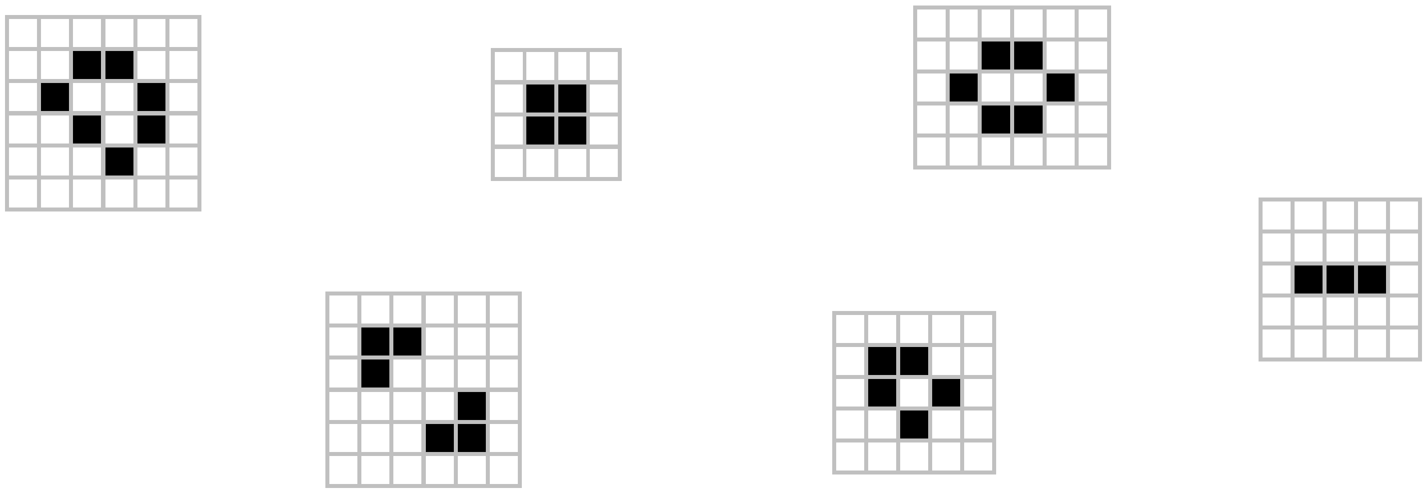
Introduction to Cellular Automata



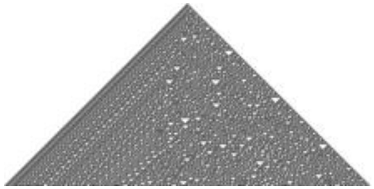
MOHAMMAD NASIRIFAR
SHAHID BEHESHTI UNIVERSITY

MAY 2015

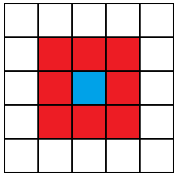
Contents

- History and Preliminaries

- Dynamical Systems



- 1 Dimensional Cellular Automata



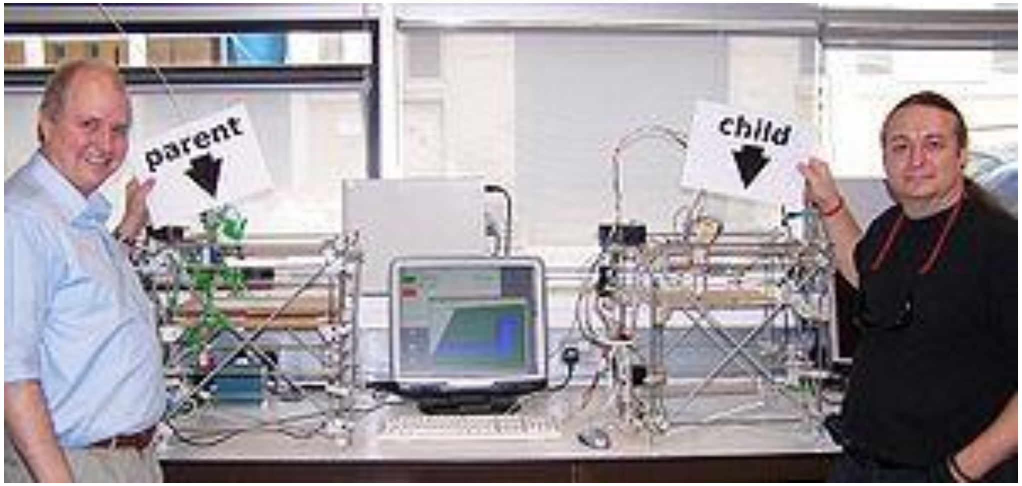
- 2 Dimensional Cellular Automata
- Models

History: First contributors

- John Von Neumann
- Stanislaw Ulam
- John Conway
- Stephen Wolfram

History: Motivations

- Self Replicating Machines
- Biological Systems
- Artificial Life

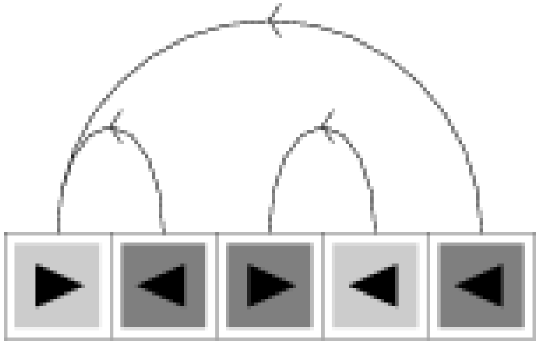


Preliminaries: Turing Machines

- Alan Turing (1936)
- Minsky's UTM : 7 head states, 4 cell states
- Wolfram using rule 110 : 2 head states, 5 cell states
- Wolfram: offering prize to a 2-3 UTM[1]

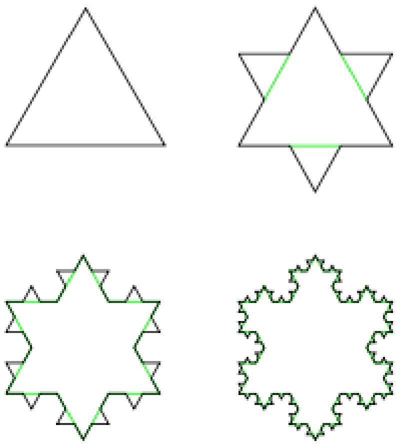
Preliminaries: Minsky Register Machine

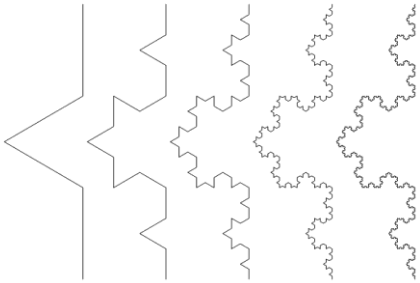
- Instructions : INC, DEC, HALT
- MRM with 2 registers: can emulate a UTM

