

Recipe: DFA correctness

A recipe for proving that $\mathcal{L}(M) = L$ for some DFA M and language L

1. Define **state invariants** for each of M 's states
 - ▶ i.e. any given string x reaches state q if and only if x has property _____
2. Define predicate $P(x)$ which asserts that string x agrees with your invariants
3. Use structural induction to prove $\forall x \in \Sigma^*, P(x)$
 - 3.1 Basis: $P(\varepsilon)$
 - 3.2 Inductive step. Assume $P(x)$ for some x , and show $P(xa)$ holds, for arbitrary $a \in \Sigma$.
 - ▶ Proof will generally be by cases. Need to consider a case for each symbol $a \in \Sigma$, and for each possible condition on x (i.e. each possible state reached by x)
 - ▶ Equivalent to showing that each transition in M respects the invariants
4. Observe that state invariants for accepting states ($F \subseteq Q$) match the definition of L