The ambiguity of the comma

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Here is a common question from students who are first learning logic: “does the comma mean ‘and’ or ‘then’?” As we discussed at the start of the course, we study to predicate logic to provide us with an unambiguous way of representing ideas. The English language is filled with ambiguities that can make it hard to express even relatively simple ideas, much less the complex definitions and concepts used in many fields of computer science. We have seen one example of this ambiguity in the English word “or,” which can be inclusive or exclusive, and often requires additional words of clarification to make precise.

There is another, more insidious example of ambiguity with which you are probably more familiar: the comma, a tiny, easily glazed over symbol that people often infuse with different meanings. Consider the following statements:

If it rains tomorrow, I’ll be sad.
David is cool, Toni is cool.

Our intuitions tell us very different things about what the commas “mean” in each case. In the first, the comma means then, indicating the conclusion of an implication. But in the second, the comma is used to mean and, the implicit joining of two separate sentences. The fact that we are all fluent in English means that our prior intuition hides the ambiguity in this symbol, but it is quite obvious when we put this into the more unfamiliar context of predicate logic, as in the formula:

\[ P(x), Q(x) \]

And this, of course, is where the confusion lies, and is the origin of the question posed at the beginning of this note. Because of this ambiguity, never use the comma to connect propositions in this course. We already have a rich enough set of symbols – including \( \land \) and \( \implies \) – that we do not need another one that is ambiguous and adds no new meaning!

That said, keep in mind that commas do have two valid uses in predicate formulas:

- immediately after a variable quantification, or separating two variables with the same quantification
- separating arguments to a predicate

You can see both of these usages illustrated below, but please do remember that these are the only valid places for the comma!

\[ \forall x, y \in \mathbb{N}, \forall z \in \mathbb{R}, \ P(x, y) \implies Q(x, y, z) \]