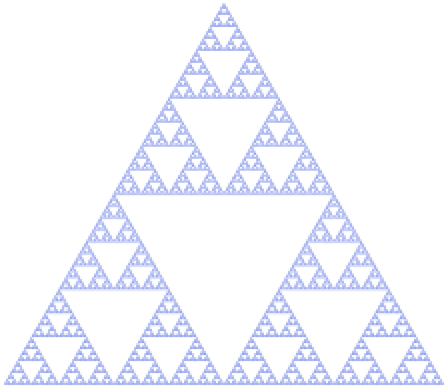


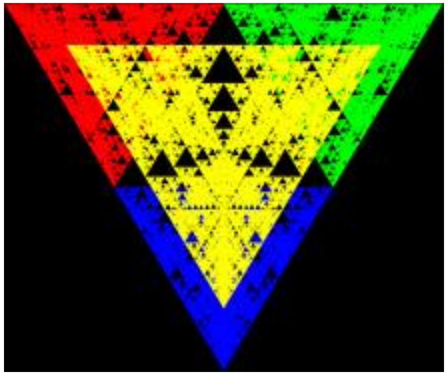
Preliminaries: Fractals

- What is a Fractal ?



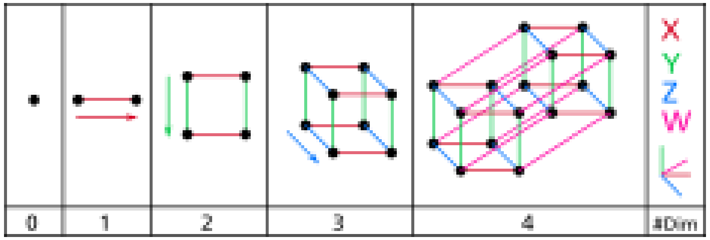






Preliminaries: Dimensions

- How can we define Dimension ?



- Similarity Dimension : $N = r^{-D}$
- Equivalently : $D = \dfrac{\log(N)}{\log(\frac{1}{r})}$ [1]

Preliminaries: Dimensions

- Fractal : Any figure with a non-integer dimension.
- Intuitively a dimension of 1.262 (Koch Curve) shows that the figure seems to be taking space more than an ordinary line [1][2]

Preliminaries: Information

- Claude Shannon (1948) : Being unlikely to occur, an event upon it's occurrence exposes more "information". (and vice-a-versa)[1]
- Being unlikely to occur : having low probability of occurrence

Preliminaries: Information

- The conveyed information by the occurrence of an event :

$$I = \log_2(\frac{1}{p})$$

- Base 2 depends on representation.
- How much info does rolling a dice/ tossing a coin convey ?

Preliminaries: Entropy

- Rudolf Clausius: *The thermodynamic entropy of a closed system increases to a maximum.*

$$H = -K \sum_{i=1}^{N} P_i \log_e P_i$$

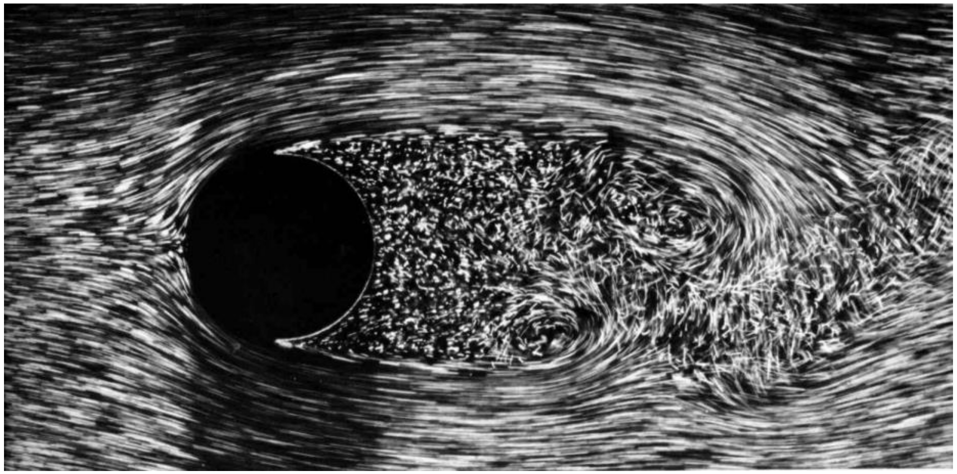
- K=1 -> Shannon entropy, information entropy
- This proceeds to define information Dimension

Preliminaries: Randomness

- High Entropy : High disorder , Randomness
- Random: unpredictable
- More technical : an infinitely long sequence is random if it's Shannon information (I) is infinite.
- Quantis, the only real random number generator
- Randomness vs pseudo-randomness

Dynamical Systems

- Modeling the changes of a system over time e.g. fluid flow,



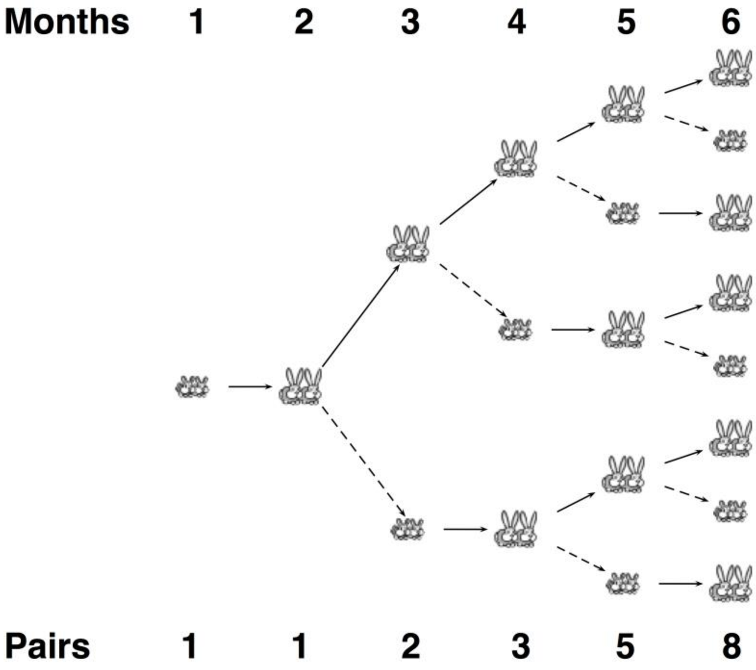
  chaos theory, …
- CA : Discrete DS

Dynamical Systems

- Fibonacci sequence is a dynamical system

$$1, 1, 2, 3, 5, 8, 13 \; \ldots$$

$$F_n = F_{n-1} + F_{n-2}$$



Dynamical Systems

- Linear growth model for generations:

$$x_{n+1} = ax_n \ , n = 0,1,2,\ldots$$

- More realistic population models:

$$x_{n+1} = (1 + a)x_n - bx_n^2 \ , n = 0,1,2,\ldots$$

$$x_{n+1} = \ x_n e^{[c(1-x_n)]}$$

- a , b , c are constants used for “tuning” the behavior of the dynamical system.

