Faculty of Arts & Science / Physical Sciences Division
University of Toronto
CSC148/A58—Introduction to Computer Science, Fall 2000

Assignment 6: "Big O" Analysis

Due: Week 13

This assignment involves no programming; all questions require pencil-and-paper solutions only.

1. Prove that n^3 is $O(6n^3 + 7n^2 + 80n + 19)$. That is, exhibit c and B s.t. $0 \leq n^3 \leq c(6n^3 + 7n^2 + 80n + 19)$ for all $n \geq B$.

2. (a) Use the definition of "Big O" to prove that 2^{n+1} is $O(2^n)$.

   (b) Use the definition of "Big O" to prove that 2^{2^n} is not $O(2^n)$.

3. Analyze the following code for its running-time complexity. Clearly outline the intermediate results.

```
k = 0;                              void DoSmth() {
while (k < p) {                         int sum;
  for (int i = 1; i < k; i++) {         for (int j = 1; j < n; j++)
    if (i == 3)                             sum += j;
      DoSmth();                     }
    else                            void DoSmthElse(int i) {
      DoSmthElse(i);                    int sum;
  }                                     if (i%2 == 0)
  k++;                                     sum = i
}                                       else
                                           sum += j
                                    }
```

4. Let $g(n)$ and $f(n)$ be two functions. Suppose you knew that $lim_{n \to \infty} \frac{f(n)}{g(n)} = a$.

   (a) Is it true that $f(n)$ is $O(g(n))$ if $a = 0$?
   (b) Is it true that $f(n)$ is $O(g(n))$ if $a = \infty$?
   (c) Is it true that $f(n)$ is $O(g(n))$ if a is some constant other than 0?
   (d) Is it true that $g(n)$ is $O(f(n))$ if $a = 0$?
   (e) Is it true that $g(n)$ is $O(f(n))$ if $a = \infty$?
   (f) Is it true that $g(n)$ is $O(f(n))$ if a is some constant other than 0?

   For the true statements, exhibit the appropriate b and c. Otherwise, prove that the statement is false.

What to Hand In

Submit legibly written (or better, typed) solutions, organized clearly and described precisely. 5% of your mark will be based on legibility and presentation of the solution.

Submit your work on paper only; there is no electronic submission for this assignment. Staple the cover sheet securely to your work. An envelope is not required, and in fact, we prefer that you not use one for this assignment.