Week 12: Normal Forms

Database Design
Database Redundancies and Anomalies
Functional Dependencies
Entailment, Closure and Equivalence
Lossless Decompositions
The Third Normal Form (3NF)
The Boyce-Codd Normal Form (BCNF)
Normal Forms and Database Design

Logical Database Design

- We have seen how to design a relational schema by first designing an ER schema and then transforming it into a relational one.
- Now we focus on how to transform the generated relational schema into a "better" one.
- Goodness of relational schemas is defined in terms of the notion of *normal form*.

Normal Forms and Normalization

- A *normal form* is a property of a database schema.
- When a database schema is un-normalized (that is, does not satisfy the normal form), it allows redundancies of various types which can lead to anomalies and inconsistencies.
- Normal forms can serve as basis for evaluating the quality of a database schema and constitutes a useful tool for database design.
- *Normalization* is a procedure that transforms an un-normalized schema into a normalized one.

Examples of Redundancy

Employee	Salary	Project	Budget	Function
Brown	20	Mars	2	technician
Green	35	Jupiter	15	designer
Green	35	Venus	15	designer
Hoskins	55	Venus	15	manager
Hoskins	55	Jupiter	15	consultant
Hoskins	55	Mars	2	consultant
Moore	48	Mars	2	manager
Moore	48	Venus	15	designer
Kemp	48	Venus	15	designer
Kemp	48	Jupiter	15	manager

Anomalies

The value of the salary of an employee is repeated in every tuple where the employee is mentioned, leading to a *redundancy*. Redundancies lead to anomalies:

- If salary of an employee changes, we have to modify the value in all corresponding tuples (*update anomaly*.)
- If an employee ceases to work in projects, but stays with company, all corresponding tuples are deleted, leading to loss of information (*deletion anomaly*.)
- A new employee cannot be inserted in the relation until the employee is assigned to a project (*insertion anomaly*.)

What's Wrong???

- We are using a single relation to represent data of very different types.
- In particular, we are using a single relation to store the following types of entities, relationships and attributes:
  - ✓Employees and their salaries;
  - ✓Projects and their budgets;
  - ✓Participation of employees in projects, along with their functions.
- To set the problem on a formal footing, we introduce the notion of *functional dependency (FD)*.

Functional Dependencies (FDs) in the Example

- Each employee has a unique salary. We represent this dependency as

  Employee → **Salary**

  and say "**Salary** *functionally depends* on **Employee**".
- This means that everywhere we have the same **Employee** attribute value, we also get the same **Salary** attribute value.
- Likewise,

  Project → **Budget**

  i.e., each project has a unique budget.

Functional Dependencies

- Given schema **R(X)** and non-empty subsets **Y** and **Z** of the attributes **X**, we say that there is a *functional dependency* between **Y** and **Z** (**Y**→**Z**), iff for every relation instance **r** of **R(X)** and every pair of tuples t_1, t_2 of **r**, if $t_1.Y = t_2.Y$, then $t_1.Z = t_2.Z$.
- A functional dependency is a statement about *all allowable relations* for a given schema.
- Functional dependencies have to be identified by understanding the semantics of the application.
- Given a particular relation r_0 of **R(X),** we can tell if a dependency holds or not; but just because it holds for r_0, doesn't mean that it also holds for **R(X)!**

Looking for FDs

Employee	Salary	Project	Budget	Function
Brown	20	Mars	2	technician
Green	35	Jupiter	15	designer
Green	35	Venus	15	designer
Hoskins	55	Venus	15	manager
Hoskins	55	Jupiter	15	consultant
Hoskins	55	Mars	2	consultant
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Kemp	48	Jupiter	15	manager

Non-Trivial Dependencies

- A functional dependency **Y→Z** is *non-trivial* if no attribute in **Z** appears among attributes of **Y**, e.g.,
  - ✓**Employee** → **Salary** is non-trivial;
  - ✓**Employee,Project** → **Project** is trivial.
- Anomalies arise precisely for the attributes which are involved in (non-trivial) functional dependencies:
  - ✓**Employee** → **Salary**;
  - ✓**Project** → **Budget**.
- Moreover, note that our example includes another functional dependency:
  - ✓**Employee,Project** → **Function**.

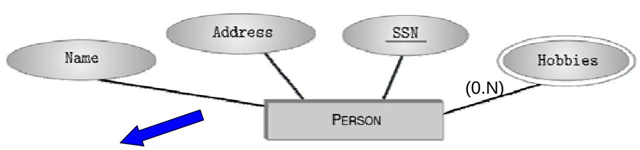
Dependencies Cause Anomalies, ...Sometimes!

