1. Let $p(k)$ be a one-dimensional discrete distribution that we wish to approximate, with support on nonnegative integers. One way to fit an approximating distribution $q(k)$ is to minimize the Kullback-Leibler divergence:

$$KL(p||q) = \sum_{k=0}^{\infty} p(k) \log \frac{p(k)}{q(k)}$$

Show that when $q(k)$ is a Poisson distribution, this KL divergence is minimized by setting $\lambda$ to the mean of $p(k)$.

$$q(k) = \frac{\lambda^k \exp(-\lambda)}{k!}$$

Solution:

2. Recall that the definition of an exponential family model is:

$$f(x|\eta) = h(x)g(\eta)\exp(\eta^\top T(x))$$

where:

$\eta$ are the parameters
$T(x)$ are the sufficient statistics
$h(x)$ is the base measure
$g(\eta)$ is the normalizing constant

Consider the univariate Gaussian, with mean $\mu$ and precision $\lambda = 1/\sigma^2$ is:

$$p(D|\mu, \lambda) = \prod_{i=1}^{N} \left( \frac{\lambda}{2\pi} \right)^{1/2} \exp\left( -\frac{\lambda}{2} (x_i - \mu)^2 \right)$$

What are $\eta$ and $T(x)$ for this distribution when it is represented in exponential family form? Solution:
3. Consider the DAG in Figure 1. List all variables that are independent of $A$ given evidence on $B$. You can use the “Bayes ball” algorithm, the $d$-separation criterion, or the method of converting to an undirected graph (all should give the same result).

Solution:

![Diagram of a directed graphical model](image)

Figure 1: A directed graphical model.

4. Variable Elimination: Murphy 20.1

Solution:

5. Consider a tree-structured factor graph over discrete variables, and suppose we wish to evaluate the joint distribution $(x_a, x_b)$ associated with two variables $x_a$ and $x_b$ that do not belong to a common factor. Define a procedure for using the sum-product algorithm to evaluate this joint distribution in which one of the variables is successively clamped to each of its allowed values.

Solution: