CSCD37H

Assignment 1

January 14, 2013 Scarborough Campus **Due:** January 28, 2013

1. A lower Hessenberg matrix is a square n by n matrix $H = (h_{ij})$ where $h_{ij} = 0$ for $j > i + 1$. Show how the standard Gaussian elimination algorithm can be modified to exploit this special structure when solving a linear system:
$$Hx = b\ ,$$
where H is lower Hessenberg. That is, outline and implement (in Matlab), an efficient modified GE algorithm for solving such a system. Give an operation count for your modified algorithm and present a well documented listing of your implementation together with examples of its performance on some test problems (of varying size). (Note that you can assume that no pivoting is necessary when performing the standard GE algorithm. bonus marks will be given for a discussion of how pivoting can be done without increasing the cost signifcantly.)

2. Let B be a given square n by n matrix and consider the problem of solving the linear system $Ax = b$, where $A = B^2$.
   (a) Describe an algorithm (present an overview in pseudo-code) for solving this problem based on first computing $A = B * B$ explicitly and then applying standard GE to solve $Ax = b$. What is the operation count and storage requirement for this algorithm?
   (b) Show that the special structure of A (being the square of a known matrix) can be exploited to produce a more efficient algorithm. What is the operation count and storage requirement of this new algorithm?
   (c) Consider, instead of a single right hand side vector (RHS), the poblem of solving $Ax^{(r)} = b^{(r)}$ for $r = 1, 2 \cdots k$ for a sequence of k different RHS vectors (but the same coefficient matrix, A). What would the operation counts be if the two algorithms discussed in (a) and (b) were use to solve this multiple RHS problem ?