# CSC384

# Introduction to Artificial Intelligence : Heuristic Search

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# Heuristic Search ($A^*$)

Primary concerns in heuristic search:

- Completeness
- Optimality
- Time complexity
- Space complexity
- Practical computational effort

We guide the search for optimality and to reduce the computational load.

# Admissibility

Let $h^*(n)$ be the cost of an optimal path from $n$ to a goal node.

- A heuristic is admissible if $h(n) \leq h^*(n)$ for all nodes $n$.
- Assumes $h(g) = 0$ for goal nodes $g$
- Assumes $c(n_1 \rightarrow n_2) > 0$ for any adjacent nodes $n_1$ and $n_2$
- Intuition: will not miss promising paths
- $f(n) = g(n) + h(n) \leq g(n) + h^*(n)$

Inadmissibile heuristics can lead to suboptimal results with $A^*$

# Consistency

Needed to ensure that the *first* time we visit a node we have an optimal path to it.

- Is a stronger condition than $h(n) \leq h^*(n)$.
- A monotone/consistent heuristic satisfies the triangle inequality (for all nodes $n_1$,$n_2$): $h(n_1) \leq c(n_1 \rightarrow n_2) + h(n_2)$
- Note that there might be more than one transition (action) between $n_1$ and $n_2$, the inequality must hold for all of them.
- Monotonicity implies admissibility.

In practice we emphasize admissible heuristics. Note $h(n) = 0$ for all $n$ is a consistent heuristic (but not a great one).

# Relaxation Heurisitics

Typical problems enforce some restrictions on how actions are taken. Removing these provides admissible heuristics. Consider the following restrictions on the 8 puzzle:

- Tiles must be moved into blank space.
- Tiles must be moved into an adjacent space.

Relaxing these conditions gives us the manhattan distance and misplaced tiles heuristics in the case of the 8 puzzle.

# Relaxation Heurisitics: Misplaced Tiles ($h_1$)

Assume that any tile can be moved to any position.
(Relax both adjacency and blank space conditions).

- Simply need to move each tile to the final position
- $h(n)$ will therefore be the numbered of misplaced tiles
- Admissible as each misplaced tile must be moved

# Relaxation Heurisitics: Manhattan Distance ($h_2$)

Assume that any tiles can overlap.
(Relax the requirement to move tiles into the blank space).

- Must move each tile to its final position independently
- $h(n)$ will be the sum of the vertical and horizontal displacement for each tile
- Admissible as any solution to the original problem is a solution to the relaxed problem

Note that $h_1(n) \leq h_2(n) \leq h^*(n)$. The manhattan distance heuristic dominates the misplaced tiles heuristics.

# Pattern Database Heuristics

Can derive heuristics from solutions to subproblems

- Perform a search on the smaller subproblem, store the result
- The maximum cost over all stored subproblems is the heuristic value
- If the subproblems are disjoint we can add the costs

This method can be very effective when feasible. When well constructed this can outperform the manhattan distance heuristic for the 8 puzzle.

# Problems with $A^*$ search

$A^*$ with an admissible heuristic provides optimality.

- Has the same space complexity as BFS!

Can we do better?

# *IDA*$^*$: Iterative Deepening $A^*$ Search

Try to gain the memory benefits of iterative deepening search

- Cutoff by $f(n)$ value instead of depth
- Next iteration should use smallest higher valued $f(n)$ seen

Potentially problematic with many distinct $f(n)$ values

# Other memory saving searches

$MA^*$, $SMA^*$ (memory bounded $A^*$, simplified memory bounded $A^*$)
Try to reduce memory consumption

- Drop the worst leaf node in the open set (highest $f(n)$ value)
- Store additional data in the parent node of the dropped node so that it can be reconstructed

Time problems can occur if we need to frequently reconstruct dropped nodes.

# Limitations enforced on search so far

All searches so far have relied on the following properties:

- Observability
- Known Environment
- Determinism
- Discrete States