VARSAT: Integrating Novel Probabilistic Inference Techniques with DPLL Search

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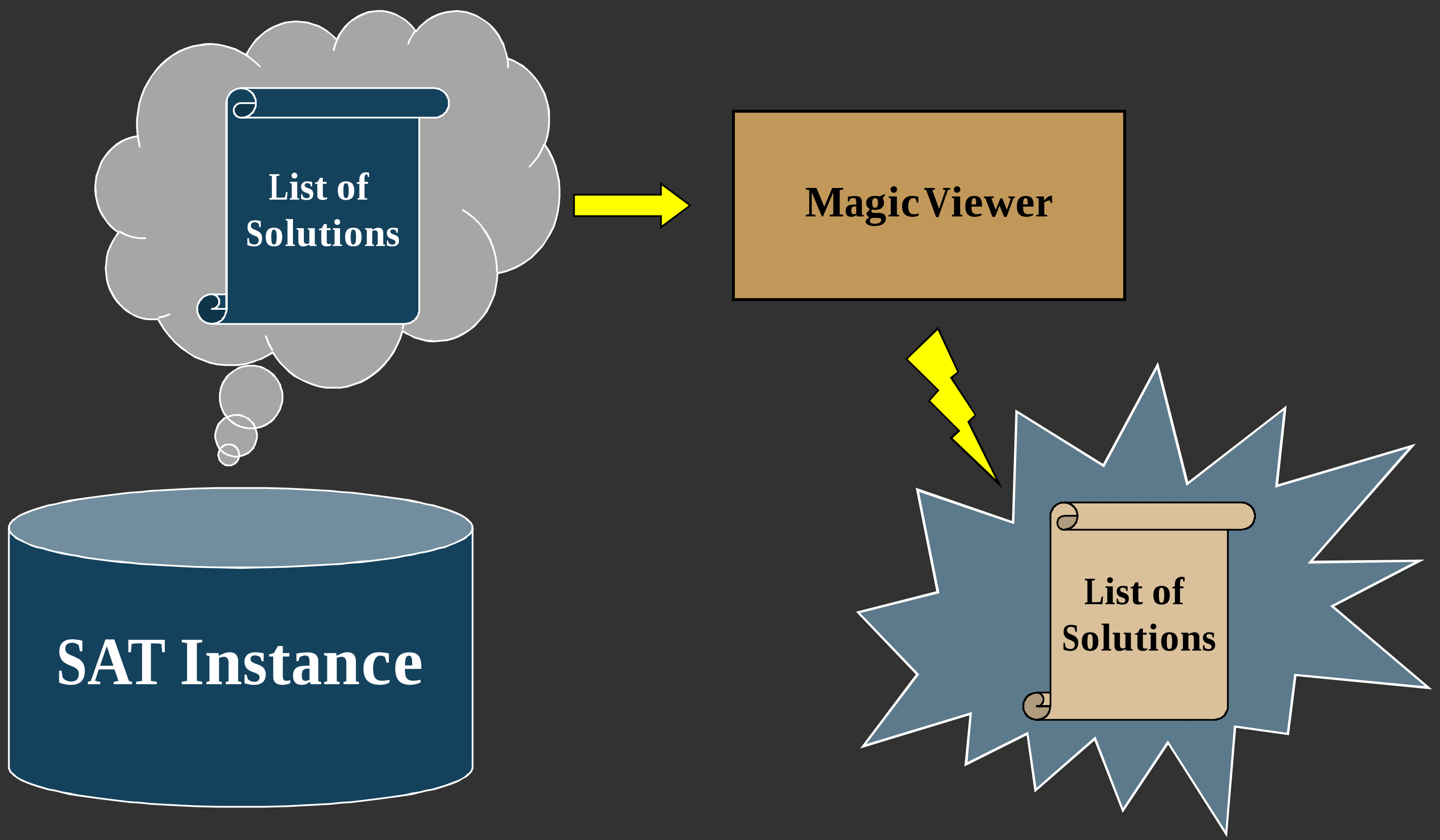
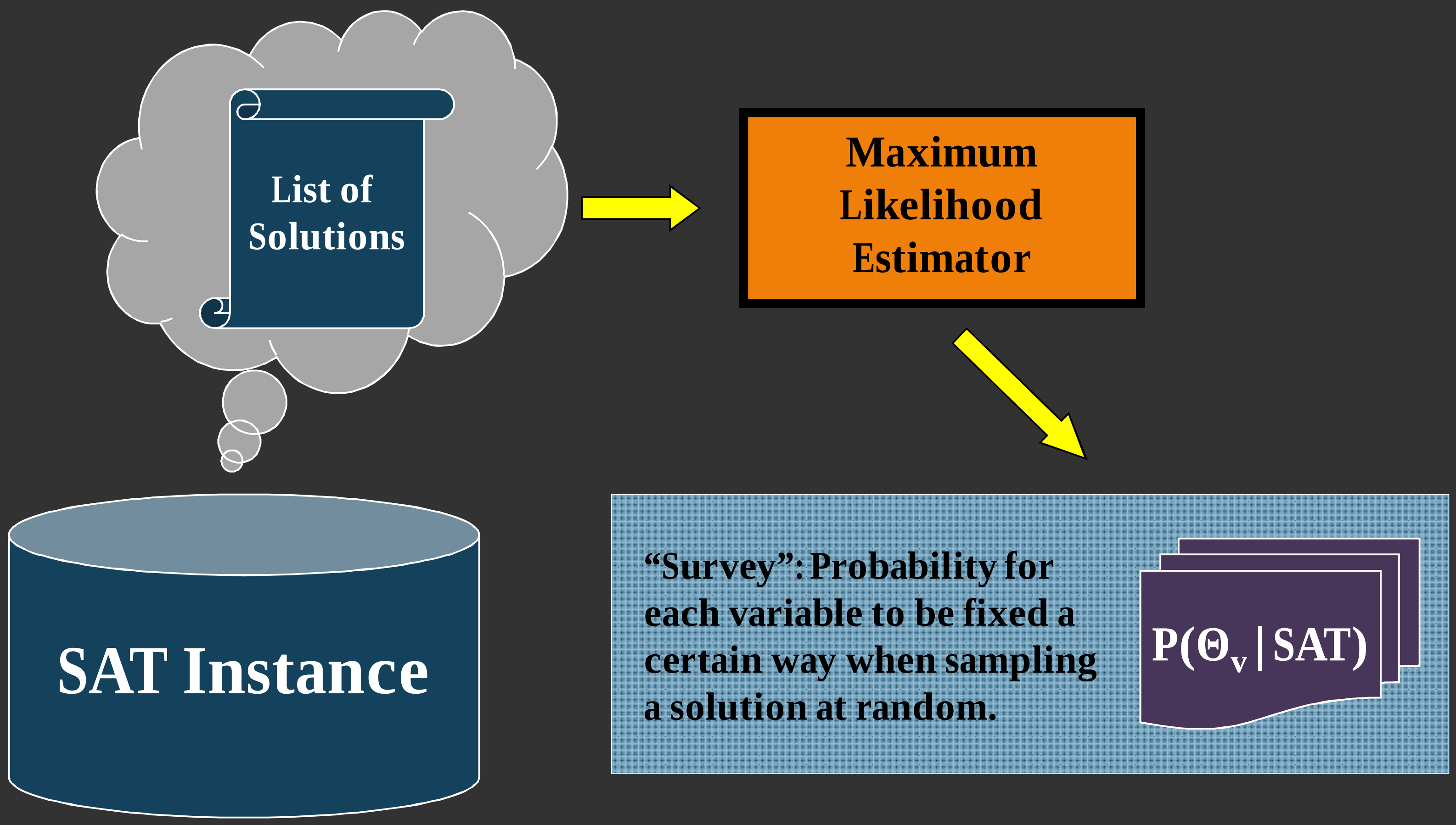
Wishful Thinking