Dynamical Systems

- Attractors, repellers, and initial values
- Fixed points : $f(x) = x$
- $|f`| < 1 : attractor$
- $|f`| > 1 : repeller$

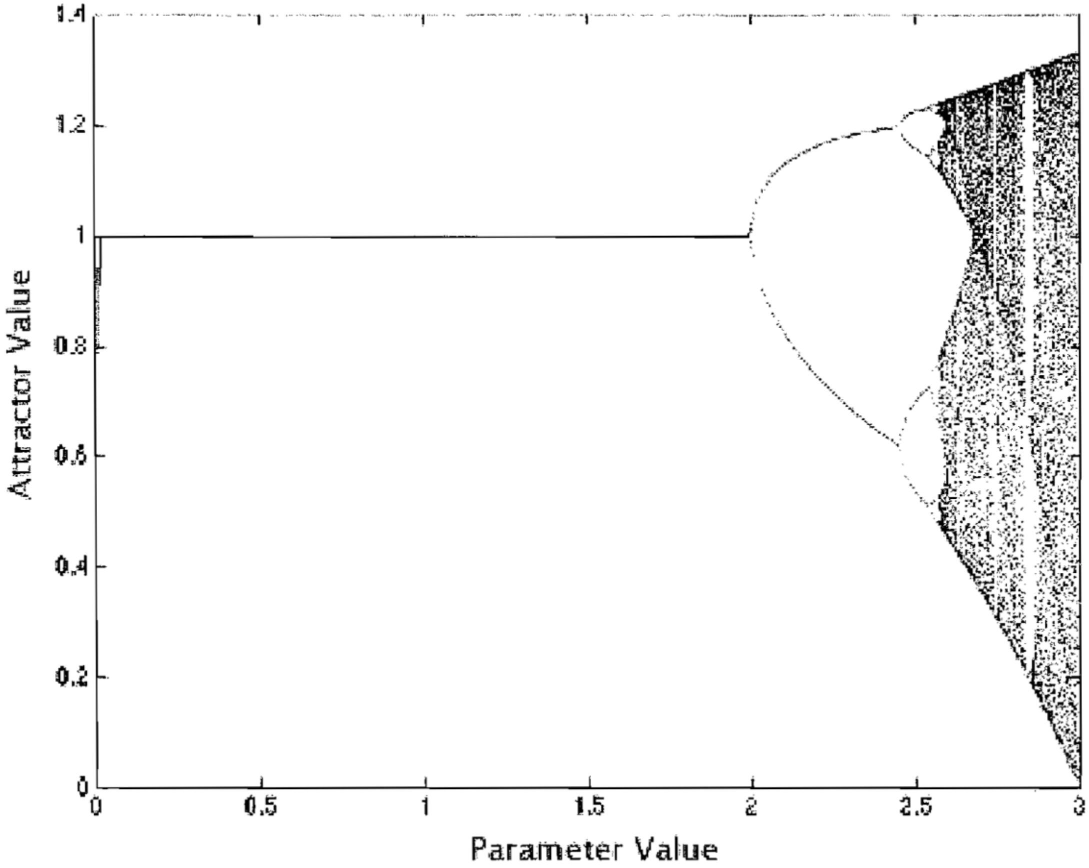
Dynamical Systems

- $f(x) = (1 + a)x \ - ax^2 \ , \ \ x_{n+1} = f(x_n)[1]$
- $0 < a < 2 : attractor\ period\ of\ 1$
- $2 < a < 2.449 : attractor\ period\ of\ 2$
- $Periods\ of\ 4, 8\ , 16\ , \ldots follow$
- And the doubling continues to … infinity ?

Dynamical Systems

- $f(x) = (1 + a)x - ax^2, \; x_{n+1} = f(x_n)$
- BUT WAIT ! THERE'S MORE !
- $2.57 < a < 3$:
- Doubling behavior breaks down and we enter the chaotic regime : infinitely many points that yield aperiodic orbits (these are hard to distinguish from long cycles)
- a = 2.840 3-cycle, which generates it's own doubling , or a=2.9605 a 4-cycle
- Finally at a = 3 the dynamical system becomes a deterministic random number generator = pseudorandom number generator

Dynamical Systems



1D Cellular Automata: Intro

- A lattice of cells usually square shaped , each of which can be in k different states, one of which is named quiescent
- Dimension and size of the lattice
- Local transition function and time steps
- State transformation and neighbors
- A cellular automaton : cells, transition function, set of states.

1D Cellular Automata: Example

Initial position (start state)

1D Cellular Automata: Example

Transition function:

if cell is white :

 if at least one neighboring cell is red, change to black

 otherwise remain white

If cell is black :

 if exactly one of the two neighboring cells

 are black, then change state to red

 otherwise change to white

If cell is red :

 change color to white

1D Cellular Automata: Example

Initial position (start state) , t = 0

White :

at least one red neighbor : black

o.w. : white

Black :

exactly one neighbor black: red

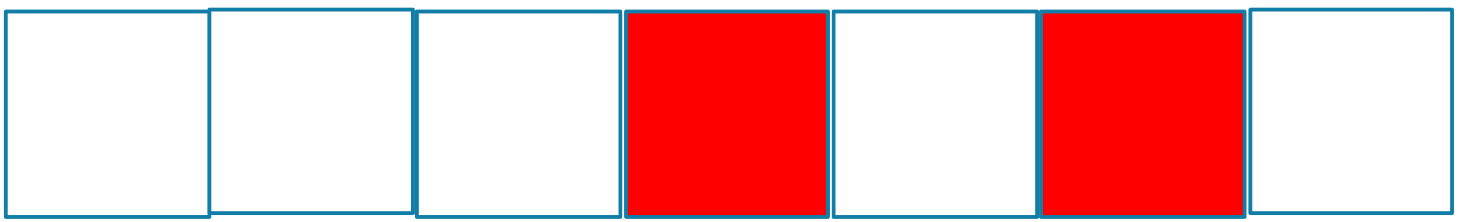
o.w. : white

Red :