- The first two dependencies cause undesirable redundancies and anomalies.
- The third dependency, however, does not cause redundancies because {**Employee,Project**} constitutes a key of the relation (...and a relation cannot contain two tuples with the same values for the key attributes.)

Dependencies on keys are OK,
other dependencies are not!

Another Example

ER Model



SI#	Name	Address	Hobbies
1111	Joe	123 Main	{biking, hiking}

This is NOT a relation

Relational Model

SI#	Name	Address	Hobby
1111	Joe	123 Main	biking
1111	Joe	123 Main	hiking
....			

Redundancy

How Do We Eliminate Redundancy?

- Decomposition: Use two relations to store Person information:
  - ✓Person1 (<u>*SI#*</u>, *Name*, *Address*)
  - ✓Hobbies (<u>*SI#*</u>, <u>*Hobby*</u>)
- The decomposition is more general: people with hobbies can now be described independently of their name and address.
- No update anomalies:
  - ✓Name and address stored once;
  - ✓A hobby can be separately supplied or deleted;
  - ✓We can represent persons with no hobbies.

Superkey Constraints

- A *superkey constraint* is a special functional dependency: Let *K* be a set of attributes of R, and *U* the set of *all* attributes of R. Then *K* is a *superkey* iff the functional dependency $K \to U$ is satisfied in R.
  - ✓*E.g.*, *SI#* → *SI#,Name,Address* (for a Person relation)
- A *key* is a minimal superkey, I.e., for each $X \subset K$, *X* is not a superkey
  - ✓*SI#*, *Hobby* → *SI#*, *Name*, *Address*, *Hobby* but
  - ✓*SI#* $\not\to$ *SI#*, *Name*, *Address*, *Hobby*
  - ✓*Hobby* $\not\to$ *SI#*, *Name*, *Address*, *Hobby*
- A *key attribute* is an attribute that is part of a key.

More Examples

- *Address* → *PostalCode*
  - ✓DCS's postal code is M5S 3H5
- *Author*, *Title*, *Edition* → *PublicationDate*
  - ✓Ramakrishnan, et al., Database Management Systems, 3rd publication date is 2003
- *CourseID* → *ExamDate*, *ExamTime*
  - ✓CSC343's exam date is December 18, starting at 7pm

When are FDs "Equivalent"?

- Sometimes functional dependencies (FDs) seem to be saying the same thing, e.g., *Addr* → *PostalCode,Str#*
  vs *Addr* → *PostalCode*, *Addr* → *Str#*
- Another example
  Addr → *PostalCode*, *PostalCode* → *Province*
  vs *Addr* → *PostalCode*, *PostalCode* → *Province*
  Addr → *Province*
- When are two sets of FDs equivalent? How do we "infer" new FDs from given FDs?

Entailment, Closure, Equivalence

- If *F* is a set of FDs on schema R and *f* is another FD on R, then *F* ***entails*** *f* (written F |= f) if every instance r of R that satisfies every FD in *F* also satisfies *f.*
  - ✓Example: $F = \{A \rightarrow B, B \rightarrow C\}$ and *f* is $A \rightarrow C$
    - ✓If *Phone#* $\rightarrow$ *Address* and *Address* $\rightarrow$ *ZipCode*, then *Phone#* $\rightarrow$ *ZipCode*
- The ***closure*** of *F*, denoted F^+, is the set of all FDs entailed by *F.*
- *F* and *G* are ***equivalent*** if *F* entails *G* and *G* entails *F.*

How Do We Compute Entailment?

- Satisfaction, entailment, and equivalence are *semantic* concepts – defined in terms of the "meaning" of relations in the "real world."
- How to check if *F* entails *f*, *F* and *G* are equivalent?
  - ✓Apply the respective definitions for all possible relation instances for a schema R …☹…
  - ✓Find algorithmic, *syntactic* ways to compute these notions.
- Note: The syntactic solution must be "correct" with respect to the semantic definitions.
- Correctness has two aspects: ***soundness*** and ***completeness*** – see later.

