CSC384
Introduction to Artificial Intelligence: Knowledge Representation and Reasoning

October 9, 2014

Knowledge representation and reasoning

- The symbolic encoding and manipulation of knowledge
- Roughly chapters 7 through 10
  We will focus on 7 and 8 for now

- **Syntax:** A grammar specifying legal respresentations
- **Semantics:** Mapping from syntactic representations to theoretic assertions

Entailment:

$$\alpha \models \beta$$

- β follows logically from α
- For worlds in which α is true, β is true

$$KB \models f$$

- f follows from the facts in the knowledge base

Syntactic Derivation:

$$\alpha \vdash \beta$$

- α derives β
- Following the derivation rules we can obtain *beta* from α

$$KB \vdash f$$

- f can be proved from the knowledge base

Soundness and completeness:

- Soundness:
  $KB \vdash f \rightarrow KB \models f$
  If we can derive f under the knowledge base, then f is true.
- Completeness:
  $KB \models f \rightarrow KB \vdash f$
  If f is true under the knowledge base, then we can derive f.

Knowledge Representation

First order logic

- *Constants*: $\{$dog, cat, red, blue, 1, 2, 3$\}$
- *Functions*: mapping of variables to a single variable ex: father(a), house(number, streetname)
- *Predicates*: mapping of variables to true/false
  ex: ison(x,y), connected(a,b), happy(c)
- *Variables*: $\{$x, y$\}$
- *Connectives*: $\neg, \vee, \wedge, \Rightarrow$
- *Equality*: $=$
- *Quantifiers*: $\forall, \exists$

Knowledge Representation

First order logic

- *Term* : A variable or constant
- *Atom*: $P(t_1, ..., t_k)$
  where t_i is a term
  and P is a predicate

We can construct formulas recursively from atoms, connectives, and quantifiers. A set of formulas for a language can be called a *knoledge base* (KB).

Knowledge Representation

First order logic

Interpretations: A 4-tuple (D, Φ, Ψ, V)
(Also known as models)

- D: A domain of individuals
- Φ: mapping $D^k \rightarrow D$ for the functions
- Ψ: mapping $D^k \rightarrow \{T, F\}$ for the Predicates
- V: variable assignment function, used to deal with quantification

Given an interpretation we can ground a knowledge base, reducing it to propositional logic. Doing so may result in an exponential increase in size.

Given interpretation I:

$I \models f$

- formula f is true under I

$I \models KB$ (I satisfies or is a model of KB)

- $\forall f \in KB$, f is true under I

$KB \models f$

- formula f is true under *all* models of KB

Conjunctive Normal Form (CNF):

- *Literal* A variable or its negation
  Ex: $A, B, \neg C, D, \neg E$
- *Clause* A disjunction of literals
  Ex: $A \vee C \vee \neg D$
- *Clausal Theory* A conjunction of clauses
  Ex: $C_1 \wedge C_4 \wedge C_5$ where $\forall i$, C_i are clauses

Resolution:

Given Clauses

$(A, B_1, ..., B_n)$
$(\neg A, C_1, ..., C_m)$

We can replace these clauses with:

$(B_1, ..., B_n, C_1, ..., C_m)$