Duration: **50 minutes**
Aids allowed: ***one single*-sided *hand*written** 8.5×11 **aid sheet**

Student number: ______________________

Last (Family) Name(s): ______________________

First (Given) Name(s): ______________________

CDF account: ______________________

Do **not** *turn this page until you have received the signal to start.*

In the meantime, please read instructions below carefully.

This term test consists of 3 questions. When you receive the signal to start, please make sure that your copy of the test is complete and fill in the identification section above. **Please write firmly and with a pen**, since pencil is not clearly visible on scanned copies.

Answer each question directly on the test paper, in the space provided, and use one of the "blank" pages for rough work. If you need more space for one of your solutions, use a "blank" page and *indicate clearly the part of your work that should be marked.*

Write up your solutions carefully! In particular, use notation and terminology correctly and explain what you are trying to do.

You will receive 20% of the points for any (sub)problem for which you write "I do not know how to answer this question." You will receive 10% if you leave a question blank. If instead you submit irrelevant or erroneous answers you will receive 0 points. You may receive partial credit for the work that is clearly "on the right track."

Marking Scheme

# 1:	____/20
# 2:	____/20
# 3:	____/20
TOTAL:	____/60

Good Luck!

Question 1 [20 MARKS]

Short answer questions. For each question circle TRUE or FALSE (exclusive). For each answer provide a brief justification or a small counter-example. Long and complicated justifications as well as unnecessarily large and complicated counter-examples will lose points. Guesses without justification or with incorrect justification receive 0 marks.

(a) If all edges in a directed graph have distinct weights, then the shortest path between two vertices is unique.

TRUE **FALSE**

Justification/counter-example:

(b) At the termination of Bellman-Ford algorithm, even if the graph has negative-weight cycles, correct shortest path length is found for each vertex for which shortest path is well-defined.

TRUE **FALSE**

Justification/counter-example:

(c) If all edge capacities in a flow network have integer values that are multiples of 5, then the value of a maximum flow will be a multiple of 5.

TRUE **FALSE**

Justification/counter-example:

(d) Let f be a maximum flow in a flow network. If we increase all edge capacities by 1, then the maximum flow value in the modified network will be at most $|f| + 1$.

TRUE **FALSE**

Justification/counter-example:

(e) Consider a linear program in standard form, i.e., $\max c^T x$ subject to $Ax \leq b$ and $x \geq 0$. If we increase some component of the b-vector then the optimal value cannot decrease.

TRUE **FALSE**

Justification/counter-example:

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Question 2 [20 MARKS]

Design as efficient algorithm as you can for the following problem.

Input: $G = ([n], E)$ — directed graph, $w : E \to \mathbb{R}_{>0}$ — positive edge weights.

Output: weight of a cycle of minimum total weight.

(a) Describe your algorithm in plain English, briefly justify why it correctly finds the minimum weight of a cycle. In your description you may use any of the algorithms covered in the lecture — if you do, make sure to state the name of the algorithm correctly.

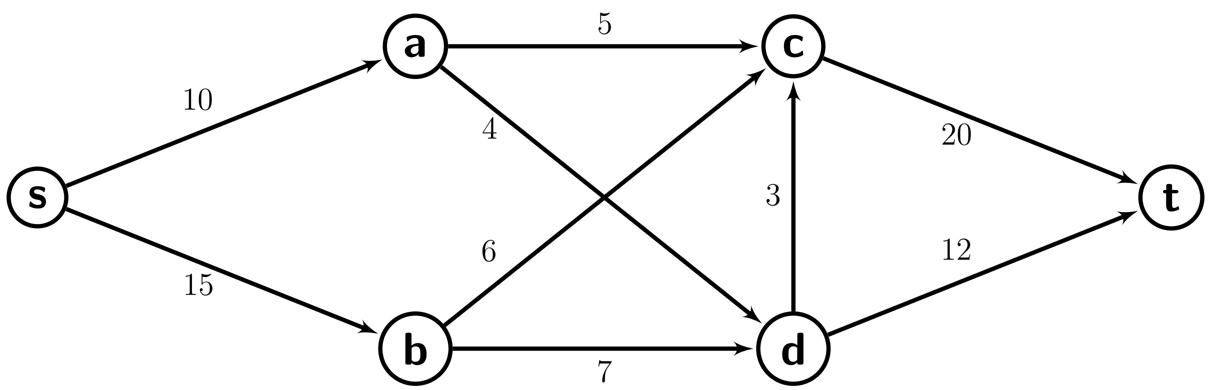
(b) Describe your algorithm *completely* in pseudocode. In other words, if you use an algorithm covered in lectures, you need to provide pseudocode for that algorithm as well. Extra space is provided on the next page.

(b) Continued. Use the space below to finish the pseudocode.

(c) State the running time of your algorithm. Justify it. State any assumptions on the implementation of abstract data types that are needed to achieve this running time.

Question 3 [20 MARKS]

(a) Consider the following flow network. Compute max $s - t$ flow and indicate it *right on the figure below* using the notation a/b near edge e to mean flow amount a is sent on edge e, which has capacity b. This is the standard CLRS notation that was also used in the lectures.



(b) Draw the residual graph corresponding to the flow computed in part (a). State the residual capacities. Find a minimum cut (S, T) and indicate it on your residual graph by circling the appropriate sets S and T.

(c) Consider the following LP in standard form. Rewrite it in slack form and solve it using Simplex. For each step of Simplex write down the entering variable, the leaving variable, and the basic solution.

$$\begin{array}{lrl}
\text{Maximize} & 5x_1 + x_2 & \\
\text{Subject to} & x_1 - x_2 & \leq 2 \\
& x_1 + x_2 & \leq 4 \\
& x_2 & \leq 3 \\
& x_1, x_2 & \geq 0
\end{array}$$

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