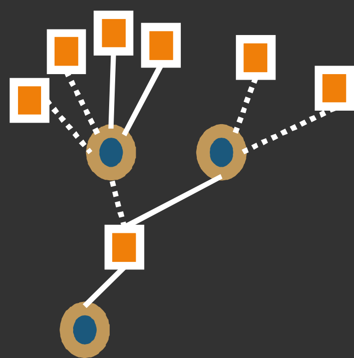
Instead: Sampling from the Solution Space



Outline

1. Message-Passing Frameworks for Computing Surveys
2. Interpreting the Four EMBP-Based Update Rules
3. Stand-Alone Performance of Bias Estimators
4. Using the Bias Estimators to Solve Problems
5. Embedded Performance with DPLL Search
6. Improving Performance and Future Work

1. Message-Passing Framework for Computing Surveys



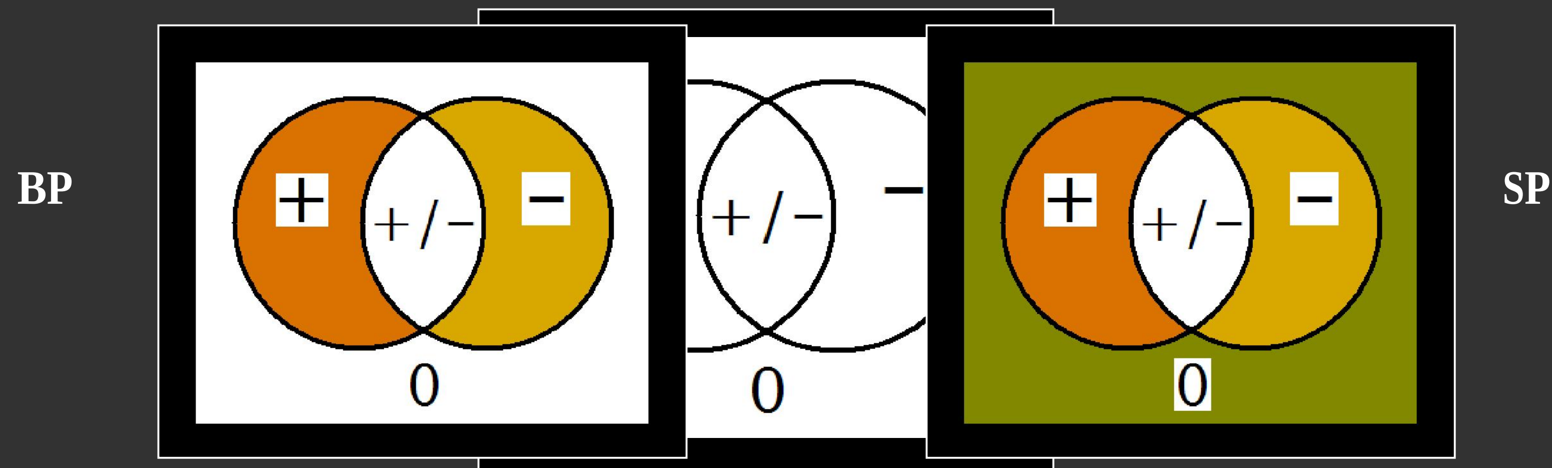
Encoding a (Uniform) Probability Distribution over Solutions

