# Electric Charge, Force, Field

September-13-11 9:03 AM

## Charging by Conduction

Charge is transferred by contact

## Charging by Induction

Charge separation is induced by nearby charge, then target is connected to ground where it becomes charged. When ground is disconnected net charge remains.

## Coulomb's Law

$$\vec{F}_{12} = \frac{kq_1q_2}{r_{12}^2}\hat{r}_{12}$$

$$k = 8.99 \times 10^9 \frac{Nm^2}{C^2}$$

Coulomb, quite large: $6 \times 10^{18}\ protons$

## Coulomb's Law:

### (1776-1806)

- Like charges repel, opposite charges attract
- $F \propto q_1q_2, F \propto \frac{1}{r_{12}^2}$

$$\vec{F}_{12} = \frac{kq_1q_2}{r_{12}^2}\hat{r}_{12}$$

$\hat{r}_{12} = \text{unit vector from 1 to 2} = \frac{\vec{\mathrm{r}}_{12}}{\|\vec{\mathrm{r}}_{12}\|}$

$k = \text{Coulomb's constant}$

$\vec{F}_{12} = \text{Force on 2 due to 1}$

$F_E$ is a central force, that is $|F_E| = F(r)$

Extends to multiple points, example for 4 points

$$\vec{F}_3 = \sum_{j \neq 3} \vec{F}_{j3}$$

# The Electric Field

September-20-11 8:35 AM

## Coordinate system

Assume Right-Handed, that is $\hat{k}$ comes out of the page.

## Electric Dipole

$$\vec{F} = q\vec{E} \Rightarrow \vec{E} = \frac{\vec{F}}{q} = \frac{kQ}{r^2}\hat{r}$$

In a field, have source charge Q, and field point q
$\hat{r}$ is always from the source point to the field point

Visualize the $\vec{E}$ field by means of electric field lines. Electric field lines always start on a positive charge and end on a negative charge, or go to infinity.
For several charges use superposition.

Example. Consider an electric dipole, $+q\ and\ -q$ separated by a fixed distance $2a$ lying along x centered on the origin. Find $\vec{E}$ along the $\vec{y}$-axis

$$\vec{E}_p = 2E_{1x}(-\hat{\imath}) = 2E_2\cos\theta\,(-\hat{\imath}) = \frac{2kq}{a^2+y^2}\times\frac{a}{\sqrt{a^2+y^2}}(-\hat{\imath}) = -\frac{2kqa}{(a^2+y^2)^{\frac{3}{2}}}\hat{\imath}$$

$p = 2aq = dipole\ moment$

$$\vec{E}_p = -\frac{kp}{(a^2+y^2)^{\frac{3}{2}}}\hat{\imath}$$

For $y \gg a$

$$\vec{E}_p \approx \frac{2kqa}{y^3}\hat{\imath}$$

## Electric Dipole in Uniform E Field

$\vec{\tau}_0 = 2aqE\sin\theta - \hat{k} = \vec{p}x\vec{E}$

## Uniform Charge Distribution

$d\vec{E} = \frac{k(dq)}{r^2}\hat{r}, \vec{E}_P = \int d\vec{E}$

Note that $dq$ could be a function of r
Will consider simple cases
Charge density?

$\rho \equiv volume\ charge\ density$
$\quad = \frac{Q}{V}\left[\frac{C}{m^3}\right]$
$dq = \rho dV$

$\sigma \equiv area\ charge\ density$
$\quad = \frac{Q}{A}\left[\frac{C}{m^2}\right]\quad dq = \sigma dA$
$\lambda \equiv linear\ charge\ density$
$\quad = \frac{Q}{l}\left[\frac{C}{m}\right]\quad dq = \lambda dl$

