

# Tern TCT      Soluzioni

1. a) Hermite cubic interpolant

$x_i$	$f(x_i)$	$f'(x_i)$	$f''(x_i)$	$f'''(x_i)$
0	0	1.0		
0	0			
1	.841	.841		
1	.841	.540		

  

$$f'(0) = 1.0$$

$$\frac{.841 - 0}{1 - 0} = .841$$

$$f'(1) = .540$$

$$\frac{.841 - 1}{1 - 0} = -.159$$

$$\frac{.540 - .841}{1 - 0} = -.301$$

$$\frac{-.301 + .159}{1} = -.142$$

$$\therefore p_3(x) = 0 + x - .159x^2 - .142x^2(x-1)$$

$$= x - .017x^2 - .142x^3$$

(check  $p_3(0) = 0$ ,  $p_3'(0) = 1$ ,  $p_3(1) = 1 - .017 - .142 = .841$   
 $p_3'(1) = 1 - .034 - 3 \cdot .142 = .540$ )

b)  $A_1 = \int_0^1 [x - .017x^2 - .142x^3] dx$

$$= \int_0^1 x dx - .017 \int_0^1 x^2 dx - .142 \int_0^1 x^3 dx$$

$$= \left. \frac{x^2}{2} \right|_0^1 - .017 \left. \left( \frac{x^3}{3} \right) \right|_0^1 - .142 \left. \left( \frac{x^4}{4} \right) \right|_0^1$$

$$= \frac{1}{2} - .017 \left( \frac{1}{3} \right) - .142 \left( \frac{1}{4} \right) = .459$$

$$A_2 = \frac{1}{6} [f_0 + 4f_{1/2} + f_1] = \frac{1}{6} [0 + 4 \cdot (.479) + .841] = 2.757/6$$

$$= .4595$$

For  $A_2$   $E^S = \frac{1}{96} \left( \frac{h^5}{2} \right) = |E^S| \leq \frac{1}{96} \left( \frac{1}{32} \right) = .000347$

for  $A_1$   $|E| \leq \left| \int_0^1 E_2^{(1)} \right| \leq |E_2^{(1)}| \leq \frac{1}{5!} \left( \frac{1}{2} \right)^4 \leq \left( \frac{1}{120} \right) \left( \frac{1}{16} \right) = .00052$

2.  $p(t) \approx x(t)$ :

$t_i$	$x(t_i)$	$x[t_{i-1}, t_i]$	$x[t_{i-2}, t_{i-1}, t_i]$
0	$1/2$		
1	1	$\frac{1-1/2}{1-0} = 1/2$	
2	$1/2$	$\frac{1/2-1}{2-1} = -1/2$	$\frac{-1/2-1/2}{2-0} = -1/2$

$\therefore p(t) = 1/2 + 1/2 t - 1/2 t(t-1)$   
 $= 1/2 + t - 1/2 t^2$

(verify  $p(0) = 1/2, p(1) = 1, p(2) = 1/2$ )

$q(t) \approx y(t)$ :

$t_i$	$y(t_i)$	$y[t_{i-1}, t_i]$	$y[t_{i-2}, t_{i-1}, t_i]$
0	0		
1	1	$\frac{1-0}{1-0} = 1$	
2	$1/2$	$\frac{1/2-1}{2-1} = -1/2$	$\frac{-1/2-1}{2-0} = \frac{-3/2}{2} = -3/4$

$\therefore q(t) = t - 3/4 t(t-1) = \frac{7}{4} t - \frac{3}{4} t^2$

(verify  $q(0) = 0, q(1) = 1, q(2) = \frac{7}{2} - 3 = 1/2$ )

(c)  $p(1/2) = \frac{1}{2} + \frac{1}{2} - \frac{1}{2^3} = \frac{7}{8}$ ;  $p(3/2) = \frac{1}{2} + \frac{3}{2} - \frac{1}{2} \cdot \frac{9}{4} = \frac{7}{8}$ ;  
 $q(1/2) = \frac{7}{4} \cdot \frac{1}{2} - \frac{3}{4} \cdot \frac{1}{4} = \frac{14}{16} - \frac{3}{16} = \frac{11}{16}$ ;  $q(3/2) = \frac{7}{4} \cdot \frac{3}{2} - \frac{3}{4} \cdot \frac{9}{4} = \frac{42}{16} - \frac{27}{16} = \frac{15}{16}$   
 $P_{1/2} = (7/8, 11/16)$   $P_{3/2} = (7/8, 15/16)$