: white

1D Cellular Automata: Example

t = 1



White :

at least one red neighbor : black

o.w. : white

Black :

exactly one neighbor black: red

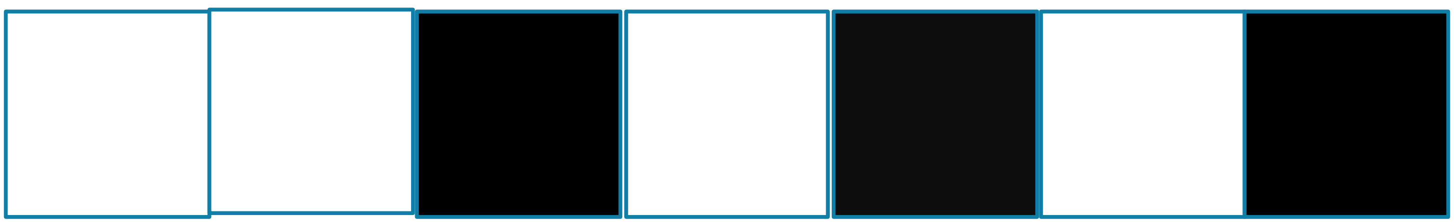
o.w. : white

Red :

: white

1D Cellular Automata: Example

t = 2



White :

  at least one red neighbor : black

  o.w. : white

Black :

  exactly one neighbor black: red

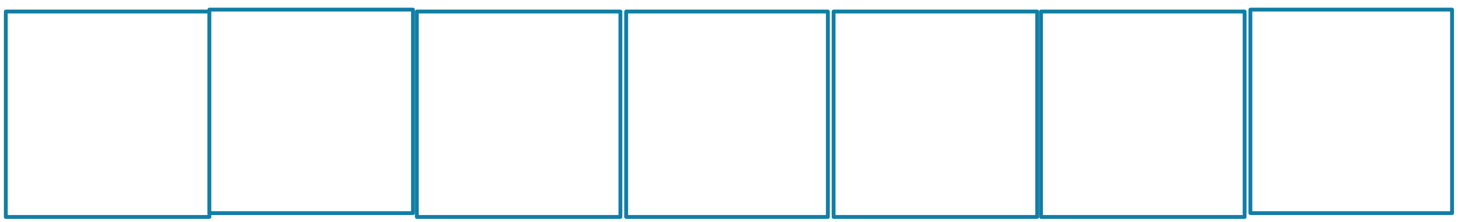
  o.w. : white

Red :

  : white

1D Cellular Automata: Example

t = 3



White :

  at least one red neighbor : black

  o.w. : white

Black :

  exactly one neighbor black: red

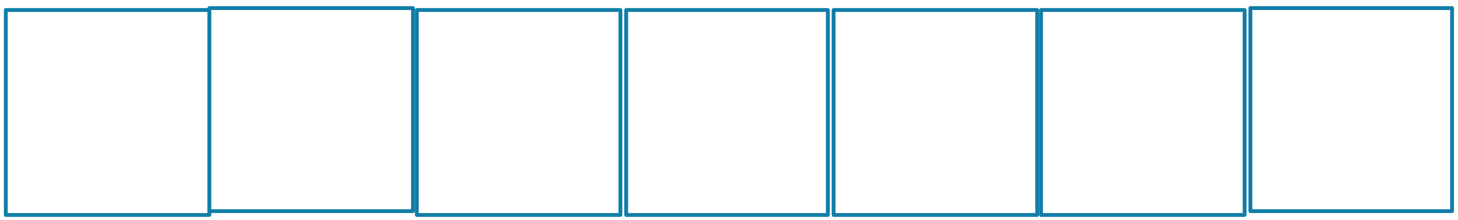
  o.w. : white

Red :

  : white

1D Cellular Automata: Example

t = 4 , 1-cycle : attractor



White :

at least one red neighbor : black

o.w. : white

Black :

exactly one neighbor black: red

o.w. : white

Red :

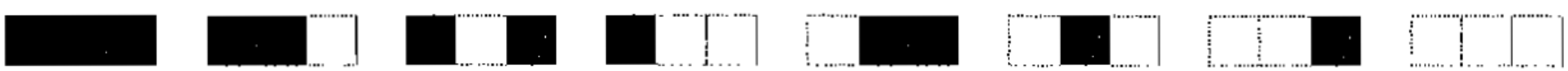
: white

1D Cellular Automata: Properties

- Uniformity
- Synchronicity
- Locality

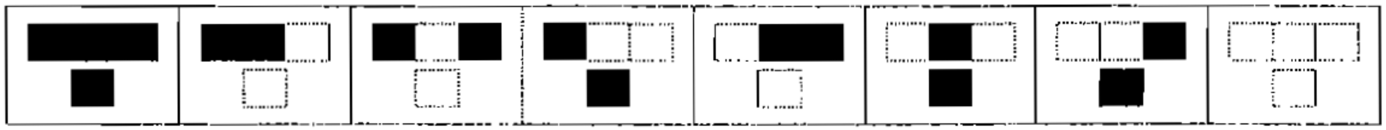
1D Cellular Automata: Transition Functions Formalism

- Example :
- $c_i(t+1) = \left(c_{i-1}(t-1) + c_{i+1}(t-1)\right) mod\, 2$
- Convenient way for radius neighborhood :

