CSC384
Introduction to Artificial Intelligence: Decision Making under Uncertainty

November 25, 2014

Final Topic: Decision making under uncertainty

- This material will not be tested
- This will be chapter 16 of the textbook. For more material building on Bayesian networks, see chapters 15 and 17.
- The material here corresponds to decisions with immediate consequences. Chapter 17 deals with sequences of decisions.

Decision Making

Introduction

Utility is a measure of an agent's preferences.

A **utility function** $U(s)$ assigns a number to express the desirability of a state. If $U(s) > U(s')$ then state s is preferable to state s'.

Expected utility $EU(a)$ is a sum of possible outcomes, weighted by probility of the outcome. We may have evidence providing additionalconditions on the probability.

$$EU(a) = \sum_{s \in states} U(s)p(s|a)$$

The **principle of maximum expected utility** states a rational agent should choose actions provided maximum expected utility.

Decision Making

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Decision Making

Preferences

$A \succ B$ the agent prefers outcome A to outcome B

$A \sim B$ the agent is indifferent between outcomes A and B

$A \succeq B$ the agent is prefers outcome A or is indifferent between outcomes A and B

A **lottery** is a set of outcomes $S_1, S_2, ..., S_n$ each with a respective probabilty of occuring $p_1, p_2, ..., p_n$. The standard notation for a lottery L is as follows:

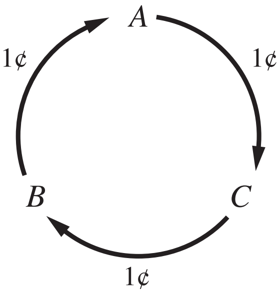
$$L = [p_1, S_1; p_2, S_2; ...; p_n, S_n]$$

Axioms of utility theory:
(Actually axioms about preferences, despite the name)

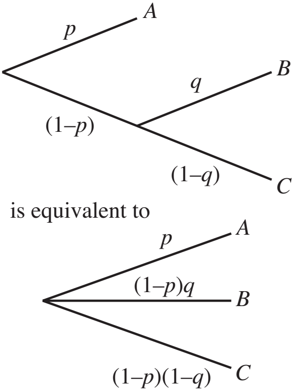
- **Orderability** one of $A \succ B, B \succ A$ or $A \sim B$ is true
- **Transitivity** $A \succ B$ and $B \succ C \Rightarrow A \succ C$
- **Continuity** $A \succ B \succ C \Rightarrow \exists p : [p, A; 1 - p, C] \sim B$
- **Substituitability** $A \sim B \Rightarrow [p, A; 1 - p, C] \sim [p, B, 1 - p, C]$
- **Monotonicity** $A \succ B \Rightarrow (p > q \Leftrightarrow [p, A; 1 - p, B] \succ [q, A; 1 - q, B])$
- **Decomposability**
  $[p, A; 1 - p, [q, B; 1 - q, C]] \sim [p, A; (1 - p)q, B; (1 - p)(1 - q), C]$

Decision Making

Preferences



(a)



(b)

Consequences of the axioms of utility theory:

- There exists a utility function U such that
  $U(A) > U(B) \Leftrightarrow A \succ B$
  $U(A) = U(B) \Leftrightarrow A \sim B$
- Lotteries have utility:
  $U([p_1, S_1; p_2, S_2; ...; p_n, S_n]) = \sum p_i U(S_i)$

The process of determining a utility function is called **preference elicitation**

Given a worst outcome $\bot$ and a best outcome $\top$ we often **normalize** a utility function so that $U(\top) = 1$ and $U(\bot) = 0$

We could assess the utility of an outcome S by asking an agent which they prefer between S and $[p, \top | 1-p, \bot]$ for various values of p.
$S \sim [p, \top | 1-p, \bot] \Leftrightarrow U(S) = p$

Decision Making

Utility Functions

Money: utility often refers to financial outcomes

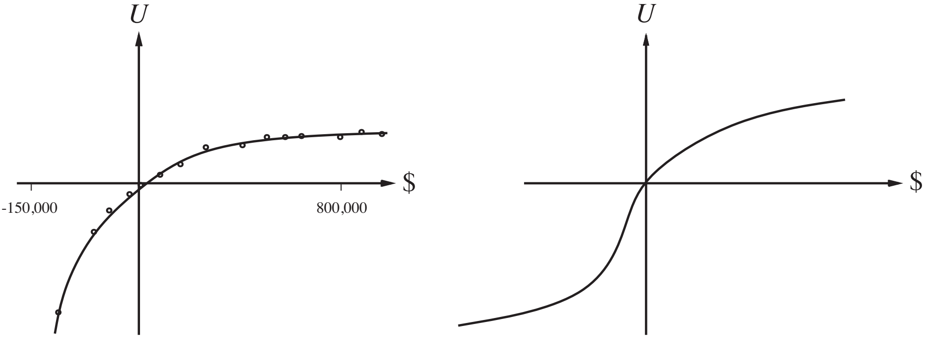
Agents are always assumed to prefer more money, but not necessarily in a linear fashion

risk-averse: preference given to more certain results, utilities are less than the payoff

risk-seeking: Utilies are greater than the payoff

Decision Making

Utility Functions



(a) (b)

The Allais paradox:

A: 80% chance of \$4000	A: 20% chance of \$4000
B: 100% chance of \$3000	A: 25% chance of \$3000

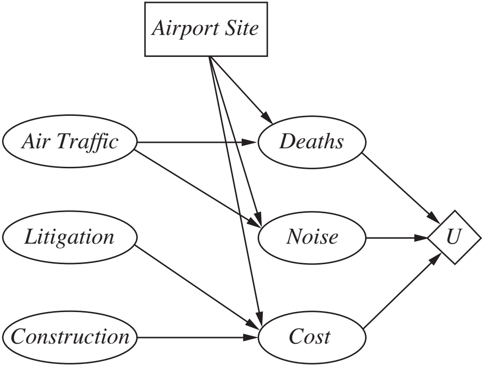
Which is preferable between A and B?
Which is preferable between C and D?

Expanding on Bayesian networks we build **decision networks**

- **Chance nodes**, represented by ovals, are random variables as in a Bayes net
- **Decision nodes**, represented by squares, represents a choice for the agent
- **Utility nodes**, represented by diamonds, represent the agents utility function

Decision Making

Decision Networks



Decision Making

Decision Networks