Height of a BST with n keys

The worst possible height:

The best possible height

Tree height	Most keys it can hold
1	1
2	3
3	7
4	15
...	...
h	$2^h - 1$

So $n \leq 2^h - 1$

Let's flip this around to get h in terms of n:

$n \leq 2^h - 1$

$n + 1 \leq 2^h$

$\log_2(n + 1) \leq h$