$$f_c(v_1, \ldots, v_n) = \begin{cases} 1, & \text{if } v_1, \ldots v_n \text{ satisfies constraint } c; \\ 0 & \text{otherwise.} \end{cases}$$

$$P(v_1, \ldots, v_n) = (1/Z) \prod_c f_c(v_1, \ldots, v_n)$$

$$P(v_1 = \text{'}+\text{'}) = (1/Z_1) \sum_{V: v_1 = v} \prod_c f_c(+, \ldots, v_n)$$

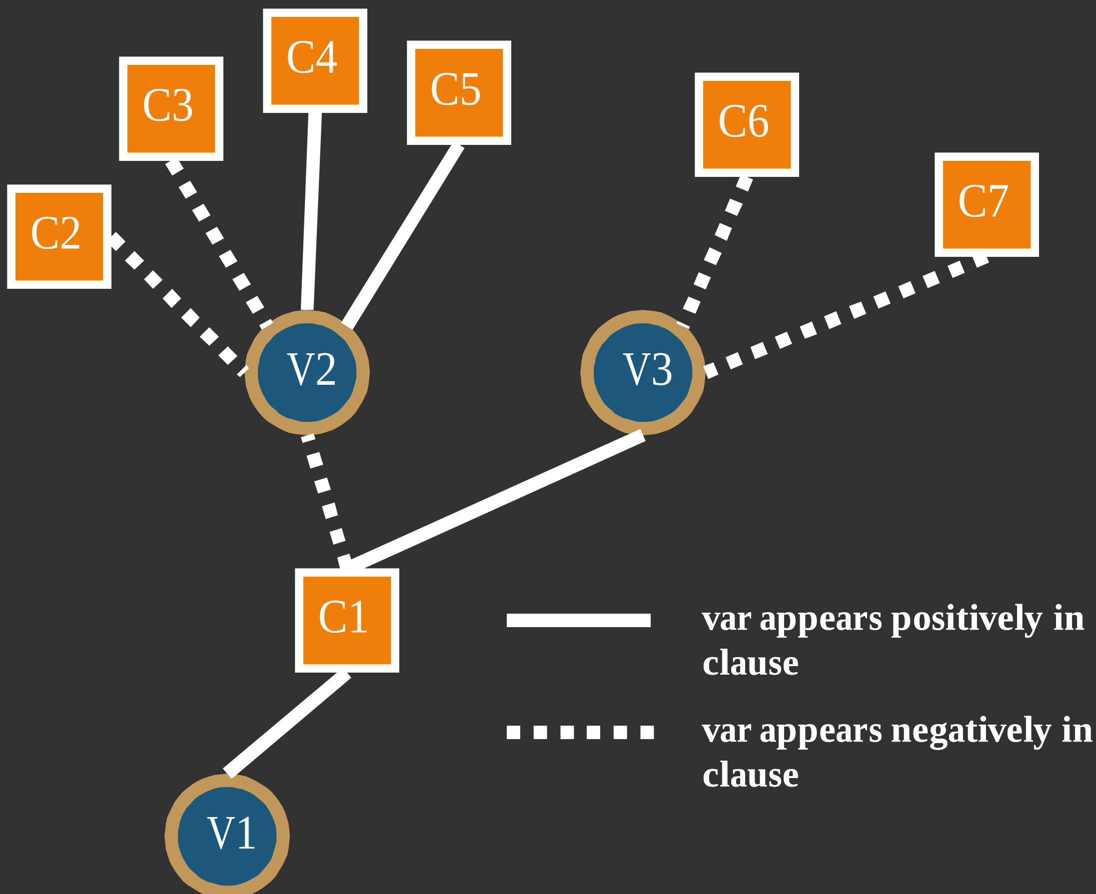
Probability Space for a Variable: "Bias Distribution" $\Theta_v()$



- Θ_v^+: variable is *constrained* to be positive.
- Θ_v^-: variable is *constrained* to be negative.
- Θ_v^*: variable is unconstrained.