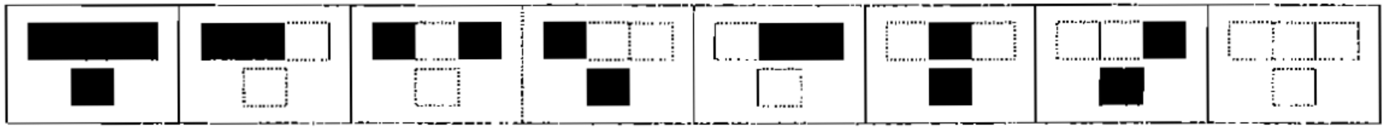


1D Cellular Automata: Rule 150

1D Cellular Automata: Rule 150

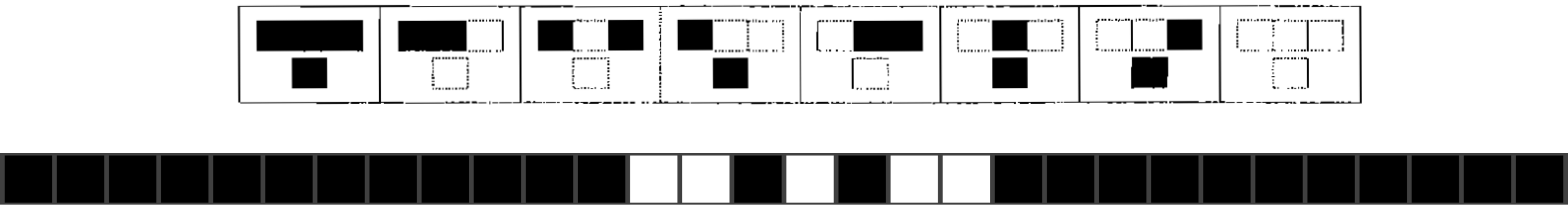


1D Cellular Automata: Rule 150

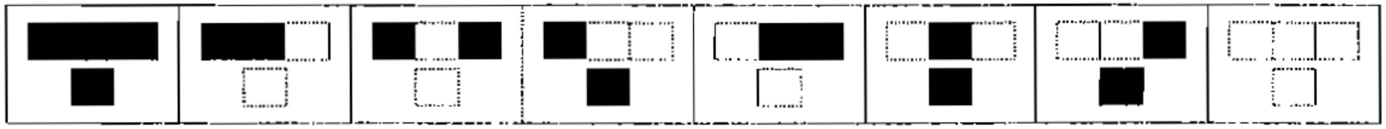


1D Cellular Automata: Rule 150

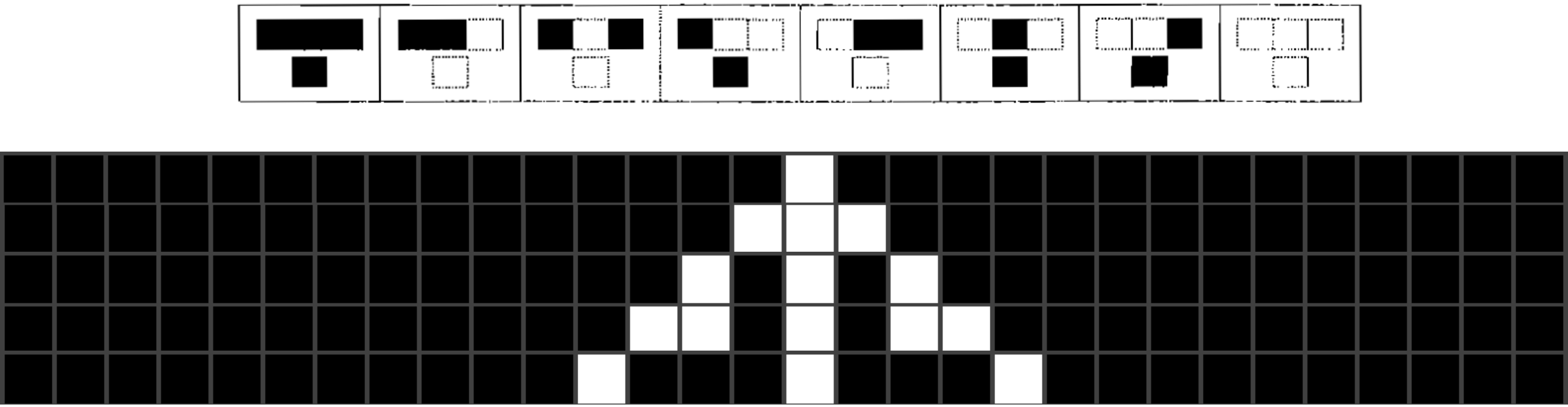
1D Cellular Automata: Rule 150



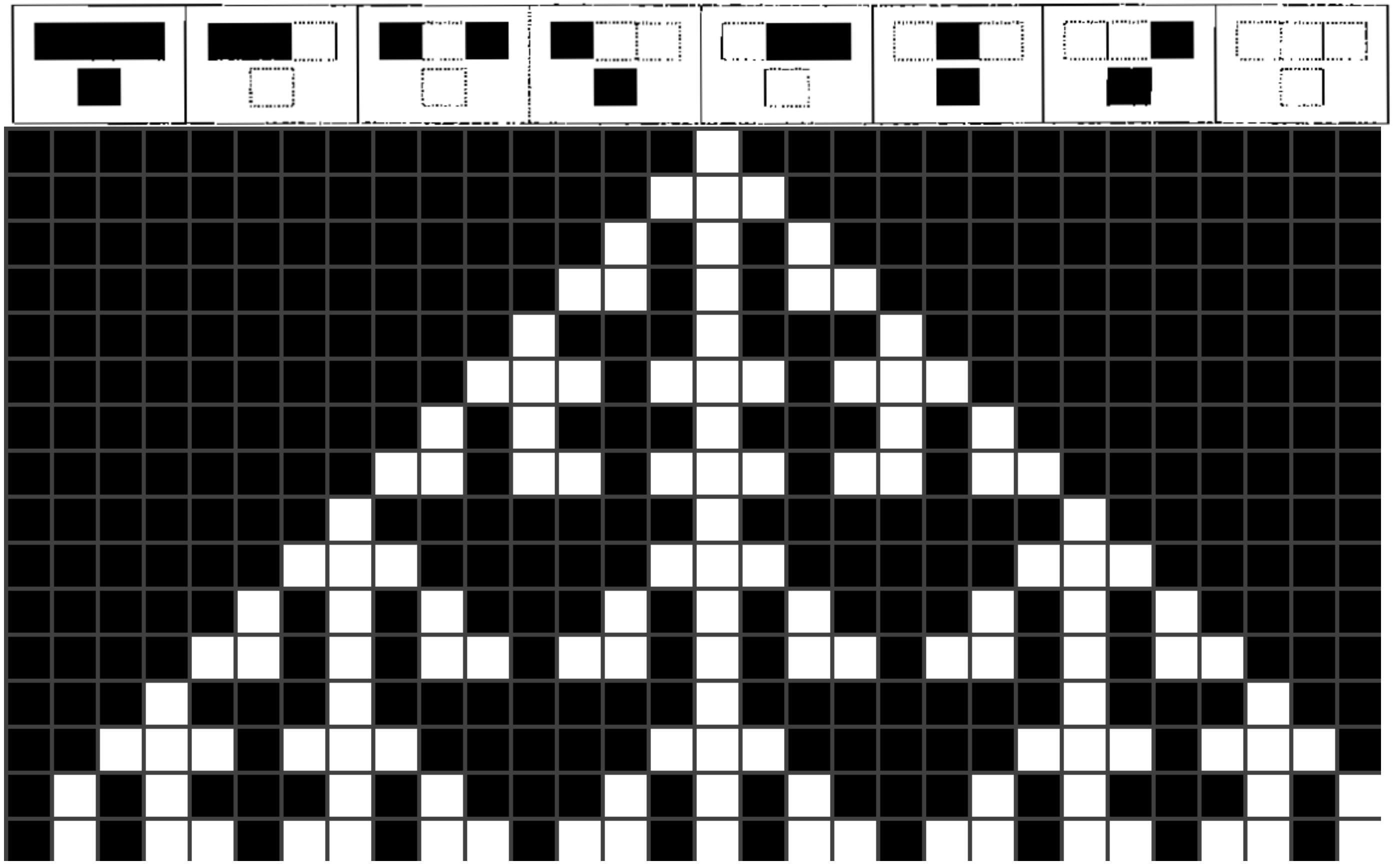
1D Cellular Automata: Rule 150



1D Cellular Automata: Rule 150



1D Cellular Automata: Rule 150



1D Cellular Automata: reversibility

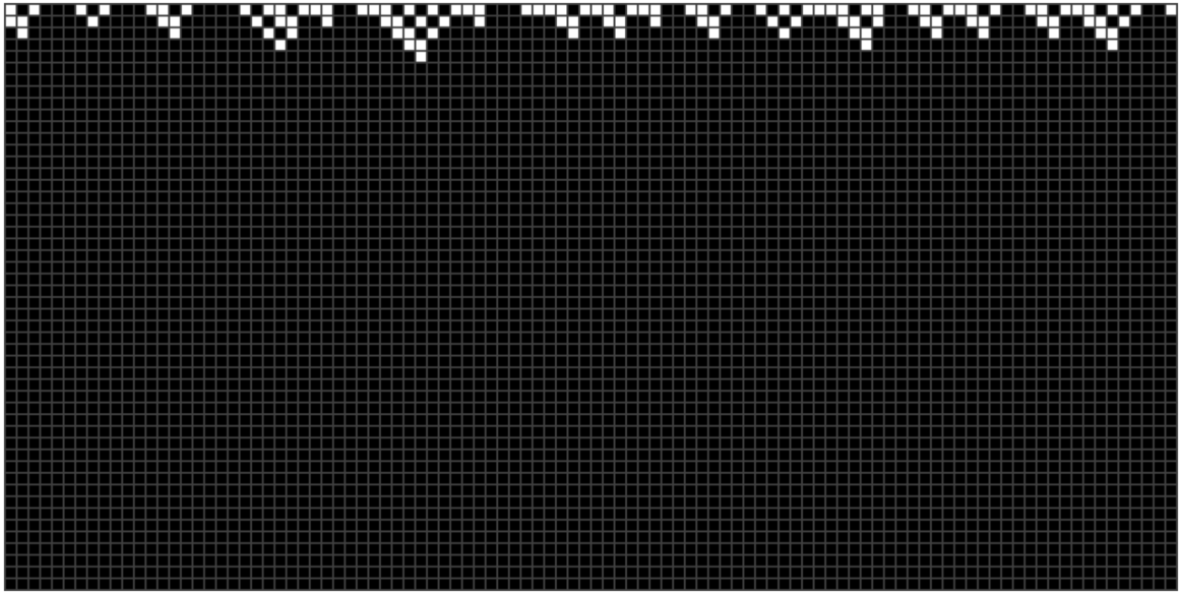
- Is cellular automata behavior reversible?
- State compression results in low entropy
- Only a small percentage of cellular automata are reversible