On exams, won't consider $\rho = \rho(x, y, z)$

# Uniform Electric Field

September-27-11 8:40 AM

### Electric field of disk

$\vec{E} = 2\pi k\sigma\left(1 - \frac{x}{\sqrt{x^2 + R^2}}\right)\hat{\imath}$

As $R \to \infty$ $(or\ x \ll R), \vec{E} \to 2\pi k\sigma$

$E = \frac{\sigma}{2\epsilon_0}$

### Coulomb's Constant

$k = \frac{1}{4\pi\epsilon_0}$

$\epsilon_0$ ≡permittivity of free space

$= 8.85 \times 10^{-12} \frac{C^2}{Nm^2}$

### Parallel Plate Capacitor

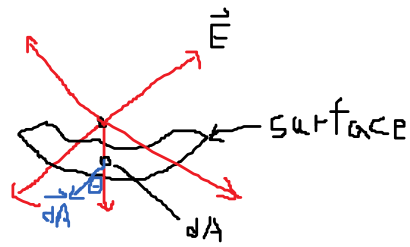
Two parallel plates of opposite charge +Q, -Q, plate dimensions >> separation

$\sigma = \pm\frac{Q}{A}$

Outside the plates, the electric fields produced by both plates are equal in magnitude and opposite in direction $(\sigma/2\epsilon_0)$. Within the plates, the electric fields add, to form a uniform electric field of
$E = \frac{\sigma}{\epsilon_0}$

### Gauss' Law:

### Electric Flux




$d\Phi = \vec{E} \cdot dA$

Flux, represents # of electric field lines passing through an area

$\Phi = \int_A \vec{E} \cdot d\vec{A}$

In problems involving Gauss' Law we are interested in finding Φ through a closed surface. Then
$\Phi = \oint \vec{E} \cdot d\vec{A} = \oint E_n dA\ (E_n is\ component \perp to\ surface)$

Not that $d\vec{A}$ is always taken to point outward.

- For $\vec{E}$ lines entering, flux is negative
- For $\vec{E}$ lines exiting, flux is positive.

Take sphere about a charge. Gaussian surface
$\Phi_{GS} = \oint E \cdot dA = \oint |E| \cdot |dA| = \left(\frac{Q}{4\pi\epsilon_0 r^2}\right)(4\pi r^2) = \frac{Q}{\epsilon_0}$

# Gauss' Law

September-29-11 8:50 AM

## Gaussian Surface

Any closed surface about some charge distribution with total charge Q has flux

$\Phi_{GS} = \frac{Q}{\epsilon_0}$

Can use Gaussian Surface flux to determine E field under special circumstances

## Spherical Symmetry

Insulator. Charge Q

Consider inside of sphere. By symmetry the E field must point along or against the radial vector.

Imagine spherical shell inside charged sphere

$\Phi_{GS} = \oint_{GS} \vec{E} \cdot d\vec{A}$

Have $\vec{E} \parallel d\vec{A}, \cos\theta = \pm 1,$

$\Phi_{GS} = \frac{q_{int}}{\epsilon_0} = 0 = \vec{E} \oint_{GS} d\vec{A} = \vec{E} 4\pi r^2 \Rightarrow \vec{E} = 0$

Outside

$|\vec{E}| 4\pi r^2 = \frac{q_{int}}{\epsilon_0} = \frac{Q}{\epsilon_0}$

$|\vec{E}| = \frac{Q}{4\pi\epsilon_0 r^2}$

As for point charge

## Line Symmetry

Find the electric field at point p a distance r from an infinitely long, thin, uniformly charged rod with linear charge density.

Draw a cylindrical Gaussian surface and consider a point on its outside surface (not caps)

Exercise: work this out

$\vec{E} = \frac{\lambda}{2\pi\epsilon_{0r}} \hat{r}$

The same answer applies equally to lines and cylindrical charge distributions.

## Plane Symmetry

Find the electric field above and below a non-conducting, infinite plane sheet of charge with uniform positive charge per unit area $\sigma$

Consider a cylinder perpendicular to the plane.

