Some Automata Theory Review Questions

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1. (a) Build an NFSA that recognizes the language

$$\mathcal{L}_1 = \{x \in \{0,1\}^\star : x = 0^m 1^n; m, n \in \mathbb{N}\}$$

   (b) Build the equivalent DFSA.

2. (a) Build an NSFA that recognizes the language

$$\mathcal{L}_2 = \{x \in \{0,1\}^\star : x \text{ starts and ends with a } 1\}$$

   (b) Use subset construction to build the equivalent DFSA.

3. Build a DFSA that recognizes

$$\mathcal{L}_3 = \{x \in \{0,1\}^\star : x \text{ contains an even number of 0's}\}$$

4. Build a DFSA that recognizes

$$\mathcal{L}_4 = \{a, ab, abe\}$$

   Your machine should have no more than 3 states.

5. Build a DFSA that recognizes

$$\mathcal{L}_5 = \{x \in \{a,b,c,d,e\}^\star : x \text{ contains the substring "}abe\text{"}\}$$

   Hint: Start from the answer you had in the previous question, and work from there. You should not need more than 3 states, but your accepting states will have to be different.

6. Draw the NFSA that recognizes the language described by the regular expression $0^\star 1^\star$.

7. Draw the NFSA that recognizes the language described by the regular expression $0^\star 011^\star$.

8. Prove or disprove each of the following statements.

   - $\mathcal{L}(0^\star 011^\star) \subseteq \mathcal{L}(0^\star 1^\star)$
   - $\mathcal{L}(0^\star 1^\star) \subseteq \mathcal{L}(0^\star 011^\star)$
   - $\mathcal{L}((a^\star b^\star c^\star)^\star) \subseteq \mathcal{L}((ab)^\star)$
   - $\mathcal{L}((ab)^\star) \subseteq \mathcal{L}((a^\star b^\star c^\star)^\star)$
   - $\mathcal{L}((0^\star 1^\star)^\star) \subseteq \mathcal{L}((0+1)^\star)$
   - $\mathcal{L}((0+1)^\star) \subseteq \mathcal{L}((0^\star 1^\star)^\star)$
   - $\mathcal{L}((xyz)^+) \subseteq \mathcal{L}((xyz)^\star)$
   - $\mathcal{L}((xyz)^+) \subseteq \mathcal{L}(((x+z)y^\star z)^\star)$
   - $\mathcal{L}((x^+ y^+ (z+\epsilon)^+)^\star) \equiv \mathcal{L}(((x^+ y^+ z^\star) + \epsilon))$
   - $\mathcal{L}((xyxy)^\star) \equiv \mathcal{L}(((xy) + \epsilon)(xy)^\star)$