1D Cellular Automata: Classification

- 1980: Wolfram began classifying cellular automata
- 4 classes, majority of automata can be classified using these classes

1D Cellular Automata: Classification

- Class I : evolves to a uniformly constant state (fixed point attractor dynamical system)
- Rule 249



1D Cellular Automata: Classification

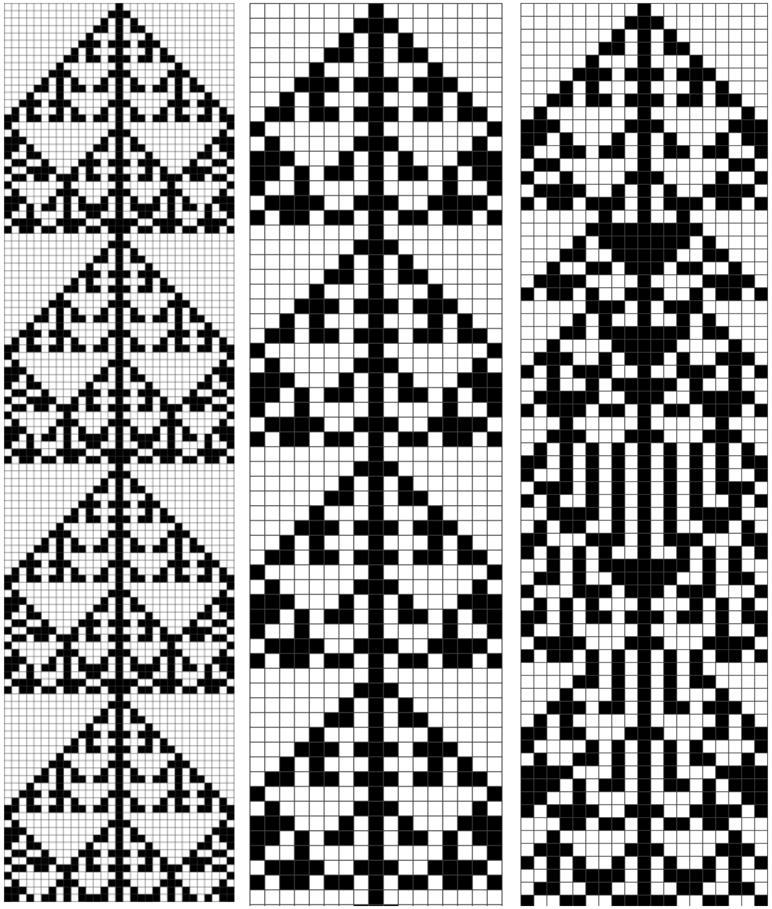
- Class II : repeats in periodic cycles.
- Similar to these are systems of finite size
- Can be used in digital image processing

1D Cellular Automata: Classification

- Class II example :

rule 150 in finite domain

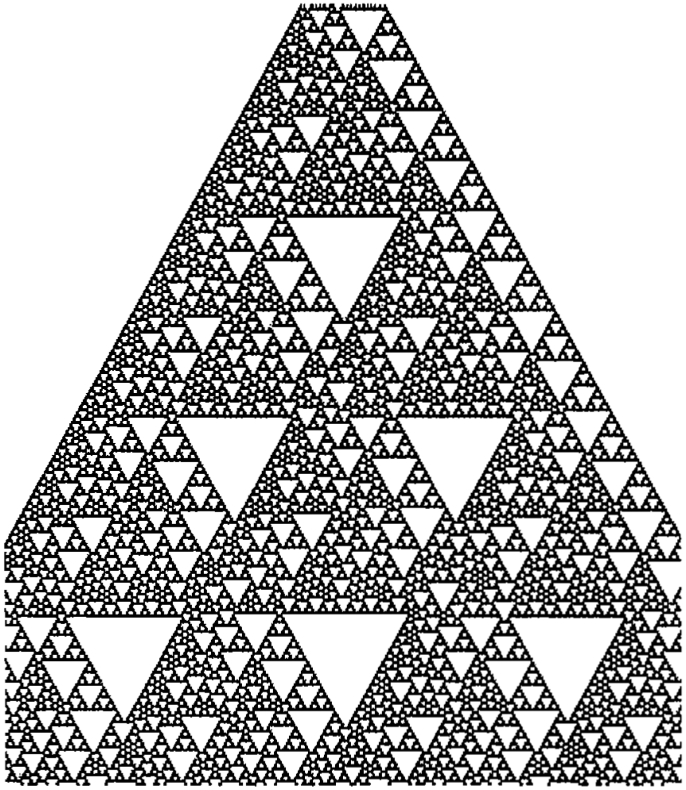
- Widths : 35, 17 , 19
- Period : 31,15, 510