$\Phi = \frac{q_{enc}}{\epsilon_0} = 2EA_0$

$E = \frac{\sigma A_0}{2\epsilon_0 A_0} = \frac{\sigma}{2\epsilon_0}$

## Conductors in Electrostatic Equilibrium

The electric field inside must be 0.

E on outside of the surface must be perpendicular to surface.

Charges accumulate more on sharper points where the radius of curvature is the largest.

# Work and Potential

October-06-11 8:36 AM

## Work

$W = -\int \vec{F}_E \cdot d\vec{s}$

For force in direction of movement

$W = -\int_i^f |\vec{F}_E||d\vec{r}| = -\int_i^f \frac{kq_0Q}{r^2} = -kq_0Q\left(\frac{1}{r_f} - \frac{1}{r_i}\right)$

### Conservative Force

$\oint \vec{F}_{ext} \cdot dr^{\rightarrow} = 0 = -\oint \vec{F}_E d\vec{r} = -q \oint \vec{E} \cdot d\vec{r}$

$\oint \vec{E} \cdot d\vec{r} = 0$

## Potential Energy

Only $\Delta U$ has physical meaning, not U

Could take any reference point in the field

Take reference point at infinity for example

$\Delta U = U_f - U_i = U_f$

$(U_i\ at\ infinity\ is\ usually\ 0)$

$U(r) = \frac{kQ_1q_0}{r}$

## Electric Potential

$V = \frac{U}{q_0}$

$V(r) = \frac{kQ_1}{r}$

$\Delta V = \frac{U_B - U_A}{q_0} = V_B - V_A = \Delta_{BA} = -\int_A^B \vec{E} \cdot \mathrm{d}\vec{r}$

Infinity as a reference point

$V(r) = -\int_\infty^r \vec{E} \cdot d\vec{r}$

### Electron Volt

$1eV = 1.6 \times 10^{-19} J$

## Potential Function of Capacitor

Plates have charge density $+\sigma, -\sigma$ and separation $d$

Want to find $V(x)$ where $x = 0$ at one plate, and $v = d$ at the other.

$\vec{E} = \frac{\sigma}{\epsilon_0}\hat{\imath}$

$V(x) = -\int \vec{E} \cdot d\vec{s}$

$V_p - V_0 = -\int_0^p \vec{E} \cdot d\vec{x} = -Ex = -\frac{\sigma}{\epsilon_0}x$

Set $V(0) = 0$

$V(p = (x, y)) = -\frac{\sigma}{\epsilon_0}x$

$\Delta V = -\frac{\sigma}{\epsilon_0 d} = -Ed \Rightarrow E = \frac{\Delta V}{d}$

## Potential to Electric Field

$\Delta V = -\int \vec{E} \cdot d\vec{s}$

$dV = -\vec{E} \cdot d\vec{s}$

$|\vec{E}| = -\frac{dV}{|d\vec{s}|}\cos\theta$

$dV = -E_s|d\vec{s}|$ , $E_s$ is component parallel to $d\vec{s}$

$E_s = -\frac{dV}{ds}, \vec{E}_s = E_s\hat{s}$

E is the -ve gradient of V

$\vec{E} = -\vec{\nabla}V = -\left(\frac{\delta V}{\delta x}\hat{x} + \frac{\delta V}{\delta y}\hat{y} + \frac{\delta V}{\delta z}\hat{z}\right)$

# Capacitance

October-27-11 8:41 AM

## Capacitance

$Q = CV \Rightarrow C = \frac{Q}{V}$

## Parallel Plate Capacitor

Charge $Q$ on plate with area $A$, charge density $\sigma = \frac{Q}{A}$

Plate separation d

$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{A\epsilon_0}$

$V = Ed = \frac{Qd}{\epsilon_0 A} \Rightarrow Q = \left(\frac{A\epsilon_0}{d}\right)V$

$C = \epsilon_0 \frac{A}{d}$

## Energy in Capacitors

Consider moving a charge $dQ$ from the negative to the positive plate.