Armstrong's Axioms for FDs

- This is the syntactic way of computing/testing semantic properties of FDs
  - ✓*Reflexivity*: $Y \subseteq X$ |- $X \rightarrow Y$ (trivial FD)
    - *e.g.,* |- *Name, Address* $\rightarrow$ *Name*
  - ✓*Augmentation*: $X \rightarrow Y$ |- $XZ \rightarrow YZ$
    - e.g., *Address* $\rightarrow$ *ZipCode* |- *Address,Name* $\rightarrow$ *ZipCode, Name*
  - ✓*Transitivity*: $X \rightarrow Y$, $Y \rightarrow Z$ |- $X \rightarrow Z$
    - e.g., *Phone#* $\rightarrow$ *Address*, *Address* $\rightarrow$ *ZipCode* |- *Phone#* $\rightarrow$ *ZipCode*

Soundness

- Theorem: F |- f implies F |= f
- In words: If FD *f:* $X \rightarrow Y$ can be derived from a set of FDs *F* using the axioms, then *f* holds in every relation that satisfies every FD in *F.*
- Example: Given $X \rightarrow Y$ and $X \rightarrow Z$ then

$$X \rightarrow XY \quad \textit{Augmentation by X}$$
$$YX \rightarrow YZ \quad \textit{Augmentation by Y}$$
$$X \rightarrow YZ \quad \textit{Transitivity}$$

- Thus, $X \rightarrow YZ$ is satisfied in every relation where both $X \rightarrow Y$ and $X \rightarrow Z$ are satisfied. We have derived the ***union rule*** for FDs.

Completeness

- Theorem: F |= f implies F |- f
- In words: If *F* entails *f*, then *f* can be derived from *F* using Armstrong's axioms.
- A consequence of completeness is the following (naïve) algorithm to determining if *F* entails *f*:

> **Algorithm**: Use the axioms in all possible ways to generate F^+ (the set of possible FD's is finite so this can be done) and see if *f* is in F^+

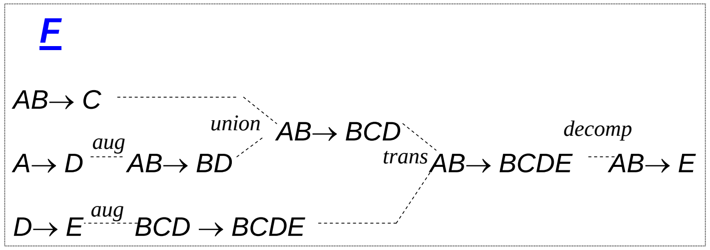
Correctness

- The notions of *soundness* and *completeness* link the syntax (Armstrong's axioms) with semantics, I.e., entailment defined in terms of relational instances.
- This is a precise way of saying that the algorithm for entailment based on the axioms is ``correct'' with respect to the definitions.

Decomposition Rule

- Another example of a derivation rule we can use in generating F^+:
- $X \rightarrow AB$, $AB \rightarrow A$ (refl), $X \rightarrow A$ (trans)
- So, whenever we have $X \rightarrow AB$, we can "decompose" this functional dependency to two functional dependencies $X \rightarrow A$, $X \rightarrow B$

Generating F^+



Thus, $AB \rightarrow BD$, $AB \rightarrow BCD$, $AB \rightarrow BCDE$, and $AB \rightarrow E$ are all elements of F^+.

Attribute Closure

- Calculating *attribute closure* leads to a more efficient way of checking entailment.
- The *attribute closure* of a set of attributes X with respect to a set of FDs F, denoted X^+_F, is the set of all attributes A such that $X \rightarrow A$
  - ✓ X^+_F is not necessarily same as X^+_G if $F \neq G$
- Attribute closure and entailment:

Algorithm: Given a set of FDs, F, then $X \rightarrow Y$ if and only if $Y \subseteq X^+_F$

Computing Attribute Closure: An Example

F: $AB \rightarrow C$
$A \rightarrow D$
$D \rightarrow E$
$AC \rightarrow B$

X	X_F^+
A	$\{A, D, E\}$
AB	$\{A, B, C, D, E\}$
AC	$\{A, C, B, D, E\}$
B	$\{B\}$
D	$\{D, E\}$

Is $AB \rightarrow E$ entailed by F? *Yes*
Is $D \rightarrow C$ entailed by F? *No*

Result: X_F^+ allows us to determine all FDs of the form $X \rightarrow Y$ entailed by F

Computing the Attribute Closure X^+_F

closure := X; // since $X \subseteq X^+_F$
repeat
  old := *closure*;
  if there is an FD $Z \rightarrow V$ in F such that
    $Z \subseteq$ *closure* **and** $V \not\subseteq$ *closure*
  then *closure* := *closure* $\cup V$
until *old* = *closure*

– If $T \subseteq$ *closure* then $X \rightarrow T$ is entailed by F

Normal Forms

- Each normal form is a set of conditions on a schema that together guarantee certain properties (relating to redundancy and update anomalies).
- First normal form (1NF) is the same as the definition of relational model (relations = sets of tuples; each tuple = sequence of atomic values).
- Second normal form (2NF) – has no practical or theoretical value – won't discuss.
- The two most used are *third normal form* (3NF) and *Boyce-Codd normal form* (BCNF).
- We will discuss in detail the 3NF.

