Hashing

Reading:

- FZR chapter 11 on basic hashing

Recap: Direct files

In a direct file:

- to access any file position is $O(1)$.

So if we have a way to go from
record key $\Rightarrow$ location in file
then we can access any record in $O(1)$ time.

NB: File access may be $O(1)$, but it's *very* slow relative to memory access.
So we want to minimize jumping around.

There are many ways to organize a direct file! ...

We've already discussed in detail **Approach B:** explicitly *store* the location of the record associated with a key.

Use some data structure to index the records.

Each element of the index stores:

- a key that is in use, plus
- the location of the record with that key.

To find a record, search data structure by key.

Bonus: can have multiple indexes.

Approach A: For any possible key, *compute* the location of the record associated with it.

We use a function to map
record key $\Rightarrow$ location in file.

We'll look at three ways to do this.

(1) Direct Mapping

The key itself *is* the position of the record in the file.
I.e., $f(key) = key$.

Problems:

(2) Directory lookup

Keep a directory (probably in a separate file). It tells you where in the record file to find the record.

To find the appropriate directory entry, use the record's key directly.
So again, $f(key) = key$.

We save space:

And we lose space:

When does this method beat Direct Mapping?

Bottom line re space:

What about time?

(3) Hashing

Don't bother with a directory. Have just one file, and use the key to find the record's location in it.
(Just like Direct Mapping.)

But this time, use a mapping function that is not "direct". Use one that takes us from
large key space $\Rightarrow$ small address space.

Hashing Issues

Must devise an appropriate hash function. Because the hash function maps a large space to a small space, we will have "collisions".

We can make each location a "bucket" that can store lots of records.

- But buckets must have fixed size, thus they can still overflow.

We will need a scheme to handle this.

Must decide on the # and size of buckets.

When file gets very full, collisions can be too numerous. May be worthwhile re-organizing the file layout to have more buckets

Hashing Performance

If everything is well designed, retrieval can be very fast — just a few file accesses.

One operation is really slow:

Hash functions

A hash function is a mathematical function that maps from

keys $\Rightarrow$ locations.

There are some standard types of hash function, including

- **mid-square:** square the number and then take some digits from the middle.
- **folding:** Divide the number in half and combine the two halves, *e.g.*, add them together.
- **modular division:** Mod by some number, preferably a prime.

See the text for more about hash functions. Note that there is a lot of interesting theory about hash functions and their properties. (csc 378 covers this.)

Examples of hash functions

Say our key is a string. Before we hash it, we need to turn it into an integer.

One solution: Concatenate together the alphabetic position of the 1st and the second character. **E.g.** "T o yota" $\Rightarrow$ 20 15.

Now we need a hash function to hash up the integer. Examples of the three general types:

- Mid-square, taking the middle 2 digits. **E.g.** $2015^2 = 406\,02\,25$.
- Folding: adding the two halves. **E.g.** $2015 \Rightarrow 20 + 15 = 35$.
- Mod by 97. **E.g.** $2015 \bmod 97 = 75$.

key		h(key)		
as string	converted	mid-square	folding	mod 97
Toyota	2015	406 02 25	35	75
Chev	0308	94 86 4	11	17
Ford	0615	37 82 25	21	33
Chrysler	0308	94 86 4	11	17
Volkswagen	2215	490 62 25	37	81
Nissan	1409	198 52 81	23	51
Plymouth	1612	259 85 44	28	60
Dodge	0415	17 22 5	19	27
Renault	1805	325 80 25	23	59
Saab	1901	361 38 01	20	58
Isuzu	0919	84 45 61	28	46
Pontiac	1615	260 82 25	31	63
Fiat	0609	37 08 81	15	27

Two kinds of collisions:

- collisions that occur because we begin with the same 2 integers. Are collisions for every hash function we might choose.
- collisions that occur even though we begin with 2 different integers. Not necessarily collisions with a different hash function.

Couldn't we use the 2015 as the hashed value?

This would be a bad idea. With 4 digits, there are 10,000 possible values (0...9999), yet only a few will be used.

- Some are unlikely to crop up
  E.g. "aa" $\Rightarrow$ 0101.
- Some *cannot* crop up
  E.g. ?? $\Rightarrow$ 2701; 27 is out of range.

Yet we need our hash table to be continuous, and therefore to have all 10,000 slots.
So our hash table will be largely empty.

With each of the 3 hash functions we looked at, the range of h(key) is 0...99 (or less with mod 97).
So our hash table only needs 100 slots.

Of course we might overflow it, but we have to deal with this anyway.

Avoiding collisions?

Upon doing a new insertion, how likely is a collision?

It's certainly more likely when many items have already been inserted.

Loading factor: $= \frac{\#records\ currently\ in\ the\ file}{\#records\ that\ the\ file\ can\ hold}$

In our cars example, the loading factor is only $\frac{13}{100} = 0.13$, yet we already have collisions!

We could reduce collisions by making the capacity of the file bigger (and hence the loading factor smaller). But ...

