CSCCD37H

Assignment 2

January 28, 2013   Scarborough Campus   **Due:** February 25, 2013

You are to use Matlab to solve the following problems and the graphing procedures of Matlab to present (visualize) the results. The first problem is from the text.

Your grade will be based partly on the clarity and readability of your results. That is, it is not only "What you say" but "How you say it".

1. Problem 3.10, p. 155 of text.

2. (A numerical investigation of least squares problems)
   The <u>continuous</u> least squares problem of approximating a function $f(x)$ on $[0, 1]$ by a polynomial of degree less than n that minimises,

   $$\int_0^1 (f(x) - p_n(x))^2 dx,$$

   leads to the following system of linear equations:

   $$Hc = b,$$

   where H is the Hilbert matrix of dimension n, $h_{ij} = \frac{1}{(i+j-1)}$, c is the vector of coefficients defining the 'optimal' polynomial approximation, $p_n(x) = \sum_{j=1}^n c_j x^{j-1}$, and $b \in R^n$, $b_j = \int_0^1 x^{j-1} f(x) dx$.
   The related <u>discrete</u> least squares problem was discussed in lectures and can be based on a given mesh of m equally spaced data points on $[0, 1], 0 = x_1 < x_2, \cdots < x_m = 1$ and corresponding function values $f_1, f_2, \cdots f_m$, (where $f(x_i) = f_i$ for $i = 1, 2, \cdots m$). When $m > n$ we can consider the solution of the linear least squares problem that minimises $\sum_{i=1}^m (\bar{p}_n(x_i) - f_i)^2$. That is, determine the coefficients $\bar{c}$ defining $\bar{p}_n(x)$, $(\bar{p}_n(x) = \sum_{j=1}^n \bar{c}_j x^{j-1})$, so that this sum of squares is minimised.

   (a) Set up this second discrete problem as a standard overdetermined LLSQ problem and solve for $\bar{c}$, in Matlab, for various functions $f(x)$ and $n = 4$, and $m = 4, 10, 100, 400$.

   (b) Compare the resulting polynomials to the polynomial solution of the continuous least squares problem for the same value of n (also obtained in Matlab) and investigate whether $A^T A$, for the discrete case, seems to be related to H for the continuous case as m increases. Also discuss whether the solution $\bar{c}$ for the discrete case, seems to be related to the solution c for the continuous case as m increases. Note that to determine $b_j = \int_0^1 x^{j-1} f(x) dx$, for this system of linear equations, you can use the analytic expression for these integrals or use an accurate quadrature from Matlab.