BP (first two) — SP (all three)

Estimating Biases via Propagation

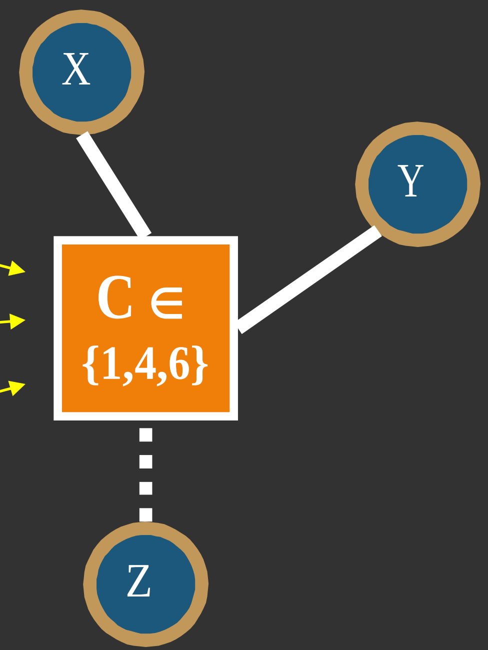


- Factor Graph Representation for Message-Passing
  - Variable->Clause: Likely Bias
  - Clause->Variable: Likely Need for Support
- Probabilities reflect status *in absence of recipient*.

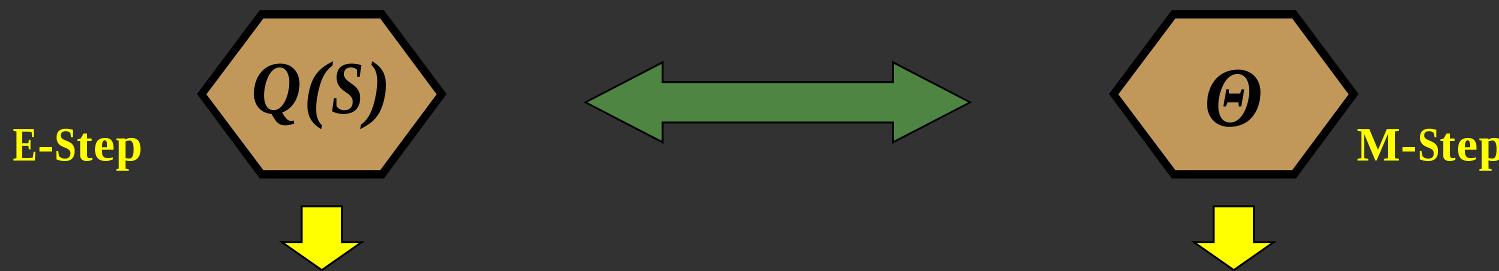
EMBP: The Dual (Extensional) Problem

- Suppose clause $c = x \vee y \vee \neg z$.
- Support profile represents which of the possible instantiations to x, y, and z was chosen to satisfy clause c.

S	X	Y	Z	
1.	**0**	**0**	**0**	**valid BP/SP**
2.	0	0	1	invalid
3.	0	1	0	SP only
4.	**0**	**1**	**1**	**valid BP/SP**
5.	1	0	0	SP only
6.	**1**	**0**	**1**	**valid BP/SP**
7.	1	1	0	SP only
8.	1	1	1	SP only



EMBP Dynamics



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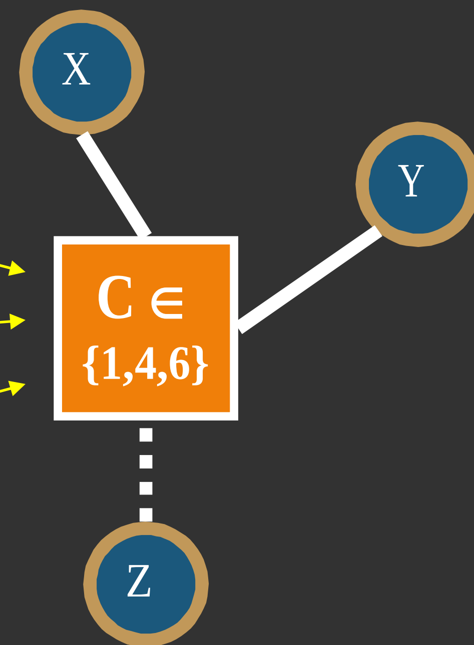


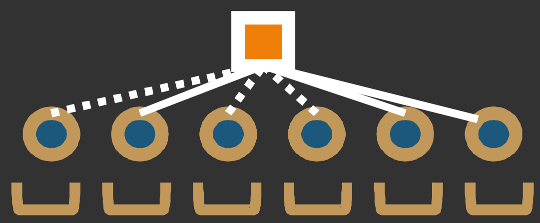
Update Rule Format

1. Randomly initialize variables' bias distributions.
2. Iterate through the variables:
   - Update current variable's bias according to an update formula based on other variables' biases:
     - Set positive, negative, and joker weights w+, w-, w*.
     - Normalize to form proper biases:

$$\theta_v^{+\prime} \leftarrow \frac{\omega_v^+}{\omega_v^+ + \omega_v^- + \omega_v^*} \qquad \theta_v^{-\prime} \leftarrow \frac{\omega_v^-}{\omega_v^+ + \omega_v^- + \omega_v^*} \qquad \theta_v^{*\prime} \leftarrow \frac{\omega_v^*}{\omega_v^+ + \omega_v^- + \omega_v^*}$$

   - Repeat until convergence (guaranteed).