Exactly how likely are collisions?

For a given file capacity, how likely are collisions as file gets more loaded?

Example: A file of 365 buckets. Let $Q(n)$ be the probability that NO collisions occur during n insertions.

$Q(1) =$

$Q(2) =$

$Q(3) =$

In general,
$Q(n) =$

Q(1) =

Solution to the recurrence relation:

$Q(n) = \frac{365!}{365^n(365-n)!}$

The probability that collisions DO occur is $1 - Q(n)$.

n	$1 - Q(n)$
10	0.1169
20	0.4114
23	0.5073
30	0.7063
40	0.8912
50	0.9704
60	0.9941

If the loading factor is only $\frac{23}{365}$, 50% chance.

If the loading factor is only $\frac{47}{365}$, $> 95\%$ chance!

So yes, collisions are a problem!

Buckets

The hash function $h(k)$ tells us where to store (or retrieve) a record with key k.

This could be a slot in an array in memory, or a slot in a file. (For this course, a file.) Either way, we often use the term "hash table".

The slots are called "buckets", because they have capacity for > 1 record. We choose the capacity based on the number of records that can be read or written in one file access.

Analogy: your address book.
key:

hash function:

bucket:

Handling collisions

What do we do when a collision occurs?

Easy case: the bucket has room for the record.

Hard case: the bucket doesn't have room for the record. We call this "overflow".
We need to figure out two things:

- A place to put the record that won't fit it its home bucket.
- A way to find that record later!

How do you handle overflow in your address book?

Handling overflow

Two kinds of approach: either compute or store where to try next. (Gee, where have we heard that before?)

Open addressing

Compute another bucket to try, based on some rule.

Closed addressing (or chaining)

Store the location of another bucket to try, using some sort of pointer.

The "overflow" records may be kept in a separate overflow area, perhaps in a separate file.

Open Addressing

Compute where to look, based on the key.

General method for insertion:
(search is analogous; why?)

Let A_i be where to look on the ith try.

- Use the hash function (h) to find the first bucket where the record might go, A_0.
  $A_0 = h(key)$
- If that bucket is full,
  use a new function (f) to find the next bucket to try. Repeat as necessary.
  $A_i = f(i, key)$
- Stop when we hit a bucket with room, or the sequence of A_i's starts to repeat (*i.e.*, there is no room anywhere).

Many ways to design the function f ...

I. Linear Probing

Step through a sequence of buckets always using the same step size.

Use mod to wrap around when we hit the end of the hash table.

$$\boxed{A_i = (A_{i-1} + stepSize) \bmod n}$$

(We can also express A_i in terms of A_0.)

Example:

$$\begin{aligned} A_i &= (A_{i-1} + 2) \bmod n \\ &= (A_0 + 2i) \bmod n \end{aligned}$$

The sequence of buckets considered is called the "probe sequence".

Linear probing example

n (# buckets) = 13 h(key) = (sum 1st 3) mod 13
bucket size = 1
step size = 2 So $A_i = (A_{i-1} + 2) \bmod 13$

key	h(key)	probe sequence
Chevrolet	16 mod 13 = 3	
Chrysler	29 mod 13 = 3	
Jaguar	18 mod 13 = 5	
Nissan	42 mod 13 = 3	
Karman Ghia	30 mod 13 = 4	

Bucket #	Bucket Contents
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	

II. Non-linear Probing

Problem: **Primary clustering**.

If several records hash to the same spot, or even *any* spot along the probe sequence, they will all follow that same probe sequence.

Example: n = 100; step size = 2.
hash to: 5 probe seq:
hash to: 9 probe seq:

Solution: Make the step size depend on the step number, i.
This is called **non-linear probing**.

E.g., $A_i = (A_{i-1} + 2i^2) \bmod n$

Example: n = 100; step size = $2i^2$.
hash to: 5 probe seq:
hash to: 9 probe seq:

III. Double Hashing

Problem: **Secondary clustering**.

All records that hash to the same spot still have the same probe sequence.

Example: n = 100; step size = $2i^2$.
key 1; hash to: 5 probe seq:
key 2; hash to: 5 probe seq:

Solution: Make step size depend on the *key* (but differently than in original hash function).
This is called **double hashing**.

E.g., $A_i = (A_{i-1} + h_2(key)) \bmod n$

Closed Addressing

Instead of computing where to look next, store it, using a "pointer".

Each full bucket has a pointer to an overflow area,

- in the same file (for example at the end)
- or in another file

There are many ways to organize this, since pointers are so flexible.

Cost vs open addressing:
Savings:

Do deletions introduce problems?

Table-assisted Hashing

With ordinary hashing,

- the hash function tells us where to start looking, and
- the collision resolution scheme tells us where to go from there if necessary.

Can we instead come up with a way to be *sure* which one bucket to look in, before we go into the file?

What would we need to know about a bucket that would tell us whether to look there?

Idea: Keep a table in memory that tells, for each bucket, what's the largest key in it.

This is called **table-assisted hashing**.