The Third Normal Form

- A relation R(X) is in *third normal form* (*3NF*) if, for each (non-trivial) functional dependency Y → Z, at least one of the following is true:
  - ✓ Y contains a key K of R(X);
  - ✓ Each attribute in Z is contained in at least one key of R(X).
- 3NF does not remove all redundancies.
- 3NF decompositions founded on the notion of *minimal cover.*

Decomposition into 3NF: Basic Idea

- Decomposition into 3NF can proceed as follows.
  - ✓ For each functional dependency of the form Y → Z, where Y contains a subset of a key K of R(X), create a projection on all the attributes Y, Z (2NF).
  - ✓ For each dependency of the form Y → Z, where Y, doesn't contain any key, and not all attributes of Z are key attributes, create a projection on all the attributes Y, Z (3NF).
- The new relations only include dependencies Y → Z, where Y contains a key K of R(X), or Z contains only key attributes.

Basic Idea

- R(ABCD), A --> D
- Projection:
  - ✓ R1(AD), A --> D
  - ✓ R2(ABC)

Normalization Through Decomposition

- A relation that is not in 3NF, can be replaced with one or more normalized relations using *normalization*.
- We can eliminate redundancies and anomalies for the example relation

  Emp(Employee,Salary,Project,Budget,Function)

if we replace it with the three relations obtained by projections on the sets of attributes corresponding to the three functional dependencies:

- ✓ **Employee → Salary;**
- ✓ **Project → Budget.**
- ✓ **Employee,Project → Function.**

…Start with…

Employee	Salary	Project	Budget	Function
Brown	20	Mars	2	technician
Green	35	Jupiter	15	designer
Green	35	Venus	15	designer
Hoskins	55	Venus	15	manager
Hoskins	55	Jupiter	15	consultant
Hoskins	55	Mars	2	consultant
Moore	48	Mars	2	manager
Moore	48	Venus	15	designer
Kemp	48	Venus	15	designer
Kemp	48	Jupiter	15	manager

Result of Normalization

Employee	Salary
Brown	20
Green	35
Hoskins	55
Moore	48
Kemp	48

Project	Budget
Mars	2
Jupiter	15
Venus	15

Employee	Project	Function
Brown	Mars	technician
Green	Jupiter	designer
Green	Venus	designer
Hoskins	Venus	manager
Hoskins	Jupiter	consultant
Hoskins	Mars	consultant
Moore	Mars	manager
Moore	Venus	designer
Kemp	Venus	designer
Kemp	Jupiter	manager

The keys of new relations are lefthand sides of functional dependencies; satisfaction of 3NF is therefore guaranteed for the new relations.

Another Example

Employee	Project	Branch
Brown	Mars	Chicago
Green	Jupiter	Birmingham
Green	Venus	Birmingham
Hoskins	Saturn	Birmingham
Hoskins	Venus	Birmingham

This relation satisfies the functional dependencies:

- ✓ **Employee → Branch**
- ✓ **Project → Branch**

A Possible Decomposition

Employee	Branch
Brown	Chicago
Green	Birmingham
Hoskins	Birmingham

Project	Branch
Mars	Chicago
Jupiter	Birmingham
Saturn	Birmingham
Venus	Birmingham

…but now we don't know each employee's projects!

The Join of the Projections

Employee	Project	Branch
Brown	Mars	Chicago
Green	Jupiter	Birmingham
Green	Venus	Birmingham
Hoskins	Saturn	Birmingham
Hoskins	Venus	Birmingham
Green	Saturn	Birmingham
Hoskins	Jupiter	Birmingham

The result of the join is different from the original relation.

We lost some information during the decomposition!

Lossless Decomposition

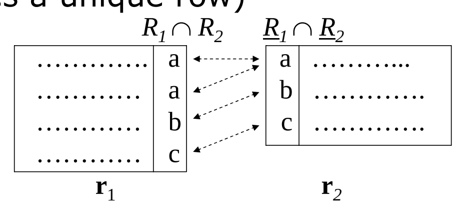
- The decomposition of a relation R(X) on X_1 and X_2 is *lossless* if for every instance r of R(X) the join of the projections of R on X_1 and X_2 is equal *to* r itself (that is, does not contain *spurious* tuples).
- Of course, it is clearly desirable to allow only lossless decompositions during normalization.

A Condition for Lossless Decomposition

- Let R(X) be a relation schema and let X_1 and X_2 be two subsets of X such that $X_1 \cup X_2 = X$. Also, let $X_0 = X_1 \cap X_2$.
- If R(X) satisfies the functional dependency $X_0 \rightarrow X_1$ or $X_0 \rightarrow X_2$, then the decomposition of R(X) on X_1 and X_2 is lossless.
- In other words, R(X) has a lossless decomposition on two relations if the set of attributes common to the relations is a *superkey* for at least one of the decomposed relations.

