CSCC51H

<u>Mid-term Examination</u>

March 2, 2011 Scarborough Campus

Time: 50 minutes.

Answer both questions showing all work in the examination booklet.

The questions are equally weighted.

Aids allowed: Approved calculator and one 8.5" x 11" page of handwritten notes.

1. (Hermite Interpolation and Numerical Integration):
   The following tabulated values of $f(x)$ and $f'(x)$ are given.

x_i	$f(x_i)$	$f'(x_i)$
0.00	0.000	1.000
0.50	0.479	0.878
1.00	0.841	0.540

   (a) Determine the Hermite cubic interpolant, $p_3(x)$, associated with the two data points $x_0 = 0$ and $x_2 = 1$.

   (b) You are to determine two approximations to $\int_0^1 f(x)dx$, where $f(x)$ is the tabulated function. The first approximation is $A_1 = \int_0^1 p_3(x)dx$, where $p_3(x)$ is the Hermite cubic you determined in part 1. The second approximation is $A_2 = S(f)$, where $S(f)$ is the approximation that corresponds to applying Simpsons Rule to approximate this integral.

      i. Compute the values of A_1 and A_2.

      ii. For each of these approximations justify and determine a bound on the magnitude of the error based on the assumption that $|f^i| \leq 1$ for $i \geq 1$.

2. (Parametric Interpolation):
   Consider the problem of interpolating the data in the following table that tabulates the x and y coordinates of points on a curve in two dimensions. The data in the table are problematic because when they are considered as being in the form $y = f(x)$, $f(x)$ is not unique for some x. This problem also exists if the data are considered to be $x = g(y)$.

i	x_i	y_i
0	1/2	0
1	1	1
2	1/2	1/2

One solution to the uniqueness difficulty is to use a *parametric* interpolant. Instead of determining an interpolant of the form $y = f(x)$, a parametric interpolant has the form $x = p(t)$ and $y = q(t)$, where t is a parameter of the curve and $p(t)$ and $q(t)$ are interpolating polynomials. It is often convenient to let the new independent variable t represent the index, i, of a data point in the table. This is not the only possible choice. (For example, in your second assignment you let t be an approximation to the arc length of the curve.) All that is required is that t be strictly monotonically increasing along the curve being interpolated. (That is, the (x_i, y_i) $i = 0, 1, 2$ are ordered tracing out the curve with the initial point (x_0, y_0) and terminal point (x_2, y_2).)

(a) Construct a parametric polynomial interpolant for the given table of data using the index as the parameter t. (That is, determine the quadratic interpolants $p(t)$, $q(t)$, where $p(t)$ interpolates the x-coordinate and $q(t)$ interpolates the y-coordinate of the curve.)

(b) Verify that your parametric polynomial interpolates the given data set.

(c) Evaluate the parametric polynomial interpolant at the additional points, $t = 1/2$ and $t = 3/2$ and sketch your results.