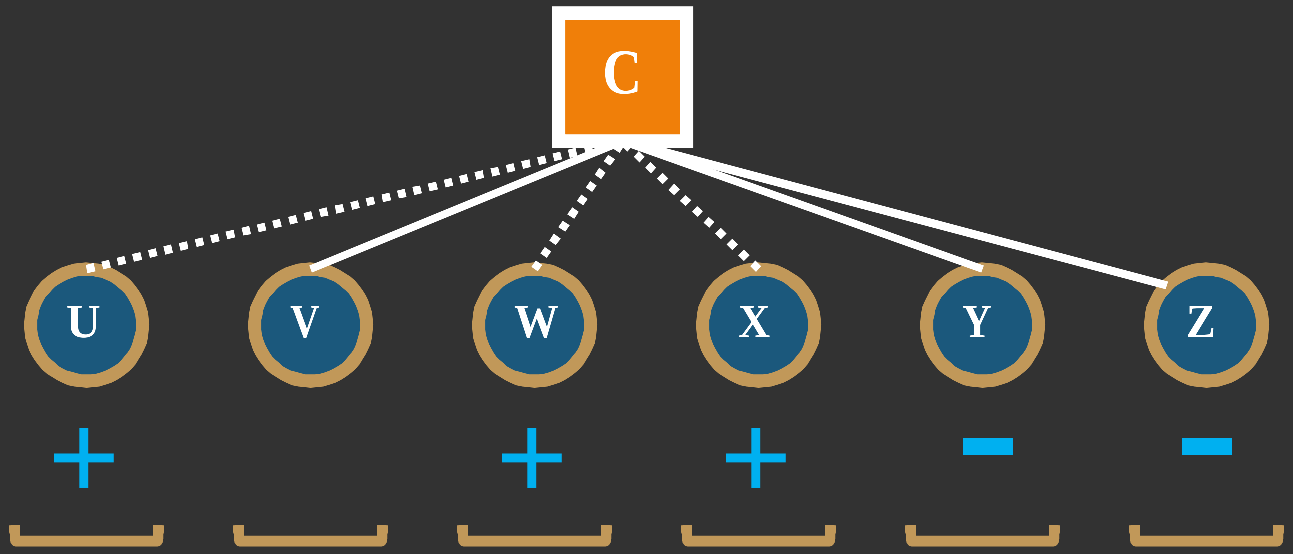
2. Interpreting the Four EMBP-Based Update Rules



Basic Building Block: Sole Support

$$\sigma(v,c) \triangleq \prod_{i \in V_c^+ \setminus \{v\}} \theta_i^- \prod_{j \in V_c^- \setminus \{v\}} \theta_j^+$$

"Variable *v* is the *sole support* of constraint *c*."



EMBP-L / EMSP-L

$$\sum_{c \in C_v^+} [1] + \sum_{c \in C_v^-} [1 - \sigma(v,c)]$$

$$\omega_v^+ = |C_v| - \sum_{c \in C_v^-} \sigma(v,c)$$

$$\omega_v^- = |C_v| - \sum_{c \in C_v^+} \sigma(v,c)$$

$$\omega_v^* = |C_v| - \sum_{c \in C_v} \sigma(v,c)$$

Local Approximation

EMBP-G / EMSP-G

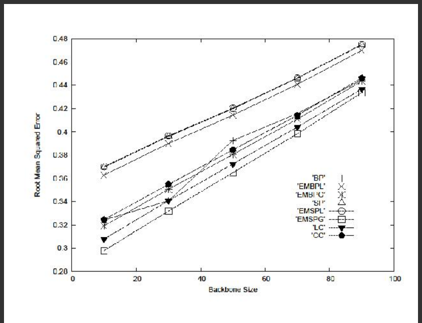
$$\sum_{c \in C_v^+} [1] + \sum_{c \in C_v^-} \left[\prod_{c \in C_v^-} (1 - \sigma(v,c)) \right]$$

$$\omega_v^+ = |C_v^-| \left[\prod_{c \in C_v^-} (1 - \sigma(v,c)) \right] + |C_v^+|$$

