CSC236 – Problem Set 9

There are two components of this problem set: a preliminary question designed to check your understanding of the basic topics covered this week, and a set of more challenging questions designed to make you think critically about the material and apply it in new contexts. *Get in the habit of starting work early* – the less time you give yourself, the most stressed you’ll find yourself each week!

**Caution:** you must submit *two* separate files in two separate locations on MarkUs, one for the Preliminary and one for the Challenge.

To avoid suspicions of plagiarism, **clearly state any resources (people, print, electronic) outside of your group, the course notes, and the course staff, you consulted at the beginning of your assignment submission.**

---

**Preliminary: due March 18, 2014 8:00 pm**

This question is an opportunity for you to check your understanding of the topics and practice writing formal solutions. This is a valuable *learning opportunity* – if you see that you’re at a loss, get help quickly!

Your goal should not be to get the right answer, but to convince the marker that you know what you’re doing. This question is marked on the following 3-point scale:

| 3: You’ve mastered this topic | 1: You don’t really know what you’re doing |
| 2: You’re almost there, but missing something | 0: You didn’t submit/had absolutely no clue |

**This question must be completed INDIVIDUALLY.**

Give regular expressions matching each of the following languages. You do not need to “prove” that your regexes are correct; however, you *must* provide one or two sentences of justification explaining your thinking.

1. \( L = \{w \in \{a, b\}^* \mid w \text{ starts and ends with the same letter}\} \)

2. \( L = \{w \in \{0, 1\}^* \mid |w| \text{ is a multiple of 3}\} \)
Challenges: due March 22, 2014 noon

Answer each question completely, always justifying your claims and reasoning. Your solution will not just be graded on correctness, but on its clarity as well. Technically correct answers that are hard to understand will not receive full marks. Mark values for each question are contained in the [square brackets].

You may work in groups of up to THREE to complete these questions.

1. [9] Recall that the reversal of a string $s$ is a string $s^R$ that has the characters of $s$ reversed. For example, $(abbc)^R = cbba$, and $\epsilon^R = \epsilon$. The reversal operator $Rev$ takes as input a regular language, and outputs a language containing the set of all reversals of words in the original language:

$$Rev(L) = \{w^R \mid w \in L\}.$$

Prove that if $L$ is a regular language, then so is $Rev(L)$. (Hint: recall the recursive definition of regular languages, and use structural induction!)

2. Give regular expressions that match each of the following regular languages. Once again, provide brief justification about why each one is correct.

   (a) [3] $L = \{w \in \{0, 1, 2\}^* \mid w$ contains both 01 and 12 as substrings$\}$

   (b) [3] $L = \{w \in \{0, 1\}^* \mid w$ is non-empty and does not contain two consecutive letters that are the same$\}$

   (c) [4] $L = \{w \in \{0, 1, 2\}^* \mid w$ does not contain two consecutive letters that are the same$\}$(Hint: divide up strings based on 2’s, and use your answer to part b)

3. [2] Consider the language $L = \{w \in \{a, b\}^* \mid w$ does not contain $ab\}$, and the regular expression $r = (aa + ba + bb)^*$. Does $L = L(r)$? Why or why not?

   Note: an implication of this question is that it’s easier to match what’s in a string rather than what’s not.