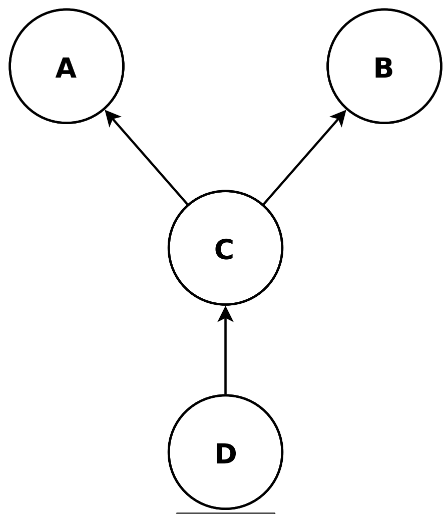
**CSC 384, Variable Elimination Example, Fall 2014**

## Description

Below is an example of an inference by variable elimination problem worked through. Bucket elimination and the chain rule equation are both referenced for descriptive purposes. For bucket elimination you can omit the equations entirely as they are implicit.

## Problem




| $f_1(A)$ | |
|---|---|
| $a$ | .8 |
| $\bar{a}$ | .2 |

| $f_2(B)$ | |
|---|---|
| $b$ | .4 |
| $\bar{b}$ | .6 |

| $f_3(A, C, B)$ | | | |
|---|---|---|---|
| $acb$ | .4 | $ac\bar{b}$ | .7 |
| $a\bar{c}b$ | .6 | $a\bar{c}\bar{b}$ | .3 |
| $\bar{a}cb$ | .1 | $\bar{a}c\bar{b}$ | .8 |
| $\bar{a}\bar{c}b$ | .9 | $\bar{a}\bar{c}\bar{b}$ | .2 |

| $f_4(C, D)$ | | | |
|---|---|---|---|
| $cd$ | .4 | $c\bar{d}$ | .2 |
| $\bar{c}d$ | .6 | $\bar{c}\bar{d}$ | .8 |

Find $P(A|B = t)$

## Solution

Note that by default the chain rule gives us the equation $P(A, B, C, D) = P(D|CBA)P(C|BA)P(B|A)P(A)$. Independence properties of the Bayes net reduces this to $P(A, B, C, D) = P(D|C)P(C|BA)P(B)P(A)$, which corresponds to our factors as $P(A, B, C, D) = f_1 \times f_2 \times f_3 \times f_4$.

To calculate $P(A|B = t)$ we must condition on $B$ and sum out all other variables in accordance with variable elimination.

$$P(A|B = t) = \sum_C \sum_D P(D|C)P(C|B = t, A)P(A)$$

.

Start by altering the original factors according to the evidence given, in this case $B = t$. Factor $f_2$ will disappear entirely, while factor $f_3$ must be reduced by $B = t$. We create the new factor $f_5(a, c)$ to replace $f_3$ by restricting ourselves to entries in which $B = t$. This is the process of replacing $P(C|BA)$ with $P(C|B = t, A)$.

| $f_5(A,C)$ | | | |
|---|---|---|---|
| $ac$ | .4 | $a\bar{c}$ | .6 |
| $\bar{a}c$ | .1 | $\bar{a}\bar{c}$ | .9 |

We can now order the variables for elimination and place corresponding factors into buckets. We will use the elimination ordering $D$, $C$:

$D$: $f_4(C, D)$
$C$: $f_5(A, C)$
$A$: $f_1(A)$

Note that this ordering corresponds to the equation:

$$P(A|B = t) = P(A)\sum_C P(C|B = t, A)\sum_D P(D|C) = \alpha f_1(A)\sum_C f_5(A, C)\sum_D f_4(C, D)$$

We now eliminate the variable $D$. We sum out $\sum_D f_4(C, D)$ to create a new factor $f_6(C)$. Note $f_6$ has not been properly normalized.

| $f_6(C)$ | |
|---|---|
| $c$ | $(0.4) + (0.2) = .6$ |
| $\bar{c}$ | $(0.6) + (0.8) = 1.4$ |

This gives us the following buckets:

$C$: $f_5(C, D)$, $f_6(C)$
$A$: $f_1(A)$

This corresponds to the equation

$$P(A|B = t) = \alpha f_1(A)\sum_C f_5(A, C)f_6(C)$$

Eliminating variable $C$ creates the new factor $f_7(a) = \sum_C f_5(A, C)f_6(C)$

| $f_7(A)$ | |
|---|---|
| $a$ | $(0.6)(0.4) + (1.4)(0.6) = 1.08$ |
| $\bar{a}$ | $(0.6)(0.1) + (1.4)(0.9) = 1.32$ |

The final bucket contains:

$A$: $f_1(A)$, $f_7(A)$

This corresponds to the equation:

$$P(A|B = t) = \alpha f_1(A)f_7(A)$$

We compute a final table, $f_8(A) = f_1(A)f_7(A)$

| $f_8(A)$ | |
|---|---|
| $a$ | $(.8)(1.08) = .864$ |
| $\bar{a}$ | $(.2)(1.32) = .264$ |

To complete the problem, we normalize the table $f_8$ so that the values sum to 1. This corresponds to the $\alpha$ term we had in previous equations.

| $f_8(A)$ | |
|---|---|
| $a$ | $(.864)/(1.128) = .766$ |
| $\bar{a}$ | $(.264)/(1.128) = .234$ |