CSC236 winter 2020, quiz week 8_2

first/given name: **last/family name:**

utorid:

Prove that the following pseudocode algorithm terminates.

Input: purse containing only nickels and pennies
while *purse is not empty* **do**
  remove two coins;
  if *at least one removed coin was a nickel* **then**
    add five pennies;
  end
end

Define loop measure $m_j = 5 \times (\#\text{ nickels}) + (\#\text{ pennies})$. In other words, m_j is the total value of the coins in the bag, measured in cents, at the end of the jth iteration. Clearly $m_j \in \mathbb{N}$ (since the number of nickels or pennies at any point is a natural number).

It remains to show that m decreases with each iteration. Consider the effect of a jth iteration ($j > 0$). There are two cases to consider:

Case 1: we remove two pennies. Then $m_j = m_{j-1} - 2$.

Case 2: we remove at least one nickel. In this case, we remove either 6 or 10 cents from the purse, and add 5 cents. Thus $m_{j-1} - m_j$ is either 1 or 5.

In every case, $m_j < m_{j-1}$. ■

Note: The while condition for the above code was actually supposed to be "while the purse has *at least two* coins". As it is, it's not clear what would happen on an iteration where the bag starts with exactly 1 coin. If we imagine that the "remove two coins" instruction falls back to removing one coin in this scenario, then the loop measure above does not decrease when the bag contains exactly one nickel (though the algorithm does still terminate). We can account for this wrinkle by treating nickels as being worth 6 cents each. The cases above still work, and the case where we remove 1 nickel and add 5 pennies still results in a 1 cent net decrease (as does the case where we remove exactly 1 penny). Solutions that failed to account for this will not be penalized.