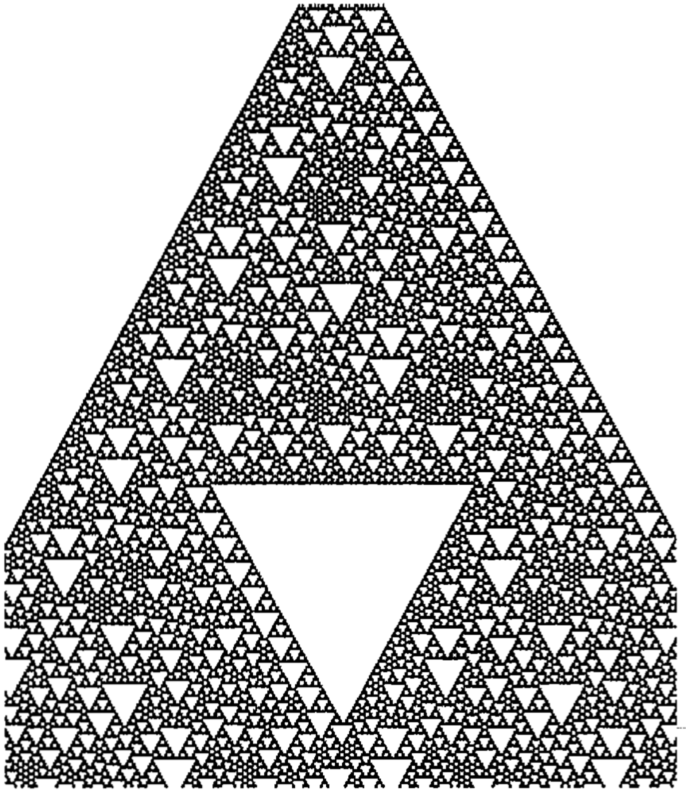


1D Cellular Automata: Classification

- Class III: random behavior, typically with triangular features present
- Analogous to chaotic dynamical systems
- Very sensitive to initial conditions
- Useful in study of randomness
- Rule 126

1D Cellular Automata: Classification

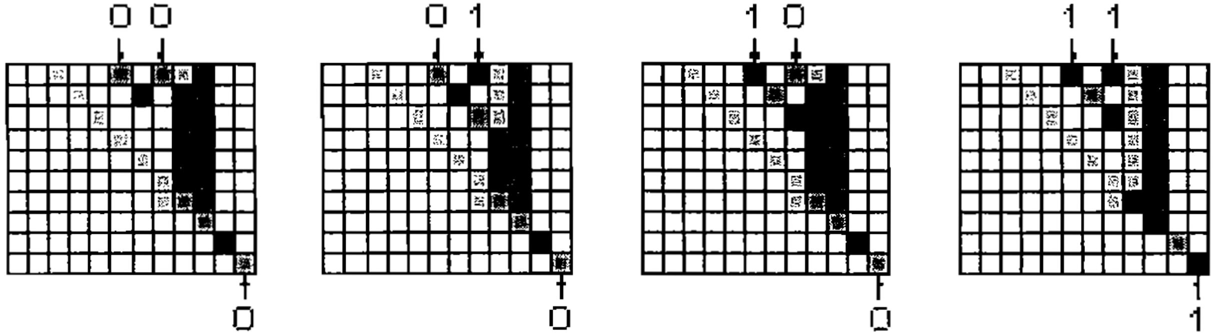




1D Cellular Automata: Classification

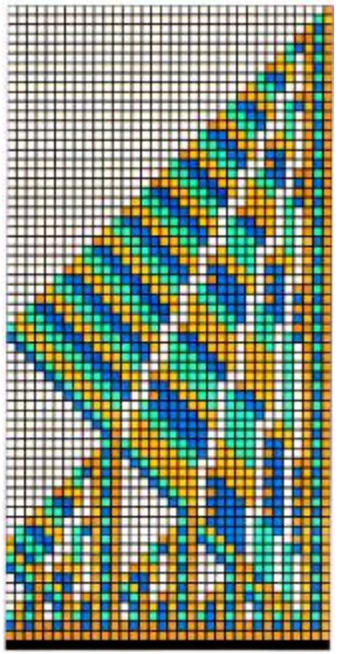
- Class IV: localized structures evolve and interact with each other
- No dynamical system equivalent, lays somewhere between class II and class III
- We can look for universal computations here
- Decidability ?

1D Cellular Automata: Universal Computation



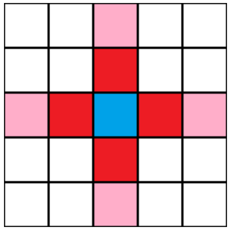
1D Cellular Automata: Notable Problem

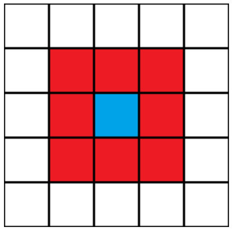
- Firing Squad Problem
- n cells : 3n -1 solution exists
- Minimal time solution : 2n-1
- Mazoyer : 6 state minimal time
- No minimal time with 4 states
- 5 or 6 states ? Still open :) [1]



2D Cellular Automata: Intro

- Essentially the same as 1D CA
- BUT IN 2D !! :D
- Two fundamental types of neighborhood
- Von Neuman neighborhood
- Moore neighborhood





2D Cellular Automata: Game of Life

- 2 states per cell
- A Dead cell becomes alive if exactly three neighbors are alive (eight-cell Moore neighborhood)
- An alive cell remains alive if either 2 or three of it’s neighbors are alive
- Many variants exist

2D Cellular Automata: Game of Life

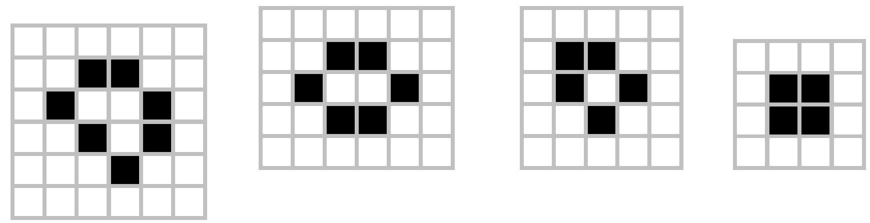
- Game of life is a class IV CA.
- No other class IV CA found (except trivial variants of life)

2D Cellular Automata: Game of Life

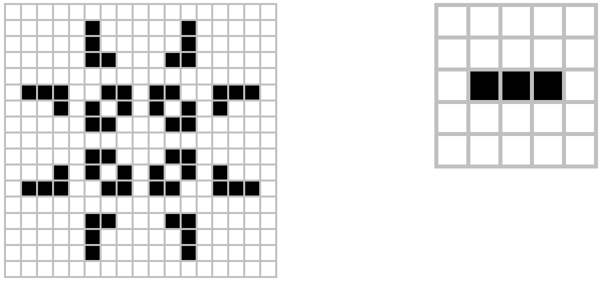
- $50 prize was offered to the person who could prove or disprove that …
- … No initial position of live cells could grow unboundedly
- What do you think ?