Intuition Behind the Test for Losslessness

- Suppose $R_1 \cap R_2 \rightarrow R_2$. Then a row of r_1 can combine with *exactly* one row of r_2 in the natural join (since in r_2 a particular set of values for the attributes in $R_1 \cap R_2$ defines a unique row)



A Lossless Decomposition

Employee	Branch
Brown	Chicago
Green	Birmingham
Hoskins	Birmingham

Employee	Project
Brown	Mars
Green	Jupiter
Green	Venus
Hoskins	Saturn
Hoskins	Venus

Notation

- Instead of saying that we have relation schema R(X) with functional dependencies *F*, we will say that we have schema

  $\mathcal{R} = (R, F)$

  , where *R* is a set of attributes and *F* is a set of functional dependencies.
- The 3NF normalization problem is then to generate a set of relation schemas $\mathcal{R}_1=(R_1,F_1)$, …, $\mathcal{R}_n=(R_n,F_n)$, such that $\mathcal{R}_i$ is in 3NF.

Another Example

- **Schema (*R*, *F*) where**

  ***R* = {*SI#, Name, Address, Hobby*}**

  ***F* = {*SI#* → *Name, Address*}**

  can be decomposed into

  R_1 = {*SI#, Name, Address*}

  F_1 = {*SI#* → *Name, Address*}

  and

  R_2 = {*SI#, Hobby*}

  F_2 = { }

since $R_1 \cap R_2$ = *SI#*, *SI#*→R_1 the decomposition is lossless.

Another Problem...

- Assume we wish to insert a new tuple that specifies that employee Armstrong works in the Birmingham branch and participates in project Mars.
- In the original relation, this update would be identified as illegal, because it would cause a violation of the **`Project`** → **`Branch`** dependency.
- For the decomposed relations, however, this is not possible because the two attributes **`Project`** and **`Branch`** have been moved to different relations.

Preserving Dependencies (Intuition)

- A decomposition *preserves dependencies* if each of the functional dependencies of the original relation schema involves attributes that appear together in one of the decomposed relation schemas.
- It is clearly desirable that a decomposition preserves dependencies because then it is possible to ensure that the decomposed schema satisfies the same constraints as the original schema.

Example

- **Schema (*R*, *F*) where**
  - $R = \{SI\#, Name, Address, Hobby\}$
  - $F = \{SI\# \rightarrow Name, Address\}$

can be decomposed into
  - $R_1 = \{SI\#, Name, Address\}$
  - $F_1 = \{SI\# \rightarrow Name, Address\}$

and
  - $R_2 = \{SI\#, Hobby\}$
  - $F_2 = \{\ \}$
- **Since $F = F_1 \cup F_2$ the decomposition is dependency preserving.**

Another Example

- Schema: $(ABC;\ F)$, $F = \{A \rightarrow B, B \rightarrow C, C \rightarrow B\}$
- Decomposition:
  - ✓ (AC, F_1), $F_1 = \{A \rightarrow C\}$
    [Note: $A \rightarrow C \notin F$, but in F^+]
  - ✓ (BC, F_2), $F_2 = \{B \rightarrow C, C \rightarrow B\}$
- $A \rightarrow B \notin (F_1 \cup F_2)$, but $A \rightarrow B \in (F_1 \cup F_2)^+$.
  - ✓ So $F^+ = (F_1 \cup F_2)^+$ and thus the decomposition is still dependency preserving

Dependency Preservation

- If *f* is a FD in *F*, but *f* is not in $F_1 \cup F_2$, there are two possibilities:
  - ✓ $f \in (F_1 \cup F_2)^+$
    - ✓ If the constraints in F_1 and F_2 are maintained, *f* will be maintained automatically.
  - ✓ $f \notin (F_1 \cup F_2)^+$
    - ✓ *f* can be checked only by first taking the join of r_1 and r_2. *...This is costly...*

Desirable Qualities for Decompositions

Decompositions should always satisfy the properties of lossless decomposition and dependency preservation:

- *Lossless decomposition* ensures that the information in the original relation can be accurately reconstructed based on the information represented in the decomposed relations.
- *Dependency preservation* ensures that the decomposed relations have the same capacity to represent the integrity constraints as the original relations and therefore to reveal illegal updates.