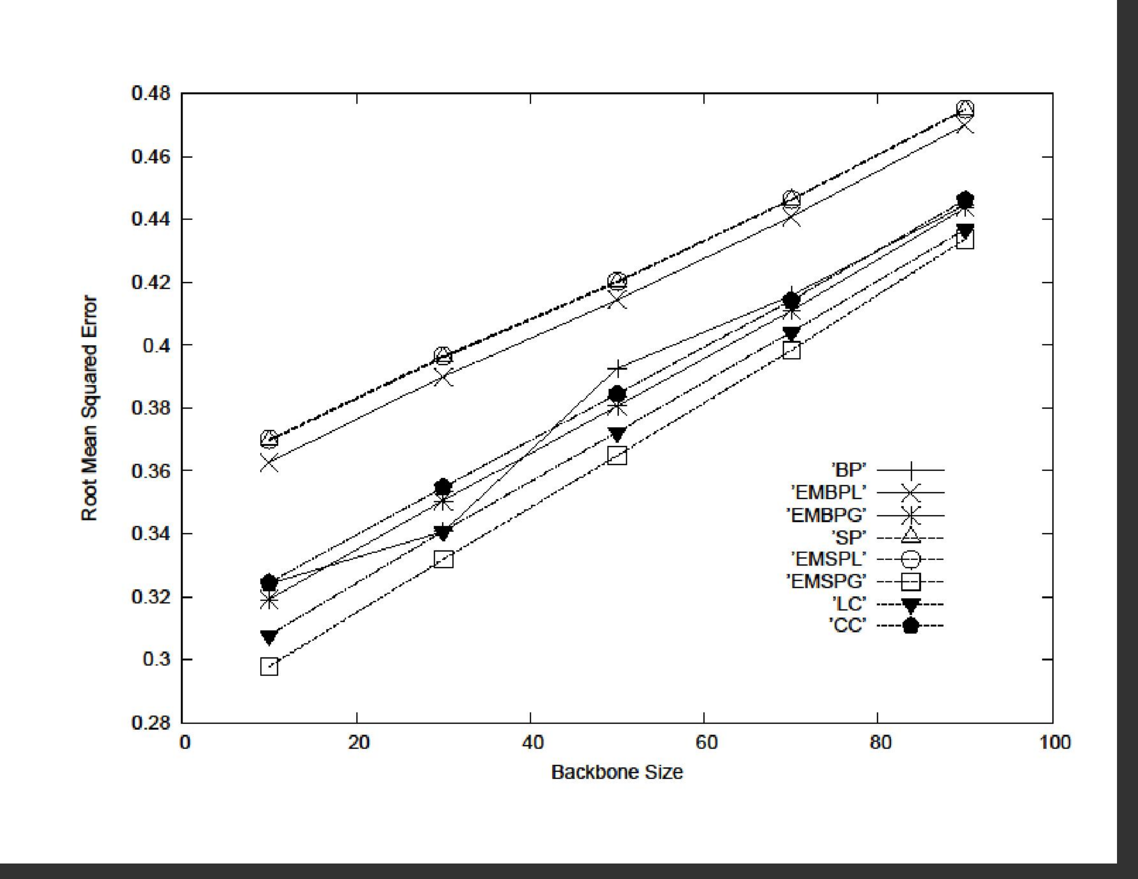
$$\omega_v^- = |C_v^+| \left[\prod_{c \in C_v^+} (1 - \sigma(v,c)) \right] + |C_v^-|$$

Globalized Approximation

3. Stand-Alone Performance of Bias Estimators

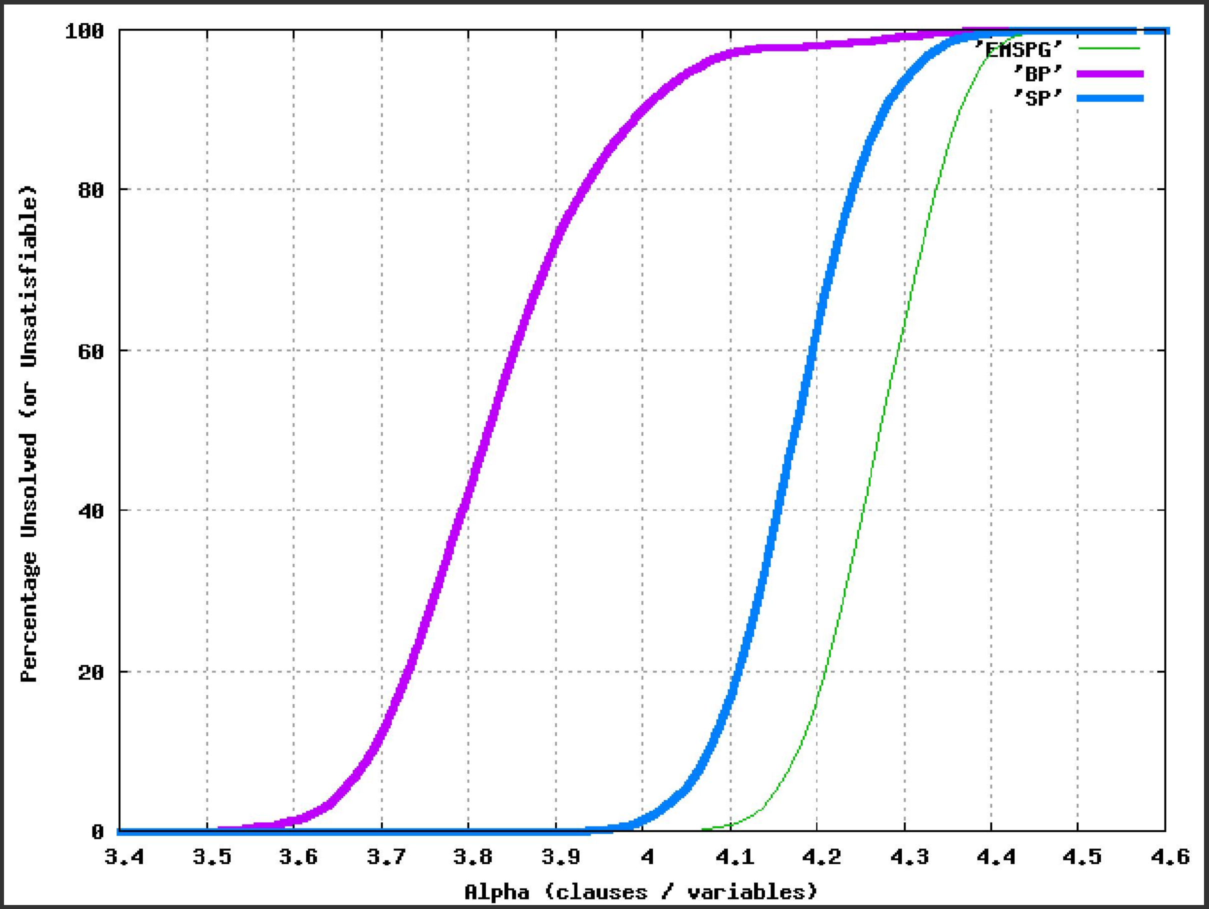


Outperforms SP in Accuracy...

