CSC384
Introduction to Artificial Intelligence: Uncertainty

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Bayesian Networks

Variable Elimination

Simply summing out probabilities left to right takes exponetial computation in the number of variables!

The fundamental strategy of *variable elimination* is to simplify the overall equation. The common way to do that is often described as:

- "Inside Out"
- "Right to Left"
- "Bottom Up"

Bayesian Networks

Variable Elimination

A **Factor** is a probability table. Before doing any variable elimination all of the conditional probablity tables in the Bayes net are factors, sometimes referred to as original factors.

- Let $f(X, Y)$ denotes a factor with variable sets X and Y
- The product of two tables can be denoted $h(X, Y, Z) = f(X, Y) \times g(Y, Z)$

$f(A, B)$		$g(B, C)$		$H(A, B, C)$			
ab	0.9	bc	0.7	abc	0.63	$ab\neg c$	0.27
$a\neg b$	0.1	$b\neg c$	0.3	$a\neg bc$	0.08	$a\neg b\neg c$	0.02
$\neg ab$	0.4	$\neg bc$	0.8	$\neg abc$	0.28	$\neg ab\neg c$	0.12
$\neg a\neg b$	0.6	$\neg b\neg c$	0.2	$\neg a\neg bc$	0.48	$\neg a\neg b\neg c$	0.12

Note that the product table must be normalized!

Bayesian Networks

Variable Elimination

We can also sum out a variable to create a new factor:

$$h(B) = \sum_A f(A, B)$$

$f(A,B)$		$h(B)$	
ab	0.9	b	1.3
$a\neg b$	0.1	$\neg b$	0.7
$\neg ab$	0.4		
$\neg a\neg b$	0.6		

Again this would require normalization before use.

Finally we can restrict a factor:

$$h = f_{A=a}$$

$f(A, B)$		$h(B) = f_{A=a}(B)$	
ab	0.9	b	0.9
$a\neg b$	0.1	$\neg b$	0.1
$\neg ab$	0.4		
$\neg a\neg b$	0.6		

Bayesian Networks

Variable Elimination

Assume we have evidence variables E with values e, query variable X, and orignal factors F. Let Z_j be the jth elimination variable in the elimination ordering.

The following is the variable elimination algorithm:

1. For all $f \in F$ that reference a variable in E, replace f with the restricted factor $f_{E=e}$
2. For each Z_j in order, eliminate Z_j:
   (A) compute $g_j = \sum_{Z_j} f_1 \times f_2 \times ... \times f_k$
   (B) replace $f_i \in F$ with g_j
3. Normalize the resulting probability table

Bayesian Networks

Variable Elimination

Complexity of variable elimination:

- Denote the size(in terms of variables) of the factor with the most variables to be the *elimination width*, k
- Worst case time and space complexity of variable elmination will be $2^{(}O(k)))$
- Note that Bayes nets have inherent elimination width, and we can not do better that this
- Finding good elimination orderings is difficult, heuristics are used in practice
- Easy to find orderings if the Bayes net is a directed tree

A Simple heuristic is *Min Fill*: eliminate the variable which will create the smallest size factor

Bayesian Networks

Variable Elimination

Not all variables in a Bayes net may be *relevant* to a particular query. Given evidence E, query variable X:

- X is relevant
- If a node is relevant, then its parents are relevant
- if $e \in E$ is the descendant of a relevant node, then e is relevant

We can restrict are attention to the subnetwork with only relevant variables