Minimal Cover

- A *minimal cover* for a set of dependencies F is a set of dependencies U such that:
  - ✓ U is equivalent to F (I.e., $F^+ = U^+$)
  - ✓ All FDs in U have the form $X \rightarrow A$ where A is a single attribute
  - ✓ It is not possible to make U smaller (while preserving equivalence) by
    - ✓ Deleting an FD
    - ✓ Deleting an attribute from an FD (its LHS)
- FDs and attributes that can be deleted in this way are called *redundant*.

Computing the Minimal Cover

Example: $F = \{ABH \rightarrow CK,\ A \rightarrow D,\ C \rightarrow E,$
$BGH \rightarrow L,\ L \rightarrow AD,\ E \rightarrow L,\ BH \rightarrow E\}$

- Step 1: Make RHS of each FD into a single attribute: Use decomposition rule for FDs.
  - ✓ Example: $L \rightarrow AD$ replaced by $L \rightarrow A$, $L \rightarrow D$; $ABH \rightarrow CK$ by $ABH \rightarrow C$, $ABH \rightarrow K$
- Step 2: Eliminate redundant attributes from LHS: If B is a single attribute and FD $XB \rightarrow A \in F$, $X \rightarrow A$ is entailed by F, then B is unnecessary.

  e.g., Can an attribute be deleted from $ABH \rightarrow C$? Compute AB^+_F, AH^+_F, BH^+_F; Since $C \in (BH)^+_F$, $BH \rightarrow C$ is entailed by F and A is redundant in $ABH \rightarrow C$.

Computing the Minimal Cover (cont'd)

- Step 3: Delete redundant FDs from F: If $F - \{f\}$ entails f, then f is redundant; if f is $X \rightarrow A$ then check if $A \in X^+_{F-\{f\}}$

  e.g., $BGH \rightarrow L$ is entailed by $E \rightarrow L$, $BH \rightarrow E$, so it is redundant
- Note: The order of steps 2, 3 can't be interchanged!! See textbook for a counterexample.

$F_1 = \{ABH \rightarrow C,\ ABH \rightarrow K,\ A \rightarrow D,\ C \rightarrow E,\ BGH \rightarrow L,\ L \rightarrow A,\ L \rightarrow D,\ E \rightarrow L,\ BH \rightarrow E\}$
$F_2 = \{BH \rightarrow C,\ BH \rightarrow K,\ A \rightarrow D,\ C \rightarrow E,\ BH \rightarrow L,\ L \rightarrow A,\ L \rightarrow D,\ E \rightarrow L,\ BH \rightarrow E\}$
$F_3 = \{BH \rightarrow C,\ BH \rightarrow K,\ A \rightarrow D,\ C \rightarrow E,\ L \rightarrow A,\ E \rightarrow L\}$

Synthesizing a 3NF Schema

Starting with a schema R = (R, F):

- Step 1: Compute minimal cover U of F. The decomposition is based on U, but since $U^+ = F^+$ the same functional dependencies will hold.
  - ✓A minimal cover for

    $F = \{ABH \to CK,\ A \to D,\ C \to E,\ BGH \to L,\ L \to AD,\ E \to L,\ BH \to E\}$

    is

    $U = \{BH \to C,\ BH \to K,\ A \to D,\ C \to E,\ L \to A,\ E \to L\}$

Synthesizing … Step 2

- Step 2: Partition U into sets $U_1, U_2, \ldots U_n$ such that the LHS of all elements of U_i are the same:
  - ✓$U_1 = \{BH \to C,\ BH \to K\}$, $U_2 = \{A \to D\}$, $U_3 = \{C \to E\}$, $U_4 = \{L \to A\}$, $U_5 = \{E \to L\}$

Synthesizing … Step 3

- Step 3: For each U_i form schema $R_i = (R_i, U_i)$, where R_i is the set of all attributes mentioned in U_i
  - ✓Each FD of U will be in some R_i. Hence the decomposition is *dependency preserving:*
  - ✓$R_1 = (BHCK;\ BH \to C,\ BH \to K)$,
  - ✓$R_2 = (AD;\ A \to D)$,
  - ✓$R_3 = (CE;\ C \to E)$,
  - ✓$R_4 = (AL;\ L \to A)$,
  - ✓$R_5 = (EL;\ E \to L)$

Synthesizing … Step 4

- Step 4: If no R_i is a superkey of R, add schema $R_0 = (R_0, \{\})$ where R_0 is a key of R.
  - ✓$R_0 = (BGH, \{\})$; R_0 might be needed when not all attributes are contained in $R_1 \cup R_2 \ldots \cup R_n$;
  - ✓A missing attribute A must be part of all keys (since it's not in any FD of U, deriving a key constraint from U involves the augmentation axiom);
  - ✓R_0 might be needed even if all attributes are accounted for in $R_1 \cup R_2 \ldots \cup R_n$

Synthesizing … Step 4 (cont'd)

- Example: (*ABCD*; {*A* → *B*,*C* → *D*}), with step 3 decomposition: R_1 = (*AB*; {*A*→*B*}),R_2 = (*CD*; {*C*→*D*}).

