CSC 373: Algorithm Design and Analysis
Lecture 3

Allan Borodin

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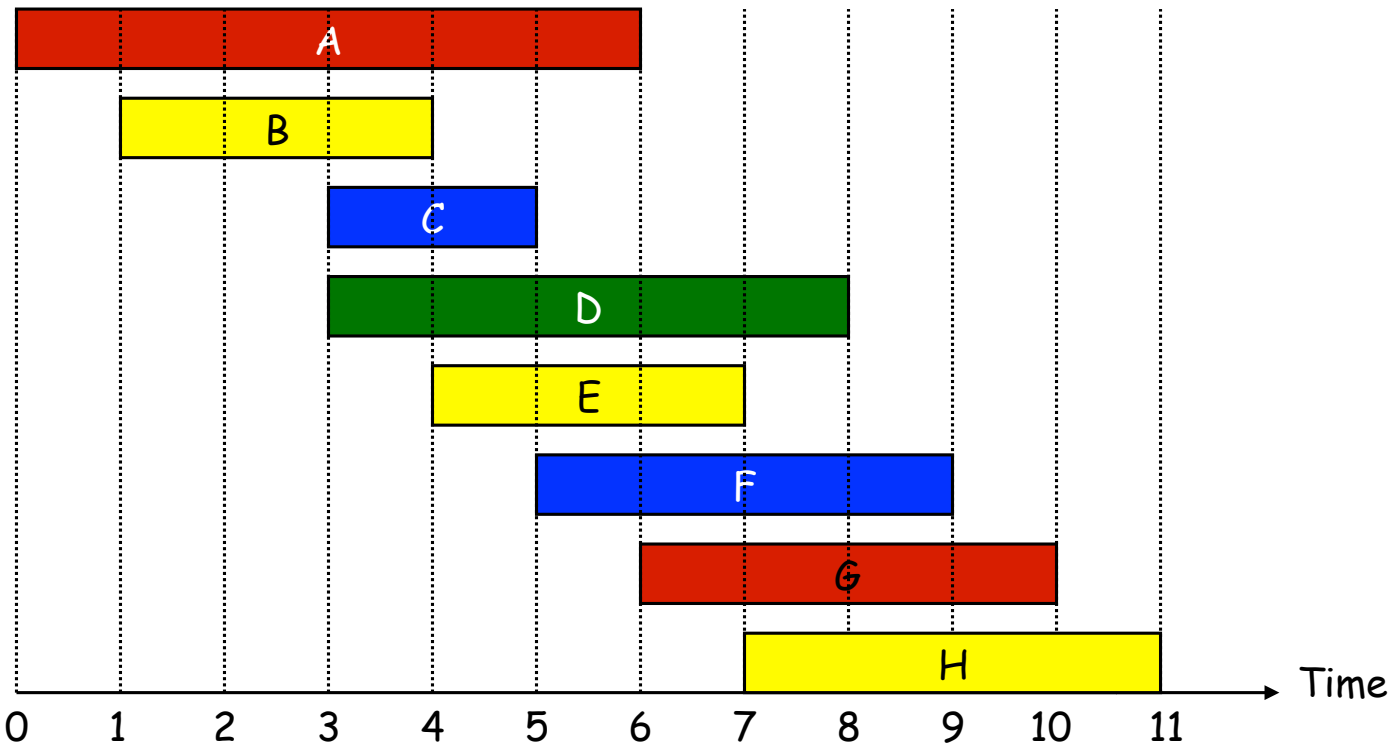
Interval colouring

Interval Colouring Problem

- Given a set of intervals, colour all intervals so that intervals having the same colour do not intersect
- **Goal:** minimize the number of colours used.



We use 4 colors in this example. **Question:** Is this optimal?

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Greedy interval colouring

- Consider the EST (earliest starting time) for interval colouring.
  - Sort the intervals by non decreasing starting times
  - Assign each interval the smallest numbered colour that is feasible given the intervals already coloured.
- Recall that EST is a terrible algorithm for ISP.
- **Note:** this algorithm is equivalent to LFT (latest finishing time first).

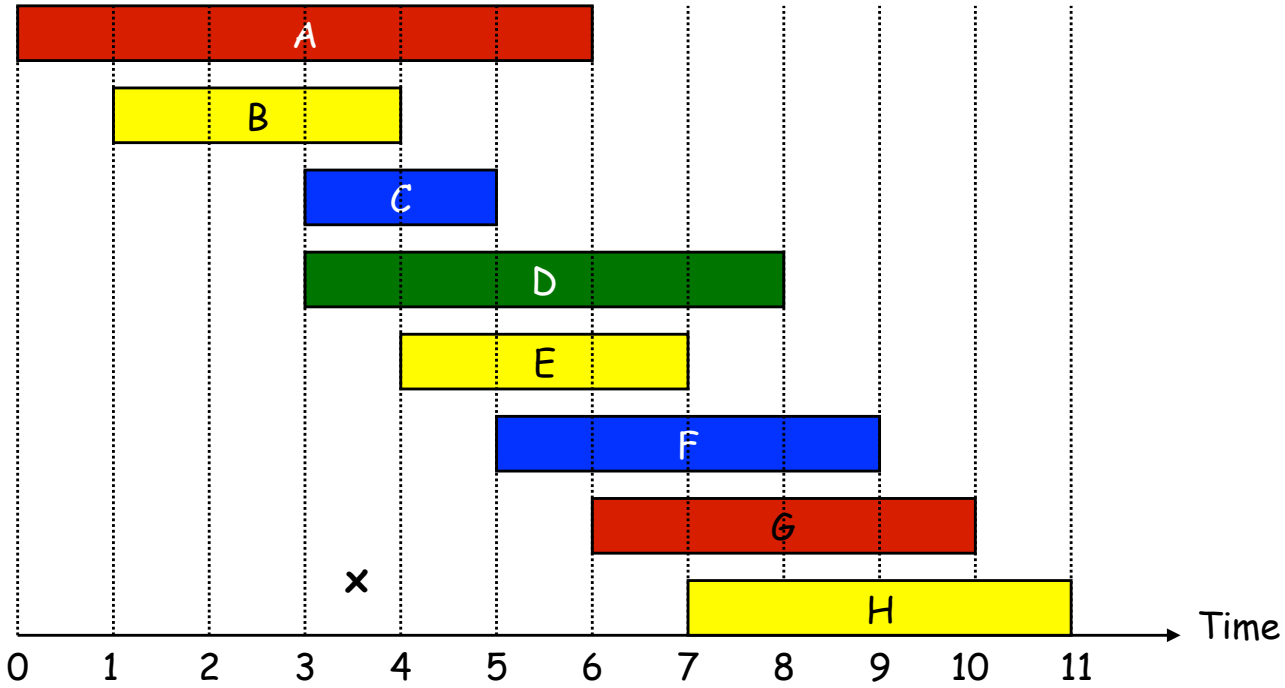
Theorem

EST is optimal for interval colouring

Greedy Interval Colouring

1: Sort intervals so that $s_1 \leq s_2 \leq \ldots \leq s_n$
2: **for** $i = 1$ to n **do**
3: $k := \min\{\ell : \ell \neq \chi(j)$ for all $j < i$ such that the j^{th} interval intersects the i^{th} interval$\}$
4: $\sigma(i) := k$
/ the i^{th} interval is greedily coloured by the smallest non conflicting colour */*
5: **end for**

An example of interval colouring



- We use the colors in the following order: red, yellow, blue, green

Idea of optimality proof

Look at the interval that (first) caused the largest colour, say k, to be used by EST. Then there must be k intervals containing a given "time" x and hence k colours are required.