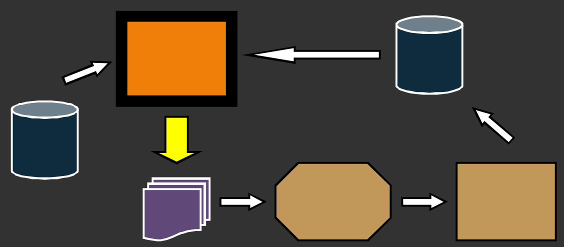


(Experiment conducted with Christian Muise)

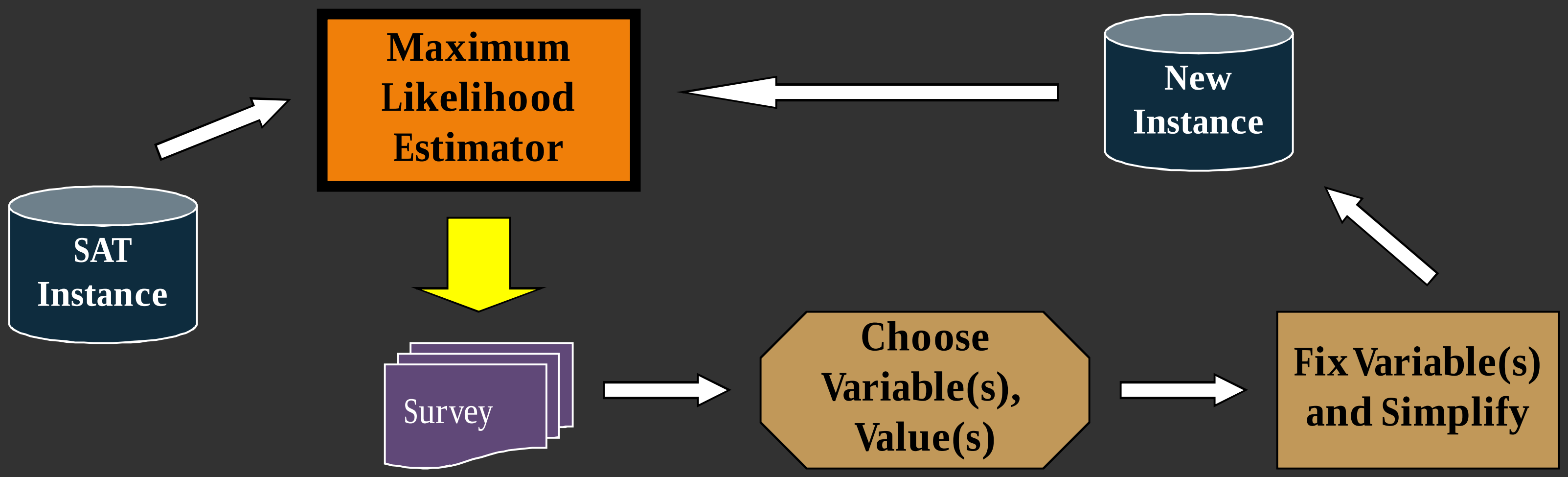
...and as a Non-Backtracking Solver



4. Using the Bias Estimators to Solve Problems

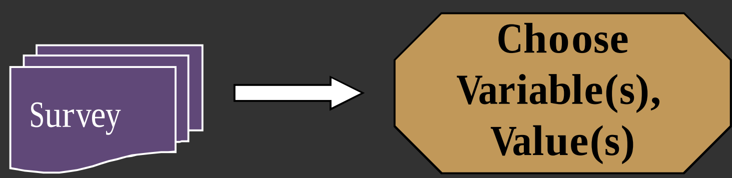


Integrating with DPLL

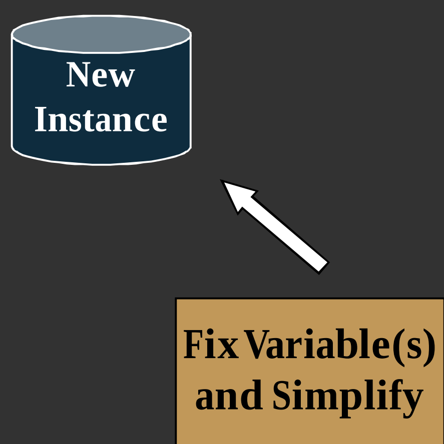


- Variable / Value-Ordering heuristic within a DPLL search
  - Pick a variable to instantiate next, by bias strength
  - Set that variable according to its stronger bias
- Revert to standard search heuristic when bias levels off

Design Issues / Parameters



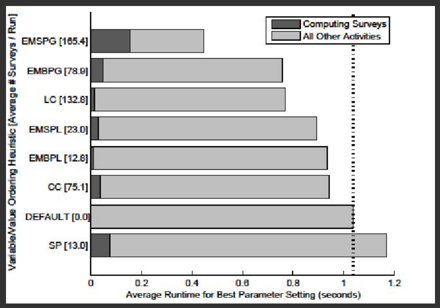
- Decimation Block Size
- Branching Rule
- Selection Criterion