2D Cellular Automata: Game of Life, Life forms

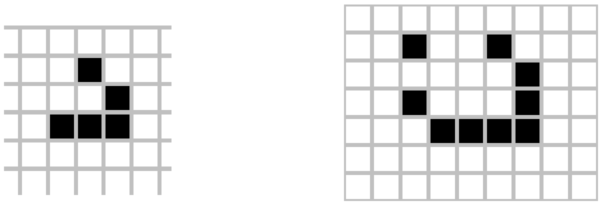
- Still life forms



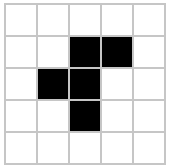
- Oscillators



- Spaceships



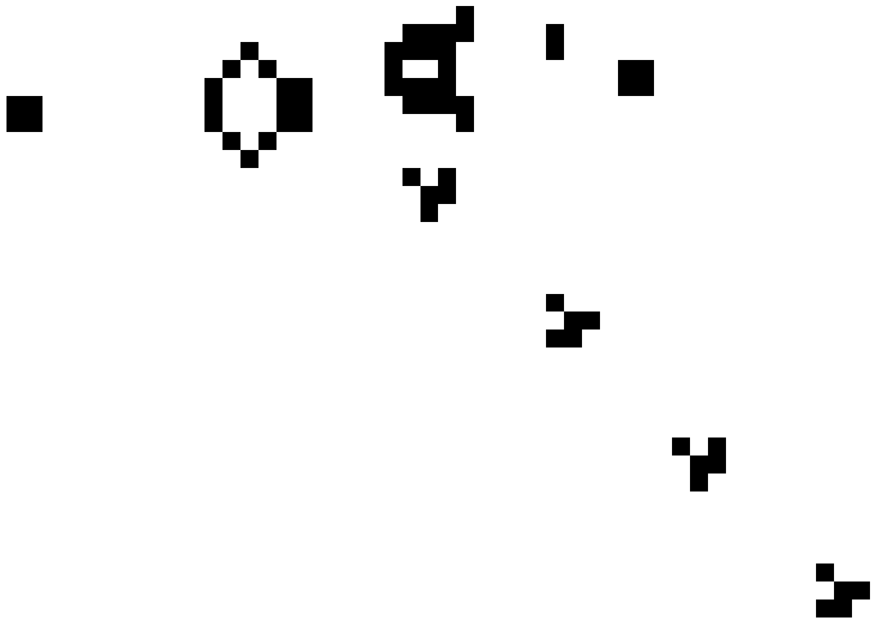
2D Cellular Automata: Game of Life, Life forms



- R-pentomino
- The most active polynomio with less than 6 cells
- 1103 generations until stabilization, with a population of 116
- Releases a glider in generation 69

2D Cellular Automata: Game of Life, Life forms

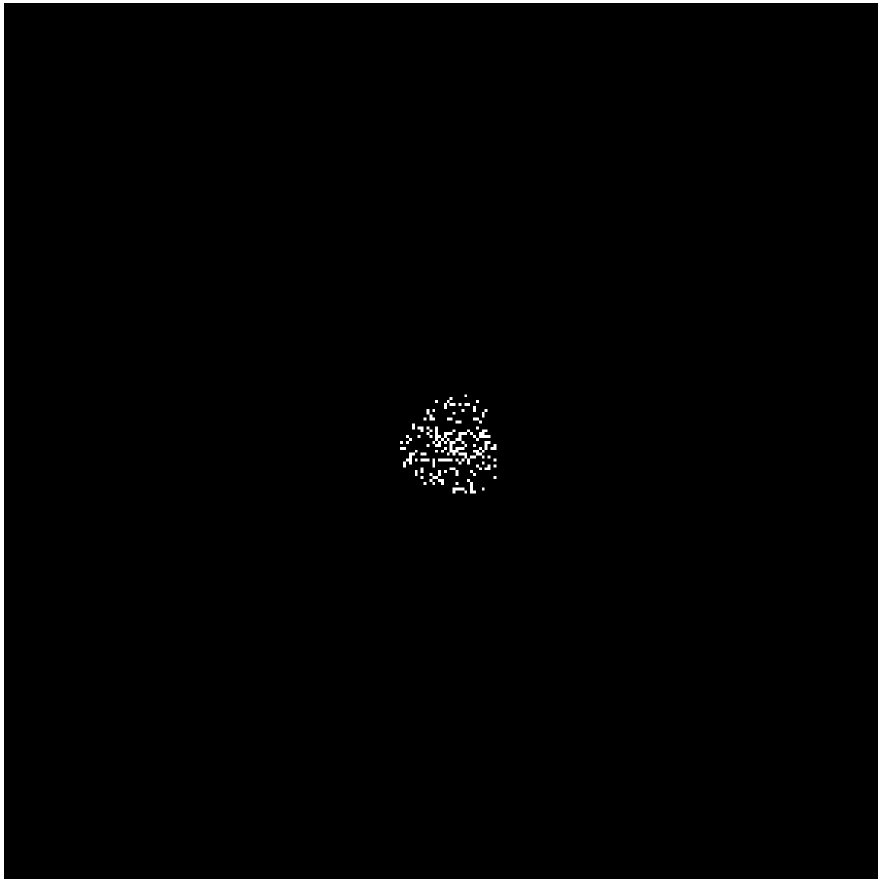
- Gosper's glider gun



2D Cellular Automata: Brian’s Brian

- 3 states : firing(alive), dying , off
- A cell which is off becomes alive in the next time step if it had had two alive neighbors
- All alive cells change into dying state and all dying cells change state to off in the next time step

2D Cellular Automata: Brian's Brian



References

[1] Schiff, J. L. (2007) References, in Cellular Automata: A Discrete View of the World, John Wiley & Sons, Inc., Hoboken, NJ, USA. doi: 10.1002/9781118032381.refs

[2] Author: **Stephen Wolfram**
Title: **A New Kind of Science**
Year: **2002**
Publisher: **Wolfram Media**
Place: **Champaign, IL**
ISBN: **1-57955-008-8**