$dW = VdQ, dQ = CdV \Rightarrow dW = CVdV$

$U = W = \int dW = \int CVdV = \frac{1}{2}CV^2$

## Dielectric

Insulators can have differing electrical permittivities (dipoles in the substance align with the electric field)

$\kappa = \frac{\epsilon}{\epsilon_0} \Rightarrow \epsilon = \kappa\epsilon_0$

For parallel plate capacitor:

$C = \frac{\epsilon A}{d} = \kappa \frac{\epsilon_0 A}{d}$

## Capacitors in Circuits

### Parallel

$C = \sum_{i=1}^{n} C_i$

### Series

$\frac{1}{C} = \sum_{i=1}^{n} \frac{1}{C_i}$

# Electric Current

October-27-11 8:41 AM

$\Delta V$ appears across ends of wire

- $\vec{E}$ set up inside wire
- free $e^-$ accelerate against 'current' direction.

$a = \frac{F_E}{m_e} = \frac{eE}{m_e}$

Electron move around mostly random with small bias against electric field.

$u = \sim 10^6 \frac{m}{s}$ = average thermal velocity, function of temperature

$\lambda$ = mean free path = average distance between collisions

$\tau$ = mean free time (between collisions)

$\tau = \lambda/u$

$v_d$ = drift velocity = average velocity due to E

Just after a collision $v_i = 0$

Just before the next collision $v_f = a\tau$

$v_{avg} = \frac{v_i + v_f}{2} = \frac{0 + a\tau}{2} = \frac{eE}{2m}\tau$

We call this the drift velocity

$v_d = \frac{eE}{2m}\tau = \frac{e\lambda}{2mu}E$

## Current

### Instantaneous Current

$I = \frac{dq}{dt}\left[\frac{C}{s} = A\right]$

$I = qnv_dA$

Where n is the number of charge carriers in some volume.

### Current density

$\vec{J} = \frac{I}{A}\left[\frac{A}{m^2}\right]$

$\vec{J} = nqv_d$

## Resistance and Ohm's Law

$J \propto v_d$ and $v_d \propto E$

$\therefore J \propto E$

$\vec{J} = \frac{1}{\rho}\vec{E}$

where $\rho = \frac{|E|}{|J|}$ =resistivity of conductor $[Ohm \cdot Meter]$

$\frac{1}{\rho} = \sigma$ =conductivity of conductor

Useful to recast the above ideas in terms of I and $\Delta V$

$E = \frac{V_{ab}}{l} = \frac{V}{l}, J = \frac{I}{A}$

$\vec{E} = \rho\vec{J}$

$V = lE = l\rho J = \frac{\rho l}{A}I$

$Resistance: R = \frac{\rho l}{A}\ [\Omega]$

$V = IR$

$R \propto \rho, R \propto l, R \propto A^{-1}$

## Power

$dU = dQ\Delta V$

$\frac{dU}{dt} = \frac{dQ}{dt}\Delta V$

$\Rightarrow P = I\Delta V\ \left[\frac{J}{S} = W\right]$

## Electromotive Force (emf, Potential Difference)

A battery has internal resistance

a,b terminals of battery, r internal resistance, $\epsilon$ chemical emf

emf across terminals:

$\Delta V = \epsilon - IR$

## Kirchhoff's Rules

Label junction points in a circuit. The algebraic sum of current at each of those points is 0.

Algebraic sum of potential difference around each loop is 0.

# Magnetic Fields

November-08-11 8:59 AM

$$\overrightarrow{F_q} = q\vec{v} \times \vec{B}$$

$$[B] = \frac{N}{C \cdot \frac{m}{s}} = \frac{N}{A \times m} = T = \frac{Wb}{m^2}$$

$$T \coloneqq Tesla$$

$$W \coloneqq Weber$$