Example: Searching for key value 30, and we hash to bucket 52.

bucket	largest key in it
26	
⋮	
52	13
⋮	
85	27
⋮	
92	36
⋮	
116	

The table tells us our record is definitely in bucket 92 (if it's anywhere).

Benefits? Costs?

Can we likely keep the table in memory?
Would it be ok to keep it in a file?

"Incremental" Hashing

Reading:

- FZR chapter 12

A Problem

Performance degrades if the file becomes heavily loaded,
i.e., if $\frac{\text{actual-number-of-recs}}{\text{num-buckets} \times \text{bucket-size}}$ gets large.

To make things better, it may be worthwhile to increase the number of buckets (and reorganize the data).

This general idea is called **incremental hashing**.

Guess what? There are many ways to do it.

Incremental Hashing

General Approach

As records are inserted, if performance becomes too low, grow the file.

- *I.e.*, "split" one bucket and disperse its records; some stay put and others go to a new bucket.
- This reduces overflow (collisions to full buckets) and hence reduces the # of file accesses during search.

As records are deleted, if space usage becomes too poor, shrink the file.

- *I.e.*, merge two buckets into one.
- This reduces the total # of buckets, and hence reduces waste.

File growth and shrinkage is incremental, *i.e.*:

- It happens on the fly.
  We do it during insertions and deletions, if needed.
- It happens in small amounts.
  We split one bucket rather than rehashing the whole file.

Possible measures of performance include:

- load factor
- average # of disk accesses per search.

Method I: Linear Hashing

Method

- When performance becomes too poor, split bucket 0. (Yes, this is arbitrary.)
- Split it by doubling the mod factor and rehashing its contents. *E.g.*,
  $h(k) = k \bmod 3$ becomes
  $h(k) = k \bmod 6$.
- Next time, split bucket 1, then 2, etc.
- Keep a counter to remember which buckets have been split.
  Unsplit ones use the old hash function.
  Split ones use the new.

Merging is analogous but opposite.

	old	new
# buckets	$T : 0 \ldots (T-1)$	$T+1 : 0 \ldots T$
hash fcn	$h(k) = k \bmod 3$ $h(k) = k \bmod T$	$h(k) = k \bmod 6$ $h(k) = k \bmod 2T$

Guarantee: Every element of bucket 0 will either stay put, or land in the new bucket T.

More generally, if we split bucket b, every record will either stay put, or land in the new bucket $T + b$.

Let k be the record's key.
If it was in bucket b originally, we know
$k \bmod T = b$.
So k must have been one of these:

$$h \quad T+b \quad 2T+b \quad 3T+b \quad 4T+b \quad 5T+b \quad \ldots$$

So when we hash k with the new hash function $h(k) = k \bmod 2T$, we get either:

- b, in which case the record stays put, or
- $T + b$, in which case it goes to the new bucket, $T + b$.

Questions

Will linear hashing work if we use open addressing to solve collisions?

Why split the "next" bucket? Why not the culprit, *i.e.*, the one we inserted to when we passed the performance threshold?

Decision: What if the split fails, *i.e.*, everything happens to stay put? We could split again.

What happens when we've split all the original buckets?

Method II: Extendible Hashing

Build a dynamic directory (in memory for speed) that copes with the varying load factor.

- Hash function takes you to a *directory* entry, rather than directly to a bucket.
- Because buckets are pointed to, needn't be consecutive in the file. So can add and remove buckets as desired.
- Directory must grow and shrink with number of buckets.
- So # of places to hash to changes. Cope by using only the first so many bits of h(key); change this as necessary to change size of directory.
- If using d bits, directory size is 2^d.
- So have capacity for 2^d buckets, but can start with fewer; even just one.

How to "grow" the file

When a bucket overflows:

- Split the one bucket in two.
- Half of the directory entries that pointed to the old bucket will still do so, and half will point to the new bucket.

Eventually, we may reach a point where we can't split a bucket this way.

- This occurs when only one directory entry points to the bucket we want to split.
- Then we double the directory size, and reorganize.

Map of the World

A quick summary of topics that we've covered concerning direct files.

Direct files ⇒ store location

- Simple static index
  (index structure never changes)
- Dynamic index
  (index structure changes)
  - BST
  - (Not covered: AVL tree; M-way tree)
  - B-tree
    * plain B-tree
    * B^* tree
    * B^+ tree

Direct files ⇒ compute location

- Direct mapping
- Directory lookup
- Hashing
  1. Need a hash function
     - folding
     - mid-square
     - modular division
  2. Must handle bucket overflow
     - overflow ⇒ compute next bucket
       ("open addressing")
       * Linear probing
       * Non-linear probing
       * Double hashing
     - overflow ⇒ store location of next bucket
       ("closed addressing / chaining")
  3. May be table-assisted
  4. May be incremental
     - Linear hashing
     - Extendible hashing

PROVIDING ACCESS BY SECONDARY KEYS

Reading:

- FZR sections 7.6–7.8

DATABASES

Reading:

- None required