Lossy! Need to add (AC; { }), for losslessness

- Step 4 guarantees lossless decomposition:

  ABCD --decomp--> AB,ACD --decomp-->AB,AC,CD

Boyce–Codd Normal Form (BCNF)

- A relation R(X) is in *Boyce–Codd Normal Form* if for every non-trivial functional dependency Y → Z defined on it, Y contains a key K of R(X). That is, Y is a superkey for R(X).
- Example: Person1(*SI#*, *Name*, *Address*)
  - ✓The only FD is *SI#* → *Name*, *Address*
  - ✓Since *SI#* is a key, Person1 is in BCNF
- Anomalies and redundancies, as discussed earlier, do not occur in databases with relations in BCNF.

Non-BCNF Examples

- Person(*SI#*, *Name*, *Address*, *Hobby*)
  - ✓The FD *SI#* → *Name*, *Address* does ***not*** satisfy conditions for BCNF since the key is (*SSN*, *Hobby*)
- HasAccount(*AcctNum*, *ClientId*, *OfficeId*)
  - ✓The FD *AcctNum* → *OfficeId* does ***not*** satisfy BCNF conditions if we assume that keys for HasAccount are (*ClientId*, *OfficeId*) and (*AcctNum*, *ClientId*); rather than *AcctNum*.

Normalization Drawbacks

- By limiting redundancy, normalization helps maintain consistency and saves space.
- *But* performance of querying can suffer because related information that was stored in a single relation is now distributed among several
- Example: A join is required to get the names and grades of all students taking CS343 in 2006F.

```
SELECT  S.Name, T.Grade
FROM  Student S, Transcript T
WHERE  S.Id = T.StudId  AND
          T.CrsCode = 'CS343'  AND  T.Semester = '2006F'
```

Denormalization

- Tradeoff: *Judiciously* introduce redundancy to improve performance of certain queries
- Example: Add attribute *Name* to Transcript --> Transcript'

```
SELECT  T.Name, T.Grade
FROM   Transcript'  T
WHERE  T.CrsCode = 'CS305'  AND  T.Semester = 'S2002'
```

  - ✓Join is avoided;
  - ✓If queries are asked more frequently than Transcript is modified, added redundancy might improve average performance;
  - ✓But, Transcript' is no longer in BCNF since key is (*StudId*,*CrsCode*,*Semester*) and *StudId* → *Name*.

BCNF and 3NF

- The Project-Branch-Manager schema is not in BCNF, but it *is* in 3NF.
- In particular, the `Project,Branch` → `Manager` dependency has as its left hand side a key, while `Manager` → `Branch` has a unique attribute for the right hand side, which is part of the {`Project,Branch`} key.
- The 3NF is less restrictive than the BCNF and for this reason does not offer the same guarantees of quality for a relation; it has the advantage however, of *always* being achievable.

3NF Tolerates Some Redundancies!

Manager	Project	Branch
Brown	Mars	Chicago
Green	Jupiter	Birmingham
Green	Mars	Birmingham
Hoskins	Saturn	Birmingham
Hoskins	Venus	Birmingham

!!

A Revised Example

Manager	Project	Branch	Division
Brown	Mars	Chicago	1
Green	Jupiter	Birmingham	1
Green	Mars	Birmingham	1
Hoskins	Saturn	Birmingham	2
Hoskins	Venus	Birmingham	2

Functional dependencies:

- `Manager → Branch,Division` **-- each manager works at one branch and manages one division;**
- `Branch,Division → Manager` **-- for each branch and division there is a single manager;**
- `Project,Branch → Division,Manager` **-- for each branch, a project is allocated to a single division and has a sole manager responsible.**

A Good Decomposition

Manager	Branch	Division
Brown	Chicago	1
Green	Birmingham	1
Hoskins	Birmingham	2

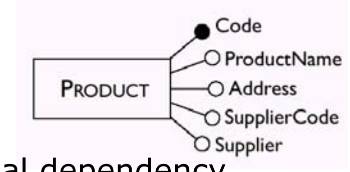
Project	Branch	Division
Mars	Chicago	1
Jupiter	Birmingham	1
Mars	Birmingham	1
Saturn	Birmingham	2
Venus	Birmingham	2

- Note: The first relation has a second key {**Branch,Division**}.
- The decomposition is in 3NF but not in BCNF; moreover, it is lossless and dependencies are preserved.
- This example demonstrates that BCNF is too strong a condition to impose on a relational schema.

