CSC2302H

Project

February 14, 2014 University of Toronto **Due:** End of Winter term 2014

As you know, part of the evaluation in this course is to be based on a project. The particular choice of topic is up to you, but two typical examples are outlined below to give you a feeling for the scope and focus that is expected. If you wish you may choose one of these projects but, to avoid duplication, please consult with me. If you choose a different project, for example one that involves comparing the performance of different methods when used to approximate the solution of a challenging problem (or class of problems) arising in an interesting application area, please have the topic approved by me in the next few weeks. In any event, The following outlines will help you in determining the depth and scope of your project.

Example Projects:

1. The Approximation of Linear Constant-coefficient IVPs

   This project will investigate numerical techniques for solving the constant coefficient IVP,
   $$y^{'} = Ay, \;\; y(0) = y_0.$$
   where A is a constant $n \times n$ matrix. The project will involve two parts.

   (a) A literature search of the relevant numerical and engineering literature and a survey of the various proposed approaches. (in this survey it is not expected that you summarize each paper, but rather you identify and summarize what you feel to be the most promising approaches). Note that the related question of approximating the matrix exponential is discussed in the literature and, although it is one approach, most of the references will not be relevent as this task is inherently more complex.

   (b) An investigation of the extent to which a good stiff ODE solver can be modified to efficiently solve this special class of problems. This will involve choosing a method (such as LSODE, VODE, RADAU or ode15s), modifying it to exploit the special structure of this problem class, and quantifying (theoretically and experimentally) the efficiency of the modified method.

2. The Magnitude of the Defect for a Multistep Method

   This project will involve an investigation of the relationship between the magnitude of the defect and TOL for an existing k-step multistep method (such as ode113, ode15s, VODE, or ODE) and some natural interpolating schemes. For any IVP solver we have discussed, one is given on each step, approximations y_i, y_i', y_{i+1} and y_{i+1}' as well as a local interpolant which can be used to determine approximations to $y(x)$ and $y'(x)$ at additional values of $x \in [x_i, x_{i+1}]$. Using this information and knowledge about the order of the method, you can investigate the relationship between TOL and the magnitude of the defect of the local interpolant (on a small set of test problems).

The method you are investigating uses only the values,

$$(x_r,\ y_r,\ y'_r)_{r=i-k}^{i+1}\ ,$$

to define the local interpolant for $x \in [x_i, x_{i+1}]$. On the other hand, once a problem has been "solved" and the discrete solution, $(x_i, y_i, y'_i)_{i=0}^{N}$, computed, one can consider alternative definitions for the local interpolant on $[x_i, x_{i+1}]$. You should define one or two such alternative interpolating schemes and investigate the relationship between the magnitude of the defect and TOL for them. Do any of the alternative schemes you have investigated appear to be better than the local interpolant associated with the method?

Note that this investigation need not involve modifying your chosen multistep method in any way. All you need do is store the associated "local information" on each step (while solving the problem) and the investigation of the associated maximum defect for the alternative interpolating schemes can be done as a subsequent "post processing" step.