Faculty of Arts & Science / Physical Sciences Division
University of Toronto
**CSC148/A58—Introduction to Computer Science, Fall 2000**

# Assignment 6: “Big O” Analysis

**Due: Week 13**

**This assignment involves no programming; all questions require pencil-and-paper solutions only.**

1. Prove that $n^3$ is $O(6n^3 + 7n^2 + 80n + 19)$. That is, exhibit $c$ and $B$ s.t. $0 \leq n^3 \leq c(6n^3 + 7n^2 + 80n + 19)$ for all $n \geq B$.

2. (a) Use the definition of “Big O” to prove that $2^{n+1}$ is $O(2^n)$.

   (b) Use the definition of “Big O” to prove that $2^{2^n}$ is not $O(2^n)$.

3. Analyze the following code for its running-time complexity. Clearly outline the intermediate results.

```
k = 0;
while (k < p) {
  for (int i = 1; i < k; i++) {
    if (i == 3)
      DoSmth();
    else
      DoSmthElse(i);
  }
  k++;
}
```

```
void DoSmth() {
   int sum;
   for (int j = 1; j < n; j++)
      sum += j;
}
void DoSmthElse(int i) {
   int sum;
   if (i%2 == 0)
     sum = i
   else
     sum += j
}
```

4. Let $g(n)$ and $f(n)$ be two functions. Suppose you knew that $lim_{n \to \infty} \frac{f(n)}{g(n)} = a$.

   (a) Is it true that $f(n)$ is $O(g(n))$ if $a = 0$?

   (b) Is it true that $f(n)$ is $O(g(n))$ if $a = \infty$?

   (c) Is it true that $f(n)$ is $O(g(n))$ if $a$ is some constant other than 0?

   (d) Is it true that $g(n)$ is $O(f(n))$ if $a = 0$?

   (e) Is it true that $g(n)$ is $O(f(n))$ if $a = \infty$?

   (f) Is it true that $g(n)$ is $O(f(n))$ if $a$ is some constant other than 0?

   For the true statements, exhibit the appropriate $b$ and $c$. Otherwise, prove that the statement is false.

## What to Hand In

Submit legibly written (or better, typed) solutions, organized clearly and described precisely. 5% of your mark will be based on legibility and presentation of the solution.
Submit your work on paper only; there is no electronic submission for this assignment. Staple the cover sheet securely to your work. An envelope is not required, and in fact, we prefer that you not use one for this assignment.