

CSC236 winter 2020, quiz week 8₁

first/given name:

last/family name:

utorid:

Prove that the following pseudocode algorithm terminates.

```
Input: bag of black and white stones
while bag is not empty do
  remove 1 stone;
  if removed stone was black then
    add 2 white stones;
  end
end
```

Let w_j and b_j denote the number of white and black stones, respectively, in the bag after the j th iteration. Define loop measure $m_j = 3b_j + w_j$. By definition $w_j, b_j \in \mathbb{N}$, so it follows that $m_j \in \mathbb{N}$.

It remains to show that m decreases. For an arbitrary j th iteration ($j > 0$), there are two cases:

Case 1: we remove a white stone. Then $m_j = m_{j-1} - 1$.

Case 2: we remove a black stone and add 2 white stones. Then $m_j = 3(b_{j-1} - 1) + (w_{j-1} + 2) = 3b_{j-1} + w_{j-1} - 1 = m_{j-1} - 1$.

In either case, $m_j < m_{j-1}$. ■