CSC2302H

Project

November 1, 2010 University of Toronto **Due:** End of Fall term 2010

As you know, part of the evaluation in this course is to be based on a project. The particular choice of topic is up to you, but two typical examples are outlined below to give you a feeling for the scope and focus that is expected. If you wish you may choose one of these projects but, to avoid duplication, please consult with me. If you choose a different project, for example one that may involve comparing the performance of different method when used to approximate the solution of a challenging problem arising in an interesting application area, or an elaboration or extension of one of the assignments, please have the topic approved by me in the next few weeks. In any event, The following outlines will help you in determining the depth and scope of your project.

Example Projects:

1. The Approximation of Linear Constant-coefficient IVPs

   This project will investigate numerical techniques for solving the constant coefficient IVP,

   $$y' = Ay, \;\; y(0) = y_0.$$

   where A is a constant $n \times n$ matrix. The project will involve two parts.

   (a) A literature search of the relevant numerical and engineering literature and a survey of the various proposed approaches. (in this survey it is not expected that you summarize each paper, but rather you identify and summarize what you feel to be the most promising approaches). Note that the related question of approximating the matrix exponential is discussed in the literature and, although it is one approach, most of the references will not be relevent as this task is inherently more complex.

   (b) An investigation of the extent to which a good stiff ODE solver can be modified to efficiently solve this special class of problems. This will involve choosing a method (such as LSODE, VODE, RADAU or ode15s), modifying it to exploit the special structure of this problem class, and quantifying (theoretically and experimentally) the efficiency of the modified method.

2. The Size of the Defect for a Stiff Method

   This project will involve an investigation of the relationship between the size of the defect and TOL for an existing stiff method (such as RADAU or ode15s) and some natural interpolating schemes. For any stiff solver we have discussed, one is given on each step, approximations y_i, y'_i, y_{i+1} and y'_{i+1} as well as a low order interpolant which can be used to determine approximations to $y'(x)$ at additional values of $x \in [x_i x_{i+1}]$. Using this information and knowledge about the order of the method, you can investigate different local interpolants (that can be computed using this information).

By keeping track of the local information discussed above on each step of the integration you would investigate the relationship between TOL and the maximum defect for some candidate interpolation schemes on both linear and nonlinear problems (over a range of tolerances). By observing this relationship and using other observations (such as the "most likely" location of the maximum defect on a given step) you should be able to recommend a particular interpolant to consider for your method. You might also be able to recommend a suitable value for $\hat{\tau}$ such that $\delta(x_i + \hat{\tau}h)$ can be used to estimate the maximum defect on step i.