1. The Corrected Trapezoidal Rule: \( CT \)
   Consider the corrected trapezoidal quadrature rule defined by the formula,
   \[
   CT \equiv \frac{(b-a)}{2} (f(a) + f(b)) + \frac{(b-a)^2}{12} (f'(a) - f'(b)),
   \]
   with an associated error expression,
   \[
   I(f) - CT \equiv E_{CT} = \frac{(b-a)^5}{720} f^{(4)}(\eta) \text{ for some } \eta \in (a,b).
   \]
   (a) Derive a formula for the corresponding composite corrected trapezoidal rule, \( CT_N \) and the corresponding exact error expression, \( E_{CT}^N \).
   (b) Derive a suitable error estimate for this formula that is valid for the case of \( N \) equal width subintervals.
   (c) Write a Matlab script which will implement this technique and produce a table of approximations, estimates, and ratios for the special case of constant interval widths (\( h_i = (b-a)/N, \ N = 2^k, \ k = 1, 2, \ldots 8 \)). Test this script out on a few problems of your own choice and comment on the order of accuracy of the results. Include in your test problems an integrand, \( f(x) \), that is periodic with a period of \( (b-a) \) and discuss the accuracy obtained on such problems.

2. An Implicit Quadrature Problem:
   Using a modified form of the Matlab script you developed to answer the first question, derive and implement in Matlab a method that, when given \( f(x) \), and an interval \([a,b]\), will return with an approximation to the first value of \( \bar{x} > a \) such that,
   \[
   \int_a^{\bar{x}} f(x)dx = 0.
   \]
   Your method should return an error flag if there is no such \( \bar{x} \leq b \). You should include a discussion of your algorithm and suitable documentation of the Matlab implementation. Test your algorithm on a few carefully chosen test problems. (Note that, in grading your solution to this question, the efficiency of your method as well as your testing and documentation will be considered.)