Database Design and Normalization

- The theory of normalization can be used as a basis for quality control operations on schemas, during both conceptual and logical design.
- Analysis of the relations obtained during the logical design phase can identify places where the conceptual design was inaccurate: such a validation of the design is usually relatively easy.
- Normalization can also be used during conceptual design for quality control of each element of a conceptual schema (entity or relationship).

Analysis of an Entity



- The functional dependency

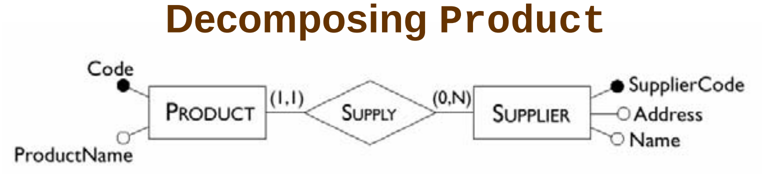
  SupplierCode → **Supplier,Address**

  holds here: all properties of a supplier are identified by its **SupplierCode**.
- The entity violates 3NF since this dependency has a left hand side that does not contain the identifier and a right hand side made up of attributes that are not part of the key.

Decomposing `Product`

- `Supplier` is (or should be) an independent entity, with its own attributes (code, surname and address)
- If `Product` and `Supplier` are distinct entities, they should be linked through a relationship.
- Since there is a functional dependency from `Code` to `SupplierCode`, we are sure that each product has at most one supplier (maximum cardinality 1).
- Since there is no dependency from `SupplierCode` to `Code`, we have an unrestricted maximum cardinality (N) for `Supplier` in the relationship.

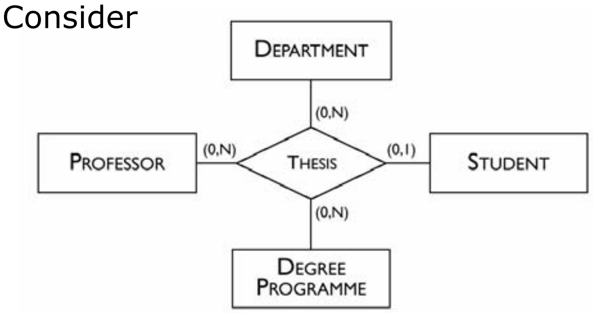
Decomposing `Product`



- This decomposition satisfies fundamental properties:
  - ✓It is a lossless decomposition, because of one-to-many relationship that allows us to recostruct the values of the attributes of the original entity;
  - ✓Moreover, it preserves dependencies because each dependency is embedded in one of the entities or can be reconstructed from them.

Analysis of a Relationship

- Now we show how to analyze n-ary relationships for n≥3, in order to determine whether they should be decomposed.
- Consider



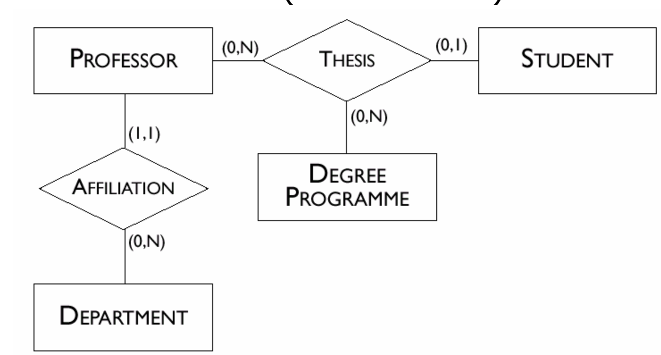
Some Functional Dependencies

- ✓`Student` → `DegreeProgramme` (each student is enrolled in one degree programme)
- ✓`Student` → `Professor` (each student writes a thesis under the supervision of a single professor)
- ✓`Professor` → `Department` (each professor is associated with a single department and the students under her supervision are students in that department)

- The (unique) key of the relationship is `Student` (given a student, the degree programme, the professor and the department are identified uniquely)
- The third FD causes a violation of 3NF.

Decomposing `Thesis`

- The following is a decomposition of `Thesis` where the two decomposed relationships are both in 3NF(also in BCNF)



More Observations...

- The relationship `Thesis` is in 3NF, because its key is made up of the `Student` entity, and its dependencies all have this entity on the left hand side.
- However, not all students write theses, therefore not all students have supervisors.
- From a normal form point of view, this is not a problem.
- However, our conceptual schema should reflect the fact that being in a degree programme and having a supervisor are independent facts.

Another Decomposition