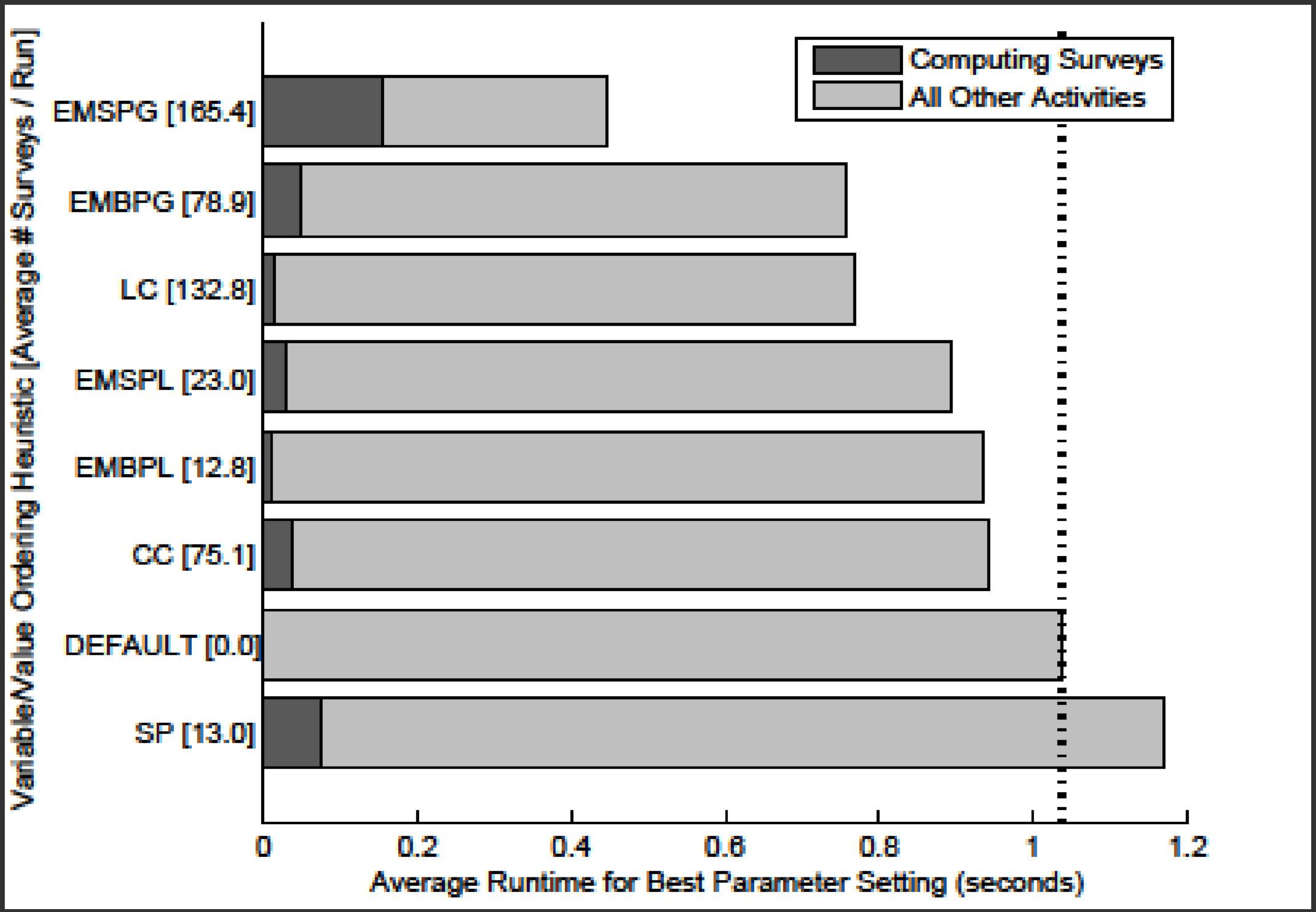
- Deactivation Threshold
- Working with Learned Clauses

5. Embedded Performance within DPLL Search

"VARSAT"



Solving Small Random Problems



Larger / Industrial Problems

- Implementation: Memory management is key issue.
- Toughest type of problem set: SAT-Race 2008.
- Comparison with default MiniSat:
  - 131 problems were solved by both.
    - Minisat Mean Runtime / STD:
      111 seconds / 154 seconds
    - VARSAT Mean Runtime / STD:
      147 seconds / 202 seconds
  - 9 problems solved by only Minisat.
  - 5 problems solved by only VARSAT.

6. Improving Performance, and Future Work

Lots of Room for Improvement

- Parameters and Design
- Algorithmic Shortcuts
- Implementational Shortcuts
- Interaction with Rest of System: Restarts, Branching, Clause Learning
- Future Opportunities:
  - MAXSAT, WSAT
  - Non-Clausal Solving!

General Conclusions

- Formally,
  - Applying EMBP with global constraints significantly improves BP / SP.
- Algorithmically,
  - Provides very valuable information (for preventing / seeking conflicts.)
- Practically,
  - Information is very expensive (in time / space.)

Thanks!

Contributions from Sheila McIlraith, Christian Muise, Fahiem Bacchus, Matthew Kitching, Chris Beck.