

CSC236 Winter 2020 term test 2 ‘cheat sheet’

This page contains reminders of some facts from weeks 4–8 which may (or may not) be useful for the second midterm. Please do not write on this sheet. It will not be collected with your test paper.

1 Master Theorem

Given $T(n)$ of the form $T(n) = aT\left(\frac{n}{b}\right) + f(n)$, where

- $f(n) \in \Theta(n^d)$ for some $d \in \mathbb{R}_{\geq 0}$
- $a \in \mathbb{N}, a > 0$
- $b \in \mathbb{N}, b > 1$

The Master Theorem says that,

$$T(n) \in \begin{cases} \Theta(n^d) & \text{if } a < b^d \\ \Theta(n^d \log_b n) & \text{if } a = b^d \\ \Theta(n^{\log_b a}) & \text{if } a > b^d \end{cases} \quad (1)$$

2 Big- Θ

For functions $f, g : \mathbb{N} \rightarrow \mathbb{R}^{\geq 0}$, $g \in \Theta(f)$ if and only if there exist constants $c_1, c_2, n_0 \in \mathbb{R}^+$ such that for all $n \in \mathbb{N}, n \geq n_0 \implies c_1 f(n) \leq g(n) \leq c_2 f(n)$.

Informally $g \in \Theta(f)$ means that g and f grow at about the same rate. The following are a few heuristics you can make free use of when reasoning about big- Θ :

- We can drop constant multiplicative factors. e.g. $100n \in \Theta(n)$
- For polynomials, we can ignore all terms except the one with highest degree. e.g. $n^3 + n + 1 \in \Theta(n^3)$

3 Geometric series

$$\sum_{i=0}^n 2^i = 2^{n+1} - 1 \quad (2)$$

$$\sum_{i=0}^n 2^{-i} = 2 - 2^{-n} \quad (3)$$

4 Loop invariant notation

If x is a variable in the program whose value changes during execution, the notation x_j denotes the value stored in x at the end of the j th iteration of the loop. By convention, $j = 0$ refers to the state of the program immediately before the first iteration.

5 Termination

A corollary of the principle of well-ordering is that if $\langle m_0, m_1, \dots \rangle$ is a strictly decreasing sequence of natural numbers, then it is finite.

6 Floor / ceil identities

For all $x \in \mathbb{R}, k \in \mathbb{N}$

$$\lfloor x \rfloor = k \iff k \leq x < k + 1 \quad (4)$$

$$\lceil x \rceil = k \iff k - 1 < x \leq k \quad (5)$$

For $n \in \mathbb{N}$

$$\lfloor n/2 \rfloor + \lceil n/2 \rceil = n \quad (6)$$

7 Python reminders

Indexing/slicing for lists and strings:

- $A[0]$ is the first element of A , $A[1]$ is the second element, etc.
- $A[-1]$ is the last element, $A[-2]$ is the second-last, etc.
- $A[i:]$ is a subsequence of A starting from index i and continuing to the end
- $A[:j]$ is a subsequence of A starting from index 0 and continuing up to but not including index j
- $A[i:j]$ is a subsequence of A starting from index i and continuing up to but not including index j
- $A[::-1]$ is a copy of A in reverse order