CSCC51H

Assignment 3

Feb. 27, 2012 Scarborough Campus **Due:** March 12, 2012

1. The Corrected Trapezoidal Rule: CT
   Consider the corrected trapezoidal quadrature rule defined by the formula,

$$CT \equiv \frac{(b-a)}{2}(f(a)+f(b)) \;+\; \frac{(b-a)^2}{12}(f'(a)-f'(b)),$$

   with an associated error expression,

$$I(f) - CT \equiv E^{CT} = \frac{(b-a)^5}{720} f^{(4)}(\eta) \text{ for some } \eta \in (a,b).$$

   (a) Derive a formula for the corresponding composite corrected trapezoidal rule, CT_N and the corresponding exact error expression, E_N^{CT}.

   (b) Derive a suitable error estimate for this formula that is valid for the case of N equal width subintervals.

   (c) Write a Matlab script which will implement this technique and produce a table of approximations, estimates, and ratios for the special case of constant interval widths ($h_i = (b-a)/N, \quad N = 2^k, \quad k = 1, 2...8$). Test this script out on a few problems of your own choice and comment on the order of accuracy of the results. Include in your test problems an integrand, $f(x)$, that is periodic with a period of $(b-a)$ and discuss the accuracy obtained on such problems.

2. An Implicit Quadrature Problem:
   Using a modified form of the Matlab script you developed to answer the first question, derive and implement in Matlab a method that, when given $f(x)$, and an interval $[a,b]$, will return with an approximation to the first value of $\bar{x} > a$ such that,

$$\int_a^{\bar{x}} f(x)dx = 0.$$

   Your method should return an error flag if there is no such $\bar{x} \leq b$. You should include a discussion of your algorithm and suitable documentation of the Matlab implementation. Test your algorithm on a few carefully chosen test problems. (Note that, in grading your solution to this question, the efficiency of your method as well as your testing and documentation will be considered.)