = a, B^\sigma = b, C^\sigma = c, D^\sigma = d$ .
2.  $R^\sigma = \{(b, a), (c, d)\}$ .
3.  $P^\sigma = \{b, c\}$ .
4.  $Q^\sigma = \{a, d\}$ .

Which of the following formulas are satisfied by  $\mathcal{M}$ .

1.  $\exists x, y. (P(x) \wedge Q(y)) \rightarrow R(y, x)$ .

# Resolution Question

Consider the following sentences:

1. Marcus was a man.
2. Marcus was a Roman.
3. All men are people.
4. Caesar was a ruler.
5. All Romans were either loyal to Caesar or hated him (or both).
6. Everyone is loyal to someone.
7. People only try to assassinate rulers they are not loyal to.
8. Marcus tried to assassinate Caesar.

# Covert to First Order Logic

1. Marcus was a man.  
*man(marcus)*
2. Marcus was a Roman.  
*roman(marcus)*
3. All men are people.  
 $\forall X. man(X) \rightarrow person(X)$
4. Caesar was a ruler.  
*ruler(caesar)*
5. All Romans were either loyal to Caesar or hated him (or both).  
 $\forall X. roman(X) \rightarrow loyal\_to(X, caesar) \vee hates(X, caesar)$
6. Everyone is loyal to someone.  
 $\forall X. \exists Y. person(Y) \wedge loyal\_to(X, Y)$
7. People only try to assassinate rulers they are not loyal to.  
 $\forall X, Y. tried\_to\_kill(X, Y) \rightarrow \neg loyal\_to(X, Y)$
8. Marcus tried to assassinate Caesar.  
*tried\_to\_kill(marcus, caesar)*

# Covert to Clauses

1. *man(marcus)*
2. *roman(marcus)*
3.  $\forall X. \text{man}(X) \rightarrow \text{person}(X)$   
 *$(\neg \text{man}(X) \vee \text{person}(X))$*
4. *ruler(caesar)*
5.  $\forall X. \text{roman}(X) \rightarrow \text{loyal\_to}(X, \text{caesar}) \vee \text{hates}(X, \text{caesar})$   
 *$(\neg \text{roman}(X) \vee \text{loyal\_to}(X, \text{caesar}) \vee \text{hates}(X, \text{caesar}))$*
6.  $\forall X. \exists Y. \text{person}(Y) \wedge \text{loyal\_to}(X, Y)$   
 *$\text{person}(f(X))$*   
 *$\text{loyal\_to}(X, f(X))$*
7.  $\forall X, Y. \text{tried\_to\_kill}(X, Y) \rightarrow \neg \text{loyal\_to}(X, Y)$   
 *$(\neg \text{tried\_to\_kill}(X, Y) \vee \neg \text{loyal\_to}(X, Y))$*
8. Marcus tried to assassinate Caesar.  
 *$\text{tried\_to\_kill}(\text{marcus}, \text{caesar})$*



# Query

1. “Who hated Caesar?”

2. First order logic:

$\exists Z.hates(Z, caesar)$

3. Negate:

$\forall Z.\neg hates(Z, caesar)$

4. Clausal form.

$\neg hates(Z, caesar) \vee answer(Z)$

# Resolution Proof

1.  $man(marcus)$
2.  $roman(marcus)$
3.  $(\neg man(X) \vee person(X))$
4.  $ruler(caesar)$
5.  $(\neg roman(X) \vee loyal\_to(X, caesar) \vee hates(X, caesar))$
6.  $person(f(X))$
7.  $loyal\_to(X, f(X))$
8.  $(\neg \text{tried\_to\_kill}(X, Y) \vee \neg loyal\_to(X, Y))$
9.  $\text{tried\_to\_kill}(marcus, caesar)$
10.  $\neg hates(Z, caesar) \vee answer(Z)$
11.  $R[9, 8a]\{X=marcus, Y=caesar\} \neg loyal\_to(marcus, caesar)$

# Resolution Proof

1.  $man(marcus)$
2.  $roman(marcus)$
3.  $(\neg man(X) \vee person(X))$
4.  $ruler(caesar)$
5.  $(\neg roman(X) \vee \text{loyal\_to}(X, caesar) \vee hates(X, caesar))$
6.  $person(f(X))$
7.  $loyal\_to(X, f(X))$
8.  $(\neg tried\_to\_kill(X, Y) \vee \neg loyal\_to(X, Y))$
9.  $tried\_to\_kill(marcus, caesar)$
10.  $\neg hates(Z, caesar) \vee answer(Z)$
11.  $R[9, 8a]\{X=marcus, Y=caesar\} \neg \text{loyal\_to}(marcus, caesar)$
12.  $R[11, 5c]\{X=marcus\} \neg roman(marcus) \vee hates(marcus, caesar)$

# Resolution Proof

1.  $man(marcus)$
2.  $roman(marcus)$
3.  $(\neg man(X) \vee person(X))$
4.  $ruler(caesar)$
5.  $(\neg roman(X) \vee loyal\_to(X, caesar) \vee hates(X, caesar))$
6.  $person(f(X))$
7.  $loyal\_to(X, f(X))$
8.  $(\neg tried\_to\_kill(X, Y) \vee \neg loyal\_to(X, Y))$
9.  $tried\_to\_kill(marcus, caesar)$
10.  $\neg hates(Z, caesar) \vee answer(Z)$
11.  $R[9, 8a]\{X=marcus, Y=caesar\} \neg loyal\_to(marcus, caesar)$
12.  $R[11, 5c]\{X=marcus\} \neg roman(marcus) \vee hates(marcus, caesar)$
13.  $R[12a, 2]\{\} hates(marcus, caesar)$

# Resolution Proof

1.  $man(marcus)$
2.  $roman(marcus)$
3.  $(\neg man(X) \vee person(X))$
4.  $ruler(caesar)$
5.  $(\neg roman(X) \vee loyal\_to(X, caesar) \vee hates(X, caesar))$
6.  $person(f(X))$
7.  $loyal\_to(X, f(X))$
8.  $(\neg tried\_to\_kill(X, Y) \vee \neg loyal\_to(X, Y))$
9.  $tried\_to\_kill(marcus, caesar)$
10.  $\neg hates(Z, caesar) \vee answer(Z)$
11.  $R[9, 8a]\{X=marcus, Y=caesar\} \neg loyal\_to(marcus, caesar)$
12.  $R[11, 5c]\{X=marcus\} \neg roman(marcus) \vee hates(marcus, caesar)$
13.  $R[12a, 2]\{\} hates(marcus, caesar)$
14.  $R[13, 10]\{Z = marcus\} answer(marcus)$